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Workshop Chair

Alastair Butler (National Institute for Japanese Language and Linguistics)

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Preface

This proceedings volume contains selected and invited papers on topics in formal semantics, formal pragmatics, and related fields, including the following:

- ✠ Formal syntax, semantics and pragmatics of natural language
- ✠ Model-theoretic and/or proof-theoretic semantics of natural language
- ✠ Computational Semantics
- ✠ Game-theoretic/Bayesian approaches to pragmatics
- ✠ Nonclassical Logic and its relation to natural language (especially Substructural/Fuzzy/Categorical/T logics)
- ✠ Formal Philosophy of language
- ✠ Scientific methodology and/or experimental design for linguistics

LENLS is being organized by an alliance of ”Establishment of Knowledge-Intensive Structural Natural Language Processing and Construction of Knowledge Infrastructure” project, funded by JST CREST Programs ”Advanced Core Technologies for Big Data Integration”.

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1st Day: November 13 (Sun), 2016. Unshared Task Workshop@NINJAL

2nd Day: November 14 (Mon), 2016. LENLS13@Keio University

3rd Day: November 15 (Tue), 2016. LENLS13@Keio University

Unshared Task at LENLS 13

Theory and System analysis with FraCaS, MultiFraCaS and JSeM Test Suites

Introduction

This gives details for the Unshared Task at LENLS 13, which is focused on undertaking theory and system analysis with FraCaS and FraCaS inspired test suites.

The FraCaS test suite was created by the FraCaS Consortium as a benchmark for measuring and comparing the competence of semantic theories and semantic processing systems. It contains inference problems that collectively demonstrate basic linguistic phenomena that a semantic theory has to account for; including quantification, plurality, anaphora, ellipsis, tense, comparatives, and propositional attitudes. Each problem has the form: there is some natural language input T, then there is a natural language claim H, giving the task to determine whether H follows from T. Problems are designed to include exactly one target phenomenon, to exclude other phenomena, and to be independent of background knowledge.

Following FraCaS, overlapping test suites are now available for a number of languages (notably in addition to the original English: Farsi, German, Greek, Japanese, and Mandarin), which together cover both universal semantic phenomena as well as language-specific phenomena. With the problem sets categorised according to the semantic phenomena they involve, it is possible to focus on obtaining results for specific phenomena (within a language or cross-linguistically), as well as strive for wide coverage.

The data

We have invited papers to apply either theoretical or computational analyses or other ideas to any of the following datasets, or subsets thereof, and describe findings:

FraCaS textual inference test suite (English)
Download machine readable version:
`http://www-nlp.stanford.edu/~wcmac/downloads`
For the original:
`ftp://ftp.cogsci.ed.ac.uk/pub/FRACAS/del16.ps.gz`

MultiFraCaS (Farsi, German, Greek, Mandarin)
Download: `http://www.ling.gu.se/~cooper/multifracas`

Japanese Semantics Test Suite (JSeM)
Download JSeM_beta.zip from:
`http://researchmap.jp/community-inf/JSeM`

Goals

Shared tasks typically provide "gold" analysed data with clear evaluation criteria for competing systems and have become popular within NLP fields. The concept of a so-called "unshared task" is an alternative to shared tasks. In an unshared task, there are neither quantitative performance measures nor set problems that have to be solved. Instead, participants are given a common ground (e.g., data) and an open-ended prompt.

With the availability of FraCaS, MultiFraCaS and JSeM Test Suites, the aim of this unshared task is for participants to put these resources to work as the basis for inspiring analysis, e.g., for showcasing a semantic theory or semantic processing system, or syntactic annotation model for the data.

We are also interested to hear about the creation of complementary data for other languages not yet represented by the existing test suites, or with work concerning properties of the existing test suites, or with cross-linguistic comparisons using the test suites, etc.

Being an unshared task, use made of the datasets is up to the authors. Any of the data sets might serve as a benchmark for testing the approach taken (or even a computational model, for participants who go that far) and reporting success levels on the problems (if applicable).

Testing the FraCaS test suite

Robin Cooper (joint work with Stergios Chatzikyriakidis and Simon Dobnik)

University of Gothenburg

In this talk I will present some of the background to the project FraCaS which led to the FraCaS test suite. I will also talk about the MultiFraCaS project and some of our ideas for extending this in the future. The examples in the original test suite were created by semanticists. I will discuss a number of ways in which one could go about verifying these examples. In particular, I will present some preliminary work we have been doing using web-based forms to collect judgements via crowd-sourcing. We will discuss the implications of some preliminary results that we have obtained, in particular the possibility of developing a probabilistic semantics.

Crowd-sourcing allows us to extend the notion of inference from strict logical inference to inference that is gradient and is prevalent in lexical meaning. The probabilities obtained through crowd-sourcing tell us the likelihood of a native speaker to make a particular conclusion. Eventually, we hope to extend the entire FraCas suite this way.

The original aim of the test suite was to provide a way of evaluating computational systems that perform natural language inference. I will talk about some work applying type theory to this task, with as yet partial coverage of the test suite.

The crowd-sourcing methods we have been using to evaluate the test suite can also be used to give an empirical basis to predictions made by semantic theories that are difficult to ascertain by only relying on intuitions of a single linguist. I will present some work we have been doing on the semantics of verbal restructuring, a phenomenon whose semantics has been debated and disagreed upon in the literature, and discuss the preliminary results we have.

Treebank annotation of FraCaS and JSeM

Alastair Butler, Ai Kubota, Shota Hiyama and Kei Yoshimoto

National Institute for Japanese Language and Linguistics and Tohoku University

This talk will describe treebank annotation of the FraCaS (English data) and JSeM (Japanese data) test suites, with a shared annotation scheme in the style of the Penn Historical family of corpora (`https://www.ling.upenn.edu/hist-corpora/other-corpora.html`). Syntactic analysis is often taken to be a necessary prerequisite for building semantic analysis, and we will argue that it is helpful to cash out what are likely to be shared syntactic assumptions with gold standard trees as transformable references of analysis. We will also detail work of transforming syntactic trees into meaning representations following Treebank Semantics (Butler 2015), and explore overlaps that arise when corresponding English and Japanese data are considered together.

Transformational Semantics on a tree bank

Oleg Kiselyov

Tohoku University

Recently introduced Transformational Semantics TS formalizes, restraints and makes rigorous the transformational approach epitomized by QR and Transformational Grammars: deriving a meaning (in the form of a logical formula or a logical form) by a series of transformations from a suitably abstract (tecto-) form of a sentence. Unlike QR, each transformation in TS is rigorously and precisely defined, typed, and deterministic. The restraints of TS and the sparsity of the choice points (in the order of applying the deterministic transformation steps) make it easier to derive negative predictions and control over-generation.

The rigorous nature of TS makes it easier to carry analyses mechanically, by a computer. The current implementation takes a form of a domain-specific language embedded in Haskell. It is intended as a 'semantic calculator', to interactively try various transformations, observe their results or failures. We report on the first experiments for using the calculator in the 'batch mode', to process tree bank data.

Improving AMR performance on FraCaS

Tim O'Gorman

University of Colorado Boulder

Abstract Meaning Representation (Banarescu et al. 2014) is a useful formalism, in part, because of its amenability to large-scale manual semantics annotation. AMR annotates the meaning of a sentence directly (rather than over a representation of the syntax), and represents the intended meaning of a sentence in context, rather than building underspecified representations of what it means out of context. Such an approach enables quick and useful annotation of even the most ungrammatical sentences, and provides a clear representation that can be easily understood and easily parsed. However, this flexibility comes at the price of relatively weak treatments of classic semantic phenomena such as quantification, tense, and monotonicity.

This talk will illustrate how the AMR-style annotation of meaning would handle the complex issues in the FraCaS test suite. The problematic portions of the test suite will then be used to discuss how AMR semantic coverage might be improved. I will first discuss the ongoing efforts to improve the AMR treatment of quantification. Secondly, I will discuss the use of event annotation methodologies to express tense, aspect and modality. These exemplify the trade-offs and hard decisions that are faced when designing a semantic representation designed for large-scale annotation.

Unshared Task of Natural Language Inference

Ran Tian and Kentaro Inui
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Tohoku University, Japan

Abstract. We propose a natural language inference engine which, at its lowest level, relies on algebraic calculations of word vectors to compose meanings of phrases and exploit semantic similarities. Upon that, it can handle negation and universal quantifiers logically, with an expressivity equivalent to first-order logic and a complete inference mechanism. We plan to use the FraCaS dataset to test the inference ability of our system.

1 Introduction

Being able to infer the logical relations of entailment and contradiction is considered a central capacity for understanding natural language. It is not much like inference in symbolic logic, due to ambiguity of word meanings and great diversity in saying a same thing. Natural language inference has to deal with these difficulties besides logical inference. To counter the problem, distributional semantics can project meanings into a continuous vector space, such that words and phrases with similar meanings have similar vectors. However, it remains open how to use the vectors in logical inference.

Recently, Tian et al. [8] show that some logical operators can be realized by algebraic calculations of word vectors, open up new possibilities in combining distributional semantics with logic. By adopting the Dependency-based Compositional Semantics (DCS) framework [4], they convert dependency parses into database queries to form sentence meaning. Then, they show that database operations such as intersection and projection can be realized as addition and linear mapping of word vectors respectively, and provide a way to train the vectors and linear mappings from unlabeled corpora. These already suffice to build compositions in the basic version of DCS.

However, the basic version of DCS as a logical representation only permits conjunctions and existential quantifiers, and most of

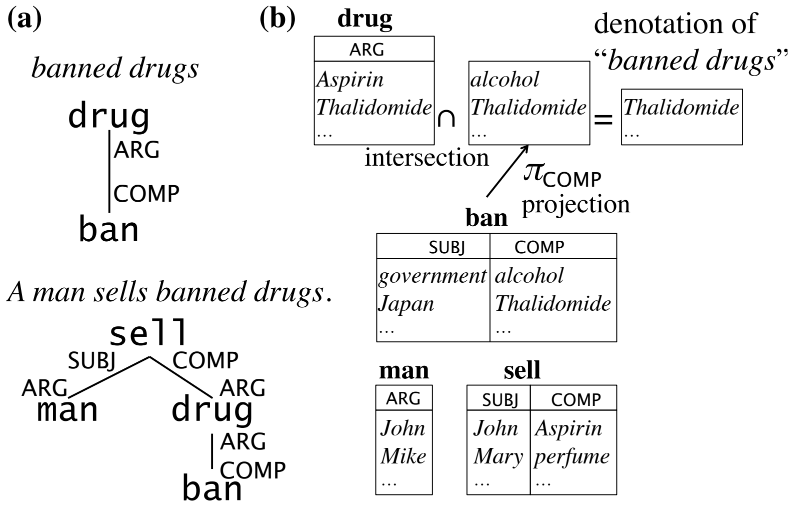

Fig. 1. (a) DCS trees and (b) database

its previous extensions [4, 7, 2] are rather ad hoc. In this paper, we extend the vector model of DCS [8] into a full-fledged natural language inference engine, using Peirce's Existential Graph (EG) [5] as an inference mechanism. EG is a logical representation implicitly assuming conjunctions and existential quantifiers, but by elegant use of negation, it acquires an expressivity equivalent to first-order logic and is equipped with a simple but complete inference algorithm. Our proposed inference engine combines the negation mechanism of EG with the vector model of DCS. At the lowest level, it relies on vector calculations to compose meanings of phrases and exploit semantic similarities; upon that, negations and universal quantifiers can be handled logically. We plan to use the FraCaS dataset [6] to test the inference ability of our system.

2 Vector Model of DCS

DCS is a framework relating dependency-like trees to database queries. A *DCS tree* (Figure 1a) is similar to a dependency tree, where each node is a content word (e.g. `ban`) and each edge (e.g. `ban`–`drug`) is labeled two syntactic-semantic roles (e.g. COMP–ARG) at its two ends respectively. Semantically, a DCS tree is a query about a (hypothetical) database (Figure 1b), where each node corresponds to a database table called its *denotation*, and labels surrounding the node correspond to columns of the table.

Formally, a denotation is a set of things, and a "thing" is represented by a tuple of features of the form $\mathsf{Label}{=}\mathit{Value}$, with a fixed inventory of labels. For example, the denotation

$$\mathbf{ban} = \{(\mathsf{SUBJ}{=}\mathit{Japan}, \mathsf{COMP}{=}\mathit{Thalidomide}), \ldots\}$$

is a set of tuples, where each tuple records participants of a banning event (e.g. *Japan banning Thalidomide*). Operations applied to sets of things generate new denotations, modeling semantic composition. One needs three operations in the basic version of DCS[1]: intersection "$\cap$", e.g. $\mathbf{pet} \cap \mathbf{fish}$ is denotation of *pet fish*; projection "π_{N}" which maps a tuple into its value of label N, e.g. $\pi_{\mathsf{COMP}}(\mathbf{ban}) = \{\mathit{Thalidomide}, \ldots\}$ consists of banned objects; and inverse image of projection $\pi_{\mathsf{N}}^{-1}(V) := \{x \mid \pi_{\mathsf{N}}(x) \in V\}$, e.g.,

$$\mathbf{D}_1 := \pi_{\mathsf{SUBJ}}^{-1}(\pi_{\mathsf{ARG}}(\mathbf{man}))$$

consists of tuples of the form $(\mathsf{SUBJ}{=}x, \ldots)$, where x is a man's name (we use the label ARG to represent names of things). Thus, $\mathbf{sell} \cap \mathbf{D}_1$ will denote men's selling events. Similarly, denotation of banned drugs as in Figure 1 is formally written as

$$\mathbf{D}_2 := \mathbf{drug} \cap \pi_{\mathsf{ARG}}^{-1}(\pi_{\mathsf{COMP}}(\mathbf{ban})),$$

and the following denotation

$$\mathbf{D}_3 := \mathbf{sell} \cap \mathbf{D}_1 \cap \pi_{\mathsf{COMP}}^{-1}(\pi_{\mathsf{ARG}}(\mathbf{D}_2))$$

consists of selling events whose SUBJ is a man and whose COMP is a banned drug.

The above calculation proceeds in a recursive manner controlled by the DCS tree of "*a man sells banned drugs*" (Figure 1a). Namely, if a DCS tree node $\mathtt{x}$ has children $\mathtt{y}_1, \ldots, \mathtt{y}_n$, and the edges $(\mathtt{x}, \mathtt{y}_1), \ldots, (\mathtt{x}, \mathtt{y}_n)$ are labeled by $(\mathsf{P}_1, \mathsf{L}_1), \ldots, (\mathsf{P}_n, \mathsf{L}_n)$ respectively, then the denotation $[\![\mathtt{x}]\!]$ of the subtree rooted at $\mathtt{x}$ is recursively calculated as

$$[\![\mathtt{x}]\!] := \mathtt{x} \cap \bigcap_{i=1}^{n} \pi_{\mathsf{P}_i}^{-1}(\pi_{\mathsf{L}_i}([\![\mathtt{y}_i]\!])). \tag{1}$$

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[1] It is noteworthy that the same operations are used in [1] to formulate database semantics in description logic.

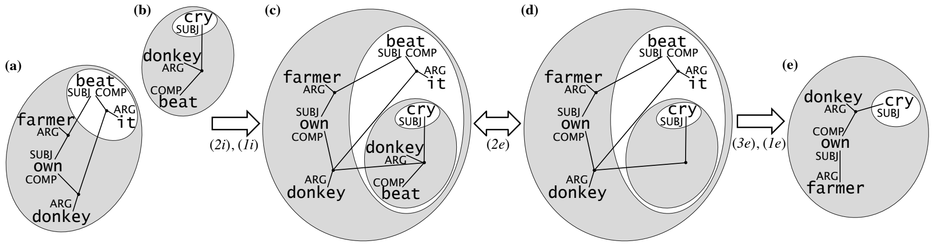


Fig. 2. Logical inference by Existential Graphs. Applied rules written under arrows.

This will result in $\mathbf{D}_3$ as the denotation of the DCS tree in Figure 1a.

To obtain a vector model of DCS, one replaces denotation of a word w with word vector $\mathbf{v}_w$, and approximate the three set operations by calculations on vectors. More precisely, one uses vector addition to approximate intersection (for its rationale we refer to [8]); then projection π_{N} should naturally be realized as a linear mapping M_{N} because logical interaction between projection and intersection resembles a linear relation:

$$\pi_{\mathsf{N}}(\mathbf{X}_1 \cap \mathbf{X}_2) \subseteq \pi_{\mathsf{N}}(\mathbf{X}_1) \cap \pi_{\mathsf{N}}(\mathbf{X}_2)$$

holds for any $\mathbf{X}_1, \mathbf{X}_2$ and is similar to

$$(\mathbf{v}_1 + \mathbf{v}_2)M_{\mathsf{N}} = \mathbf{v}_1 M_{\mathsf{N}} + \mathbf{v}_2 M_{\mathsf{N}}.$$

Finally, inverse image π_{N}^{-1} is realized by the inverse matrix M_{N}^{-1} because one has $\pi_{\mathsf{N}}(\pi_{\mathsf{N}}^{-1}(V)) = V$ for any set V. Thus, as parallel to (1), one can use the following vector to model denotation of a DCS tree:

$$\mathbf{v}_{[\![\mathsf{x}]\!]} := \mathbf{v}_x + \frac{1}{n}\sum_{i=1}^{n} \mathbf{v}_{[\![\mathsf{y}_i]\!]} M_{\mathsf{L}_i} M_{\mathsf{P}_i}^{-1}.$$

3 Existential Graphs

When written in a logical formula, DCS tree nodes are predicates and edges are variables. A denotation is a set of possible values of some variables when other variables are bound to existential quantifiers. For example, denotation of *banned drugs* is the set

$$\{y \mid \exists x : \mathrm{ban}(x, y) \wedge \mathrm{drug}(y)\}.$$

It is easy to see that the basic version of DCS only allows conjunctions and existential quantifiers as logical operators, which is a limitation the same as many graph-based semantic representations proposed in 1960s. However, the *Existential Graph* (EG) invented by Peirce in 1897 provides a brilliant solution: while graphs implicitly represent conjunctions and existential quantifiers, an oval enclosure marks negation and its scope; ovals can be nested but not overlapped. With negation, EG acquires the same expressivity as first-order logic. For example, Figure 2a shows an EG representation for the sentence "*every farmer who owns a donkey beats it*". The nested ovals allow quantifiers in the antecedent of an implication to include consequent within their scope. It is noteworthy that this is exactly the same assumption that Kamp [3] made about the scope of quantifiers.

In EG, a graph node is a predicate, and equal variables are joined by lines. Lines can be prolonged into nested ovals (i.e. the variables can be used there). Oval enclosures separate a blank sheet into disjoint areas, and an area nested inside an odd (resp. even) number of ovals is called negative (resp. positive). By convention, negative areas are shaded. Graphs inside a negative area are negated, so by De Morgan's Law, existential quantifiers become universal and conjunctions become disjunctions.

The inference rules for EG are as follows. One can (*1i*) add graphs in negative area and (*1e*) erase subgraphs in positive area; (*2i*) copy subgraph into nested area and (*2e*) erase duplicated graphs from nested area; (*3i*) add or (*3e*) erase two nested ovals with nothing in between (i.e. double negation). For the soundness and completeness of these rules we refer to [5].

In the Figure 2 example, EG **(b)** ("*every beaten donkey cries*") is copied into the inner positive area of **(a)**, and a line joining the two donkey variables is added, resulting **(c)**; then, the duplicate nodes `donkey-` and `beat-` are removed, revealing **(d)**; finally, one erases a double negation surrounding `cry` and part of a positive area to get **(e)**, "*every donkey owned by a farmer cries*".

4 Inference with Vectors

An essential step in the previous example is to find graphs inside a negative area that are duplicates of some subgraphs outside, then erase them using rule (*2e*). This is a basic pattern in most non-trivial inference. Logically, a “duplicate” can be characterized as the denotation of any variable in the negative area being a super set of some denotation outside. In Section 2, denotations are assigned vectors, so one can calculate their similarities; our proposed inference engine will then regard similar denotations as duplicates, besides exact super sets. Hopefully, this will integrate knowledge about phrase similarity into the inference engine.

References

1. D. Calvanese, G. D. Giacomo, and M. Lenzerini. On the decidability of query containment under constraints. In *Proceedings of PODS*, 1998.
2. Y. Dong, R. Tian, and Y. Miyao. Encoding generalized quantifiers in dependency-based compositional semantics. In *Proceedings of PACLIC*, 2014.
3. H. Kamp and U. Reyle. *From Discourse to Logic*. Springer Netherlands, 1993.
4. P. Liang, M. I. Jordan, and D. Klein. Learning dependency-based compositional semantics. *Computational Linguistics*, 39(2), 2013.
5. J. F. Sowa. Peirce’s tutorial on existential graphs. *Semiotica*, 186, 2011.
6. The Fracas Consortium. Using the framework. FraCaS: A Framework for Computational Semantics, 1996.
7. R. Tian, Y. Miyao, and T. Matsuzaki. Logical inference on dependency-based compositional semantics. In *Proceedings of ACL*, 2014.
8. R. Tian, N. Okazaki, and K. Inui. Learning semantically and additively compositional distributional representations. In *Proceedings of ACL*, 2016.

How ccg2lambda solves FraCaS/JSeM

Koji Mineshima (joint work with Pascual Martínez-Gómez, Ribeka Tanaka, Yusuke Miyao and Daisuke Bekki)
Ochanomizu University/JST CREST

I present an on-going work on developing formal compositional semantics and inference system for English and Japanese wide-coverage statistical CCG parsers. The focus of the talk is on how our system can solve linguistically challenging inference problems compiled in the FraCaS and JSeM datasets. I will introduce a pipeline ("ccg2lambda") presented in Mineshima et al. (EMNLP2015, 2016) and Martínez-Gómez et al. (ACL2016) and evaluate the current system on FraCaS and JSeM. I will also discuss how to extend the system with semantic underspecification using the idea of Dependent Type Semantics (DTS; see ESSLLI2016 lecture course: `http://esslli2016.unibz.it/?page_id=216`), and thereby illustrate one way in which a logic-based system solves some of the most challenging problems in FraCaS, in particular, those in the "nominal anaphora" section of the dataset.

FraCaS meets transformational grammar

Yusuke Kubota
University of Tsukuba

I aim to do two things in this talk: (i) attempt (at least some beginnings of) a meta-evaluation of the linguistic significance of the work reported in Mineshima et al. (EMNLP 2015, 2016) and (ii) try to motivate the use of a more powerful linguistic formalism than the one employed in Mineshima et al.'s work for more or less the the same task. For the latter component, I discuss possible advantages (and disadvantages) of using Hybrid Type-Logical Categorial Grammar (Hybrid TLCG) as a replacement for the CCG syntax of Mineshima et al.'s system. I will argue that the use of Hybrid TLCG is especially promising in dealing with complex linguistic phenomena (such as ellipsis) with which Mineshima et al.'s system struggles due to the inflexibility of the CCG syntax it adopts. In particular, Hybrid TLCG enables incorporating various analytic techniques introduced in the 'mainstream' transformational generative syntax and semantics over the last several decades much more straightforwardly than is practically possible with CCG. This has the further potential advantage of bringing computational linguistics and mainstream 'pencil-and-paper' theoretical linguistics much closer to each other than they currently are.

Can semantics contribute to neural machine translation?

Masaaki Nagata
NTT Communication Science Laboratories

Neural machine translation (NMT) is a recently developed translation technology which outperformed "conventional" statistical machine translation (SMT). Unlike SMT which requires syntax to overcome word order difference between distant languages such as Japanese and English, NMT is good at word reordering and it seems no linguistic theory is required. In this talk, I will illustrate by examples that it is semantics that can contribute to solve the remaining problems in NMT such as zero pronoun resolution and article generation.

Solving the Proportion Problem: A Plea for Selectivity

Hsiang-Yun Chen

Institute of European and American Studies, Academia Sinica

Abstract. [12] argues that quantificational adverbs are unselective binders over individuals. The Lewisian analysis, however, fails to recognize the ambiguity in some quantificationally modified conditionals. That the Lewisian approach cannot predict some attested reading is known as the "proportion problem." I propose a solution based on the following ideas: (a) quantificational adverbs bind selectively; (b) a singular indefinite and its anaphoric pronoun may introduce a plural discourse referent, and (c) plural predication is elusive.

Keywords: Proportion Problem, Donkey Anaphora, Dynamic Semantics

1 The Proportion Problem

[12] argues that quantificational adverbs (hereafter Q-adverbs) such as 'always' and ''usually' are unselective binders. The Lewisian analysis consists of the following theses:

(1) a. Conditionals are analyzed as having a tripartite structure (i.e. Q-adverb: restrictor: nuclear scope)[1]
b. Indefinites introduce free variables in the logical form.
c. Q-adverbs range primarily over individuals.
d. Q-adverbs are unselective binders that bind all variables in their scope.

The analysis, however, suffers from the so-called "proportion problem." ([5]) To illustrate, consider (2). Is it true given (3)?[2]

(2) If a farmer owns a donkey, he usually beats it.

[1] Lewis' analysis of 'if' is in fact three-way ambiguous: (a) 'if'-clauses in quantificationally modified conditionals are analyzed as restrictors ([12]); (b) indicative conditionals are analyzed as material implication ([13]); (c) counterfactuals are analyzed as variably strict conditionals ([11]).

[2] Underline indicates that a donkey is beaten by its owner.

(3)

Farmer	Donkey
A	d_1, d_2, d_3, d_4, d_5
B	d_6
C	d_7
D	d_8

According to the Lewisian analysis, (2) is true iff the majority of the admissible value assignments that satisfy the restrictor satisfy also the nuclear scope. That is, most of the farmer-donkey pairs in which the farmer owns the donkey, the former beats the latter. (2) is true given (3). Yet there is a very natural reading that it is false. The Lewisian analysis is problematic because, if we take (2) to be ambiguous, it under-generates; if (2) is not ambiguous but simply false, the analysis is plainly wrong.

Now consider a different scenario:

(4)

Farmer	Donkey
A	d_1,...,d_{10}
B	d_{11}, d_{12},d_{13}
C	d_{14}, d_{15},d_{16}

Here again, the traditional Lewisian analysis predicts that the Q-adverb modified donkey sentence (2) is false, since merely 4 out of the 16 farmer-donkey pairs satisfy the condition specified in the nuclear scope. But this answer is lacking and shows insufficient sensitivity to the complexity of the model. To be sure, for those who think that a donkey-owing farmer is a donkey-beater just in case he beats all his donkeys, (2) is false. On the other hand, (2) is true for those that think mistreating just one donkey is enough for the bad name. For example, those who fights for animal rights or work for animal welfare and protection would not hesitate to call out a farmer who beats at least one of his donkeys. The question is, ultimately, what qualifies a donkey-owning for a donkey-beating farmer.

2 Diagnosis

Quantifying over farmer-donkey pairs, as the Lewisian analysis predicts, yields just the symmetric reading. The proportion problem shows that we need also the asymmetric reading where the farmers somehow carry more weight. However, there is no straightforward quantification over the (donkey-beating) farmers, and we do not want to quantify over just the donkeys that are mis-treated. Drawing on Heims idea that the occurrence of pronouns inserts indirect pressure on how the restrictor should be interpreted, we argue that all variables in the nuclear scope must be quantificationally bound; however, the quantification cannot be the standard unselective binding.

Meanwhile, it is unfortunate that previous studies on the proportion problem focus on consistent scenarios[3] only. Due to the oversight of non-consistent scenarios, the elusiveness of plural predication has yet to received its due attention. The puzzle about how the Q-adverb modified donkey sentence is to be evaluated with respect to (4) is essentially how relational predication works for plurals. If one thinks, as we do, that donkey sentences such as S1 is not just about farmers that own one donkey only, we need to consider not only if a relational predicate, such as 'beat, holds between a farmer and each of his donkey, but also if the predicate holds between a farmer and the collection of his donkeys as a whole.

That native speakers do oscillate their judgements regarding (4) is suggestive.[4]

It indicates a sentence like 'a farmer beats his donkeys can be made true by various types of scenarios. That there is some grey area in the truth conditions is evidence that there is a fundamental indeterminacy in plural predication. It is unfortunate that pervious studies of the proportion problem focus on scenarios where the relation described by the matrix predicate holds consistently, that is, each farmer beats either all or none of his donkeys. Due to the oversight of non-consistent scenarios (where a farmer does not beat all of his donkeys), the elusiveness of plural predication does not receive its due attention.What is puzzling about (2) when evaluated with respect to (4) is essentially how relational predication works for plurals. Once we abandon the assumption of relative uniqueness, we need to seriously consider the possibility of non-consistent relationship between a 'boss' and its many 'dependents.' If we deem it is possible that a farmer owns more than one donkey, we need to consider not only if a relational predicate (e.g. 'beat') holds between a farmer and each of his donkey, but also if it holds between a farmer and the whole collection of his donkeys.

To summarize, the proportion problem exemplifies an ambiguity in what the Q-adverbs should bind, which is connected to how the restrictor should be understood. The standard Lewisian analysis predicts only the symmetric reading, where the Q-adverbs binds unselectively; the preferred asymmetric reading, on the other hand, rests on an alternative interpretation of the restrictor according to which it is not about the farmer-donkey pairs. Therefore, to obtain the asymmetric reading, the Q-adverbs needs to bind more selectively. We maintain that the proper construal must take into account two points. First, all the variables in the consequent need to be quantificationally bound. Second, once we admit the

[3] By consistent scenarios I mean where the relation described by the matrix predicate holds consistently; e.g. a farmer beats all of his donkeys.

[4] Here is a relevant quote from [16]: "[regarding "Most farmers who own a donkey beat it."] does it mean that most farmers who own a donkey beat all of the donkeys they own, that most farmers who own a donkey beat most of the donkeys they own, or that most farmers who own a donkey beat some of the donkeys they own? I am simply not sure, and informants I have consulted have not expressed strong or consistent opinions. This does not obviate the need for an analysis, since people do have intuitions about certain situations. My own rationalization of the data is that people have firm intuitions about situations where farmers are **consistent** [my emphasis] about their donkey-beating." p.256.

need to collapse some farmer-donkey pairs, we must address plural predication. As a synthesis of these two points, the binding of the ‘donkey variable will have to be special.

3 Solution

We propose a new solution based on the following ideas:

(5) a. (a) Q-adverbs bind selectively;
b. (b) singular indefinites and their anaphoric pronouns may introduce plural discourse referents;
c. (c) plural predication is elusive.

Our formal analysis is couched in DRT. In standard DRT, a singular indefinite such as ‘a farmer’ introduces a discourse referent/variable, which is matched to a single individual via the embedding function. We keep the indefinite-as-variable thesis intact but argue that a singular indefinite may introduce into DRS a *plural* or *sum* discourse referent. An indefinite description can receive a ‘collective’ reading where it introduces a set-indicating variable. Such a set is maximal in the sense that its members are all the individuals satisfying the relevant conditions.[5]

Here is the construction rule: a singular indefinite such as ‘a donkey’ introduces invariably into K a sub-DRS K_1, and then there are three options:

(i) the sub-DRS remains unchanged, resulting in relative uniqueness *à* la Kadmon

(ii) the sub-DRS converges to the main DRS K

(iii) the sub-DRS triggers a “plural introduction” such that a sum discourse referent *Z* is added to the universe of the main DRS

(i) $\begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline \mathrm{y} \\ \hline \mathrm{donkey(y)} \\ \hline \end{array}_{K_1} \\ \hline \end{array}_{K}$ (ii) $\begin{array}{|c|}\hline \mathrm{y} \\ \hline \mathrm{donkey(y)} \\ \hline \end{array}_{K}$ (iii) $\begin{array}{|c|}\hline \mathrm{Z} \\ \hline \mathrm{Z}{=}\sum \mathrm{y} \begin{array}{|c|}\hline \mathrm{y} \\ \hline \mathrm{donkey(y)} \\ \hline \end{array}_{K_1} \\ \hline \end{array}_{K}$

Solving the proportion problem calls for the asymmetric reading, which the third option facilitates. The verification conditions for various DRS conditions follows the interpretation of plurals in ([7]).

For our purpose, the only verification condition that needs to be noted is:[6]

(6) f verifies Y$= \sum$y K iff f(Y)$= \sigma a \exists g[f \subseteq g \& Dom(g) = Dom(f) \cup U(K) \&$ $g(d) = a \& g$ verifies K in M]

[5] This way, we can attribute the ambiguity between the symmetric reading and the asymmetric reading to the ambiguity in interpreting indefinites.

[6] We adopt the following from chapter 4 in [7] and [18].

Following our proposal, (2) is analyzed as:

(7)
$$\begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline \text{x, Z} \\ \hline \text{farmer(x)} \\ \text{Z}=\sum\text{y}\begin{array}{|c|}\hline \text{y} \\ \hline \text{donkey(y)} \\ \text{own(x,y)} \\ \hline \end{array} \\ \\ \hline \end{array} \quad \left\langle \begin{array}{c} \textit{most} \\ x \end{array} \right\rangle \quad \begin{array}{|c|}\hline \text{u, V} \\ \hline \text{u=x} \\ \text{V=Z} \\ \text{beat(u,V)} \\ \hline \end{array} \\ \hline \end{array}$$

An embedding function *f* verifies (7) iff:

most extensions g of *f*, where Dom(g)=Dom(f)$\cup\{x, Z\}$ that verify the conditions in K_1, farmer (x) and $\text{Z}=\sum\text{y}\begin{array}{|c|}\hline \text{y} \\ \hline \text{donkey(y)} \\ \text{own(x,y)} \\ \hline \end{array}$

can be extended to h, where in this case h=g, such that h verifies K_2, i.e.,

$$\begin{array}{|c|}\hline \text{u, V} \\ \hline \text{u=x} \\ \text{V=Z} \\ \text{beat(u,V)} \\ \hline \end{array}$$

A function *f* verifies $\text{Z}=\sum\text{y}\begin{array}{|c|}\hline \text{y} \\ \hline \text{donkey(y)} \\ \text{own(x,y)} \\ \hline \end{array}$ in a model M=$< D, I, \sqcup >$ iff:

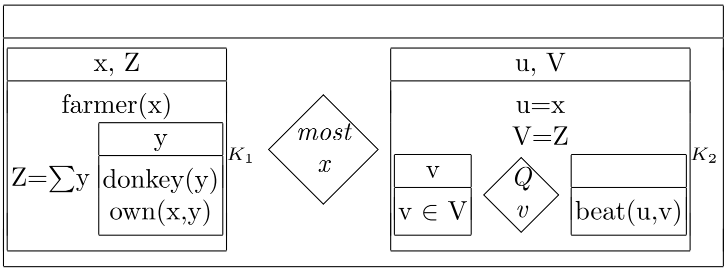
f(Z)=$\sigma a \exists g[f \subseteq g \& Dom(g) = Dom(f) \cup \{y\} \& g(y) = a \& g(y) \in I(donkey) \&$
$< g(x), g(y) > \in I(own)$

Given the verification conditions so sketched, it should be clear that the *primary quantification* involved in (7) is the quantification over x, or the farmers. However, the verification condition of DRS K_2 is tricky. This is due to the problem of plural predication. We may address the issue using two approaches. The first is to be minimalistic, the second explicit. Suppose a function maps u to an individual A, and his three donkeys d_1, d_2 and d_3. We then state:

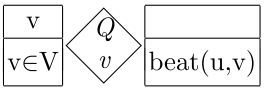
(8) beat(u,V) is true iff A beats d_1, d_2 and d_3.

(8) acknowledge the underlying looseness in plural predication and declare that it is not the job of the semanticists to decide when a beating relation holds between a man and his donkeys. That is the job for people who study the metaphysics of beating. This is the minimalistic approach.

Alternatively, we may expand (7) like this:

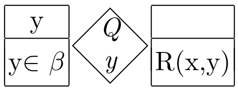
(9) 

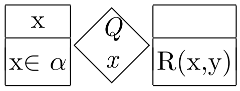
The crucial difference between (7) and (9) is the transition from 'beat(u,V)' to the duplex condition

(10) 

More generally, we postulate a construction rule for "plural elimination":

(11) Let x and y stand for singular discoure referents and α and β stand for plural discourse referents:

a. A condition R(x,β) is transformed to 

b. A condition R(α,y) is transformed to 

The representation delineated in (9) has the advantage of making the *second* quantification explicit, confirming the idea that all pronouns occurring in the nuclear scope must be quantified over. The pronoun 'he', represented by u, is quantified by 'most'; the pronoun 'it', represented by V, is quantified indirectly via the secondary quantification Q over v. The second quantifier Q is left unspecified in (9). This underspecification is intended to reflect the flexibility with respect to the secondary quantification over the donkeys. While we prefer this second, more explicit solution, we remain neutral whether the first approach is feasible.

Below we illustrate how our proposal would deal with a number of different scenarios. Take (12):

(12)

Farmer	Donkey
A	d_1,...,d_{10}
B	$\underline{d_{11}}$, d_{12},d_{13}
C	$\underline{d_{14}}$, d_{15},d_{16}

If someone judges the donkey sentence 'If a farmer owns a donkey, he usually beats it' to be true with respect to (12), we can infer that for her, the secondary quantifier Q is equivalent to the existential quantification '∃.' In contrast, if someone judges the sentence to be false, then we know that for her, Q has a quantificational force stronger than '∃.'

Consider again (2), which is repeated here as (13):

(13)

Farmer	Donkey
A	$d_1,...,d_{10}$
B	$\underline{d_{11}, d_{12}}, d_{13}$
C	$\underline{d_{14}, d_{15}}, d_{16}$

With respect to (13), if one judges the adverbially modified donkey conditional to be false, it means that for her, Q is not existential. On the other hand, if she judges the sentence to be true, then Q might be '∃' or something stronger, such as 'most.' Note that reelative to the same scenario, different speakers may have divergent criterion regarding Q, and one speaker can have varying specifications of Q relative to different scenarios. Furthermore, depending on the relational predicate in question ('beat' in the current example), one may have a particular preference for some Q.

4 Final Remarks

The analysis we advance here have four central theses: (i) every pronoun that appears in the nuclear scope of a conditional must be quantified over; (ii) a singular indefinite and its anaphoric pronoun may introduce a plural discourse referent; (iii) when a conditional contains two pronouns, the corresponding discourse referents may receive distinct quantifications, and finally (iv) plural predication is elusive.

Recall Heim's observation that the presence of a pronoun in the nuclear scope exerts pressure on how the restrictor should be constructed. Our first point that all the pronouns in the nuclear scope must receive some quantification is much like an extended argument stemming from that idea.

Our second point addresses the challenge presented by conditionals with two pronouns. We argue that singular indefinites and their anaphoric pronouns may receive a collective reading and introduce into DRS a sum discourse referent. This effectively echoes Neale's (1990) claim that singular donkey pronouns are semantically numberless.[7] What we argue is that for the sake of obtaining the proper asymmetric reading, we adopt the "plural introduction" strategy so that a singular indefinite and its anaphoric pronoun can be understood as possibly plural.

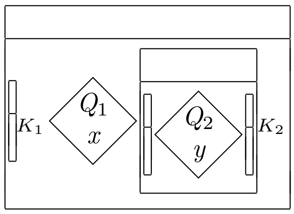
In the foregoing discussion, we only interpret 'a donkey' collectively, but it is possible that 'a farmer' receives a pluralization treatment. If we consider

[7] See [14], Chapter 6, especially section 6.3. However, due to the difficulties [9] raises to salience, I remain neutral about applying the number neutrality account across the board.

scenarios where the co-ownership of donkeys are relevant, this is what we will need. In principle, we can have either ‘a farmer,’ ‘a donkey,’ or both to be read as introducing a plural individual. However, if we confine ourselves to scenarios where each farmer owns at most one donkey and any donkey is owned by at most one farmer, it does not matter which indefinite description receives the collective reading or if they both do.

Regarding the third point, we believe the split of quantification should be welcomed. Given a uniform treatment of conditionals as triggering duplex conditions on a par with sentences with quantified noun phrases, we now have a general scheme for their DRS representation:

(14) For any conditional ‘Q-adverb, if ϕ, ψ,’ let K_1 represent the restrictor and K_2 represent the nuclear scope. The DRS for the conditional is:

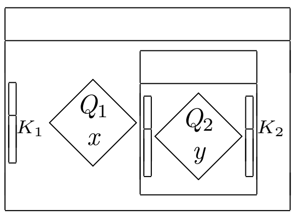


where Q_1 corresponds to the quantificational force of the Q-adverb in use.

The secondary quantification Q_2 is optional in two senses. First, when there is only one donkey pronoun in the nuclear scope, nothing will trigger the secondary quantification. Secondly, even if there are two donkey pronouns, we may choose to be parsimonious with respect to the representation and leave the secondary quantification to interpretation (and metaphysics).

Complex DRSs triggered by quantified phrases also undergo minor changes. $K_1(Q_1x)K_2$ now becomes:

(15)



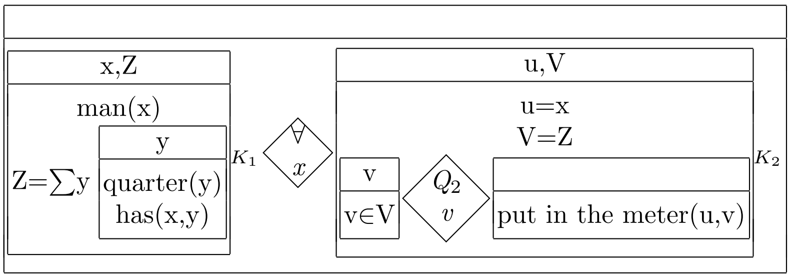
where the secondary quantification Q_2 is needed only when there is a donkey pronoun. When there is no donkey pronoun in the quantified sentence, e.g., ‘Most farmers who own a donkey are rich,” nothing will trigger the secondary quantification.

Besides contributing to the desired asymmetric reading to handle the proportion problem, the separation of quantification naturally lends itself as an explanation of the difference between the weak and strong readings.[8]

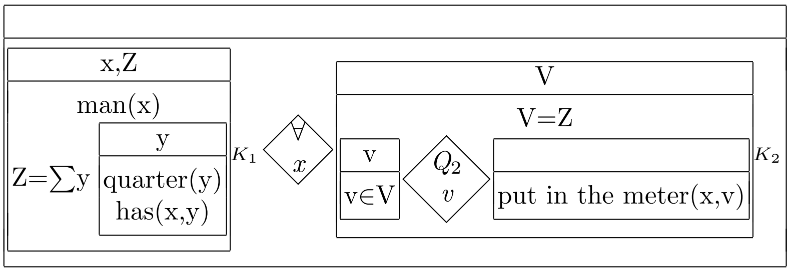
(16) a. If a man has a quarter, he puts it in the meter.
b. Every man who has a quarter puts it in the meter.

The respective representation for (16a) and (16b) are:

(17) a.



b.



The strong reading results from taking the secondary quantification Q_2 to be universal, and the weak reading results from taking Q_2 to be existential.

In short, while the proportion problem demonstrates the need to separate the quantification so that the (non-universal) Q-adverb binds more selectively, the demand for splitting the quantification is already present when we are charged with accounting for the weak reading. It is desirable to have a unified machinery that provides the required division.

Finally, regarding the fourth point, that plural predication is loose is clearly exemplified in the following:

(18) a. At the end of the press conference, the reporters asked the president questions.[9]
b. At the end of the press conference, the reporters asked the presidents questions.
c. At the end of the press conference, the reporters asked the president a question about gun control regulations.

[8] For more discussions on the weak and strong readings, see [15], [1], [8], [10] and [3], among others.
[9] This is from [17], which he attributes to [2].

The truth of (18a) does not depend on every single reporters at the press conference asked the president a question. One or more reporters might have asked one or more questions, but there might be one or more reporters who did not asked any. Similarly, the truth of (18b) does not require that every reporters asked a question and/or that every president was asked a question. Perhaps bearing more directly on my proposal is (18c). It is not transparent the singular indefinite ‘a question’ entails that there was only one question asked about the new stimulus package. Our intuition is that several reporters could have each asked such a question, but other speakers might have a different judgement.

Returning to the initial Lewisian account of adverbs of quantification and conditionals, we have come to realize that in order to handle the proportion problem, besides the assumption of unselective binding, other modifications are necessary:

(19) a. Conditionals are analyzed as having a tripartite structure.

b. Indefinites introduce free variables in the logical form. ⇒ **singular** indefinites may introduce **plural** variables.

c. Q-adverbs range primarily over individuals.

d. Q-adverbs are unselective binders that bind all variables in their scope. ⇒ Q-adverbs bind **selectively** one of the variables in their scope.[10]

The separation of quantification together with the elusiveness of plural predication suggests that there may be an inherent indeterminacy of donkey sentences. The indeterminacy is subject to various constraints such as world knowledge, the predicate in question, the Q-adverb in use, and the logical properties (monotonic features) of the determiner, to name but a few. After all, “it may be sometimes be futile if not wrong to suppose that donkey sentences must have a definite reading.”[11] Nevertheless, the representation helps to elucidate, given a particular reading (i.e., a determinate truth or falsity with respect to a scenario), what the discourse content, or structure of information, must and might be.

[10] A different formulation is this: Q-adverbs **may** bind multiple variables, but need not do so; when the primary and secondary quantification coincide in their quantificational force, we have what appears as unselective binding from one single quantifier.
[11] [4], p.151.

References

1. G. Chierchia. Anaphora and dynamic binding. *Linguistics and Philosophy*, 15:111–183, 1992.
2. D. Dowty. Collective predicates, distributive predicates and all. In *Proceedings of the Eastern Proceedings of the Eastern States Conference on Linguistics*. ESCOL3, Cascadilla Press., 1986.
3. P. Elbourne. *Situations and Individuals*. Cambridge: MIT Press, 2005.
4. B. Geurts. Donkey business. *Linguistics and Philosophy*, 25:129–156, 2002.
5. N. Kadmon. *On Unqiue and Non-Unique Reference and Asymmetric Quantification*. PhD thesis, University of Massachusetts at Amherst, 1987.
6. N. Kadmon. Uniqueness. *Linguistics and Philosophy*, 13(3):273 – 324, 1990.
7. H. Kamp and U. Reyle. *From Discourse to Logic*. Kluwer, Dordrecht., 1993.
8. M. Kanazawa. Weak vs. strong readings of donkey sentences and monotonicity inference in a dynamic setting. *Linguistics and Philosophy*, 17(2):109–158, 1994.
9. M. Kanazawa. Singular donkey pronouns are semantically singular. *Linguistics and Philosophy*, 24(3):383–403, 2001.
10. M. Krifka. At least some determiners aren't determiners. In K. Turner, editor, *The semantics/pragmatics interface from different points of view. (= Current Research in the Semantics/Pragmatics Interface Vol. 1).*, pages 257–291. Elsevier Science B.V.,, 1999.
11. D. Lewis. *Counterfactual*. Harvard University Press, 1973.
12. D. Lewis. Adverbs of quantification. In E. Keenan, editor, *Formal Semantics of Natural Language. CUP*. Cambridge University Press, 1975.
13. D. Lewis. Probabilities of conditional and conditional probabilities. *Philosophical Review*, 8:297–315, 1976.
14. S. Neale. *Dsecriptions*. Cambridge: MIT Press, 1990.
15. F. J. Pelletier and L. K. Schubert. Mass expressions. In F. Guenthner and D. Gabbay, editors, *Handbook of philosophical logic,*, volume 4, pages 327–407. Kluwer: Dordrecht, 1989.
16. M. Rooth. Noun phrase interpretation in montague grammar, file change semantics, and situation semantics. In P. Gardenfors, editor, *Generalized Quantifiers*. Dordrecht: Reidel, 1987.
17. Y. Winter. Atoms and sets: a characterization of semantic number. *Linguisitic Inquiry*, 33:493–505, 2002.
18. Y.-E. Yoon. *Weak and Strong Interpretations of Quantifiers and Definite NPs in English and Korean*. PhD thesis, The University of Texas at Austin, 1994.

Coordination and Anaphora in Attitude Contexts

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Abstract. In this paper, I offer a DRT analysis of cross-attitudinal anaphora (intentional identity) in which the coordination relation between mental files plays a central role.

Keywords: intentional identity, cross-attitudinal anaphora, propositional attitude reports, discourse representation theory, coordination, mental files

1 Introduction

As Geach (1967) pointed out, cases of cross-attitudinal anaphora like (1), where 'she' in the second sentence is anaphoric to 'a witch' in the first sentence, have readings on which both attitude reports in them are true.

(1) Hob believes that a witch blighted Bob's mare. Nob believes that she killed Cob's sow.

For example, (1) is true in the following situation (cf. Edelberg, 1986, p. 2).

> **The Hob-Nob case**: In a village, because of an epidemic, many livestock are blighted and dead. A newspaper of the village spreads a rumor that a witch (exactly one witch) is threatening the village and trying to kill the livestock there. Both Hob and Nob read the newspaper and believe the rumor. Hob finds Bob's mare getting sick and believes that a witch blighted Bob's mare. Nob finds Cob's sow dead and then believes that a witch killed Cob's sow.

It is often claimed that the anaphoric relation between 'a witch' and 'she' in the reading in question is not properly explained by analyzing 'she' as an individual variable bound by an existential quantifier corresponding to its antecedent 'a witch'. First consider (2) (For simplicity, I ignore the complexity in VPs in the embedded sentences and treat them as simple predicates).

(2) a. $\exists x(witch'(x) \wedge BEL(h, blight\text{-}bm'(x)) \wedge BEL(n, kill\text{-}cs'(x)))$
b. $\exists x(BEL(h, witch'(x) \wedge blight\text{-}bm'(x)) \wedge BEL(n, witch'(x) \wedge kill\text{-}cs'(x)))$

Since there exists no witch, for the reading in question to be true, it is not necessary that Hob's belief and Nob's belief are about the same particular existent object. However, (2a) entails this, and (2b) does too.[1]

The following analysis for the reading of (1) avoids this problem, since the existential quantifier appears within the scope of the belief operator.

(3) $BEL[h, \exists x(witch'(x) \wedge blight\text{-}bm'(x) \wedge BEL(n, kill\text{-}cs'(x)))]$

This is not a correct analysis, since the reading of (1) in question can be true even when Hob believes nothing about Nob, Cob and his sow (Edelberg, 1986, pp. 3-4). In (3), '$BEL(n, kill\text{-}cs'(x))$' appears within the scope of 'Hob believes that', and thus it implies that Hob believes something about Nob's belief and Cob' sow. But (1) doesn't entail this. For a similar reason, the E-type analysis of pronouns, which is illustrated by (4), does not give appropriate treatment of the reading of (1) in question (cf. Geach, 1967, p. 630, Edelberg, 1986, pp. 3-4): (1) can be true even when Nob believes nothing about Hob, Bob, and his mare, but (4) can not.

(4) a. $BEL[h, \exists x(witch'(x) \wedge blight\text{-}bm'(x))] \wedge BEL[n, kill\text{-}cs'(\iota x[witch'(x) \wedge blight\text{-}bm'(x)])]$
b. $BEL[h, \exists x(witch'(x) \wedge blight\text{-}bm'(x))] \wedge BEL[n, kill\text{-}cs'(\iota x[BEL(h, witch'(x) \wedge blight\text{-}bm'(x))])]$

In this manner, it is difficult to analyze the pronoun 'she' in (1) as an individual variable bound by existential quantifier or as an E-type pronoun.

In the Hob-Nob case, Hob's belief and Nob's belief contain the concept *witch*. Regarding this as a common feature of the situations which make (1) true, one may propose the following second-order analysis of the reading of (1) in question.

(5) $\exists F(F = witch' \wedge BEL[h, \exists x(F(x) \wedge blight\text{-}bm'(x))] \wedge BEL[n, \exists x(F(x) \wedge kill\text{-}cs'(x))])$

However, this is not correct, since for the reading of (1) in question to be true, it is not sufficient that the descriptive content of Hob's belief overlaps the one of Cob's (cf. Edelberg, 1986, pp. 8-9). For instance, suppose that no rumor concerning witch-crisis has spread in the village. When Hob finds Bob's mare to get sick, the belief that a witch blighted it happens to come to his mind. Independently from this, Nob finds Cob's sow dead and happens to believe that a witch killed Cob's sow. In this case, even though their beliefs contain the same descriptive content, say, $witch(x)$, (1) is not true—at least it sounds quite weird.[2]

[1] This is so, if we understand $\exists$ as 'existential' quantifier in the standard manner. Note that, according to Meinongianism, quantifiers need not to be existentially-loaded. Does this anti-Quinean understanding of quantifier solve the problem in question? Unfortunately, as Edelberg (1986) shows, anti-Quinean quantification is not sufficient to give a correct analysis of cross-attitudinal anaphora.

[2] This doesn't mean that such overlap is not required. For example, suppose Nob believes that a wolf man, not a witch, killed Cob's sow. In such a case, I tend to say that (1) is not true. But, in this paper I leave this matter open.

In this way, it is not sufficient for the truth of (1) that Hob's belief and Nob's belief share the same descriptive content. Then, what else is required? It is widely accepted that for the reading of (1) in question to be true, it must be the case that Hob's belief and Nob's belief can be said to be 'about the same thing' in some sense, in other words, their beliefs have 'a common focus, whether or not here actually is something at that focus.' (Geach, 1967, p. 627). Asher says, anaphoric links exemplified by (1) 'only make sense if the agent's attitudes are *coordinated* together, whether by means of communication or some other mechanism, in such a way that the two agents can be said to have the "same" individual in mind' (Asher, 1987, p. 127. My emphasis). For example, in the Hob-Nob case, Hob's belief and Nob's belief are coordinated together because their beliefs are based on the common source of information—the local newspaper. As Asher claimed, such coordination will be established by means of communication between the agents in question (see also EXAMPLE 1 in Edelberg, 1986, p. 1). There can be other ways of establishing coordination among mental states. For some reasons, a mental state is coordinated with some other mental state without their being about some particular existent object. And such coordination between mental states is crucial for the truth of the cases of problematic cross-attitudinal anaphora like (1).

This paper explores a way to develop a semantic analysis of cross-attitudinal anaphora in which coordination among mental states plays a central role. The aim of this paper is not to elucidate the nature of coordination. Rather, putting aside the question of what the nature of coordination is, in this paper, I show that by incorporating the coordination relation between mental states into semantics as its primitive notion, we can give a relatively simple and adequate semantics for cross-attitudinal anaphora of the kind in question. Extending DRT semantics for propositional attitude reports presented in Kamp, van Genabith and Reyle (2011), I propose the following: Firstly, indefinites and pronouns appearing in attitude contexts introduce what I call *meta discourse referents*, that is, discourse referents which represent mental files; and secondly, assuming that coordination is a relation between mental files, anaphoric links between indefinites and pronouns in attitude contexts are recorded by using meta discourse referents and *the coordination predicate*, which represents coordination between mental files.

In section 2, we quickly review DRT semantics for cross-attitudinal anaphora based on Kamp et al. (2011), a useful survey of recent development of DRT. Section 3 gives a brief exposition of the notion of coordination. Section 4 provides a semantics for cross-attitudinal anaphora by using DRT extended with meta discourse referents and the coordination predicates.

2 Kamp et al. (2011) on cross-attitudinal anaphora

At the beginning of the history of DRT, Kamp stated the view that DRSs are mental representations a hearer of a discourse constructs as her interpretation of the discourse (cf. Kamp, 1981). In particular, discourse referents are taken as mental representations of entities (cf. Kamp, et al. 2011, pp. 326-327). Even though the idea of DRS as mental representation is not a core doctrine of DRT, it has motivated DRT semantics for propositional attitude reports which describes mental states of reportees of attitude reports by using DRSs.

A mental state is described as a pair of its *attitudinal mode* like belief, desire, or doubt and a DSR which specifies the content of the state. For example, a belief that a delegate arrived is described as follows, where BEL is the indicator of the attitudinal mode of *believe*.

(6) $$\left\langle BEL, \begin{array}{|l|} \hline x \\ \hline delegate'(x) \\ arrive'(x) \\ \hline \end{array} \right\rangle$$

An *attitude ascription* ascribes one or more mental states to a subject For example, we can ascribe to Mary a belief that a delegate arrived and a doubt that *the same delegate* registered. The following DRS condition corresponds to this attitude ascription (I use 'DOU' as the mode indicator for doubt).[3]

(7) $$Att(m, \left\{ \left\langle BEL, \begin{array}{|l|} \hline x \\ \hline delegate'(x) \\ arrive'(x) \\ \hline \end{array} \right\rangle, \left\langle DOU, \begin{array}{|l|} \hline \\ \hline registered'(x) \\ \hline \end{array} \right\rangle \right\})$$

$Att(s, \{\langle \Phi, K\rangle\})$ is an *attitude ascription condition* which means that a subject s has a mental state whose mode—belief, desire, intention, and so on—is Φ and whose content is specified by the DRS K.

Kamp et al. (2011) claims that in cases of interpersonal cross-attitudinal anaphora like (1), what supports the anaphoric link in question is that two subjects has some mental state with *the same content* (cf. Kamp, et al, 2011, p. 383). In the Hob-Nob case, both Hob and Nob believe that there is a witch, and this is supposed to support the anaphoric link between 'a witch' and 'she' in (1). Based on these considerations, Kamp et al. (2011) proposes an analysis of interpersonal cross-attitudinal anaphora illustrated by (8) for (1) (again, for simplicity, I ignore the complexity in VPs in the embedded sentences and treat them as simple predicates).

[3] The following exposition omits the complexity concerning the notion of *external anchor* (Kamp, et al. 2011, in particular, pp. 332-343), which is required to treat cases of *de re* ascriptions.

(8)
$$\begin{array}{|l|}\hline h, n \\ \hline Hob'(h) \\ Nob'(n) \\ Att(h, \left\{\left\langle BEL, \begin{array}{|l|}\hline x \\ \hline witch'(x) \\ blighted\text{-}Bob\text{'s-}mare'(x) \\ \hline \end{array} \right\rangle\right\}) \\ Att(n, \left\{\left\langle BEL, \begin{array}{|l|}\hline x' \\ \hline witch'(x') \\ \hline \end{array} \right\rangle\right\}) \\ Att(n, \left\{\left\langle BEL, \begin{array}{|l|}\hline u \\ \hline u = x' \\ killed\text{-}Cob\text{'s-}sow'(u) \\ \hline \end{array} \right\rangle\right\}) \\ \hline \end{array}$$

However, this analysis fails to explain the anaphoric link between 'a witch' in the first sentence and 'she' in the second sentence of (1). As we have seen, Kamp et al. (2011) claims that in cases of interpersonal cross-attitudinal anaphora like (1), anaphoric links of the kind in question are supported by subjects' having some mental state with the same content. In (8), the content that there is a witch is shared by Hob and Nob, and this shared content is assumed to be support the anaphoric link in question. However, as we have seen in Section 1, sharing the same descriptive content in this sense is not sufficient for interpersonal cross-attitudinal anaphora.

3 Coordination and cross-attitudinal anaphora

In the passage quoted in Section 1, Asher claims that the anaphoric links between NPs in attitude contexts can be supported by what he called *coordination* between mental states, which can be *inter*personal. Kamp et al. (2011) fails to capture this feature of cross-attitudinal anaphora. But, what is coordination between mental states? Even though it is hard to give a precise characterization of coordination, the following consideration will give us an intuitive understanding of it.

As Geach pointed out, it sometimes makes sense to regard a mental state of a subject as being directed toward a 'common focus' as a different mental state of a different subject is (Geach, 1967, p. 627). For example, suppose that John told to Mary about a man who he met yesterday and Mary believed what John said. In this cases, even if Mary had no idea about who the man John mentioned, Mary's belief is reasonably regarded as being directed towards a common focus as John's mental states. As Asher says, the coordination relation holds 'by means of communication or some other mechanism': Communication is a good reason for coordination, but it is not the only reason. As we have seen, in the Hob-Nob case, Hob's belief and Nob's belief are coordinated together because their beliefs are based on the common source of information—the local newspaper. In another example, let us suppose that John and Mary together see a mirage of an oasis in a desert. In this case, it is reasonably said that John's perceptual experience is directed toward a common focus as Mary's, and *vice versa*. This relation can be *intra*personal as well. For example, when you see an apple in front of you and think that it looks delicious, your thought is reasonably regarded as being directed toward a common focus as your perceptual experience is. The same thing holds even when you have a hallucination of an apple. Roughly speaking, two mental states are coordinated when they have the common focus in the sense illustrated here. Let us say that a mental state x *is coordinated with* a mental state y if x is reasonably regarded as being directed toward a common focus as y is directed toward.

The mirage case and the hallucination case above show that the coordination relation between mental states is *referent-independent*, in the sense that in order that a mental state of a subject is coordinated with other mental state of the same/other subject, it is not necessary for these mental states to be about a particular external existent object.

I proposed to say that a mental state x is coordinated with a mental state y when x is reasonably regarded as being directed toward a common focus as y is directed toward. I don't present this as the definition of coordination. This, together with examples, gives at most intuitive understanding of it. Many things are left unexplained in the clause 'being reasonably regarded as being directed toward a common focus as': Who does regard so?; what is a common focus?; and, what may other mechanism than communication establish coordination between mental states? In this paper I can not answer to these questions. Rather, I propose to take coordination as a *primitive* notion in semantics of cross-attitudinal anaphora, and use it to record cross-attitudinal anaphoric linkage.

To do this, we need to represent such coordination between mental states in a DRS. But how? In (8) the discourse referent x can be regarded as a mental representation of a witch for Hob and u as the one for Nob. Given this, it seems natural to represent coordination between Hob's belief and Nob's belief as a relation between x and u (cf. Asher, 1986, pp. 151-159). However, within a standard framework of DRT, it is not clear how to record information about coordination between attitudes. Let us call a DRS containing some attitude ascription condition as its DRS condition a *matrix* DRS, and a DRS which appears in attitude ascription condition and represents the content of some mental state a *content* DRS.[4] Within the standard framework of DRT, information about linguistic anaphora between two NPs is recorded by the identity between the discourse referents introduced by these NPs. Given this framework, there are only two possible ways to record anaphoric links. The first one is to put the identity in question in a matrix DRS, and the second one is to put it in a content DRS. Unfortunately, both ways cannot properly treat cases of interpersonal cross-attitudinal anaphora. If we put '$x = u$' in a matrix DRS, then x and u represent some individual objects, not mental representations, and the result

[4] For simplicity, I ignore cases where the complement of an attitude verb is itself an attitude report.

entails that there is a particular object of which Hob's mental state and Nob's mental state in question are. On the other hand, it is also problematic to put the identity in a content DRS. According to this option, the condition $x = u$ appears in the content DRS which specifies the content of Hob's belief or one which specifies the content of Nob's belief. If the former was the case, then Nob's mental representation would appear in Hob's mental representation, but this is impossible. What can appear in Hob's mental representation is, at most, a representation of Nob's mental representation (and this is not useful in the present context, since the truth of the reading of (1) in question doesn't require Hob to believe anything about Nob). The same problem would arise if the latter was the case.

In the next section, we explore a way to write down the coordination relation between mental states in matrix DRSs (that is, outside the scope of an attitude), not in content DRSs, so as to record anaphoric links by this relation. To do this, we need something different from the discourse referents x and u appearing in content DRSs. A key notion to do this is the notion of *meta discourse referent*. In DRS, the coordination is represented by using meta discourse referents.

4 DRT with coordination

In 4.1 I extend DRT language so that anaphoric links are recorded by meta discourse referents and the coordination predicate in matrix DRSs, and give an informal exposition of what extended DRSs means. In 4.2 we have a brief look at a model theoretic semantics for DRSs containing meta discourse referents and the coordination predicate. In section 4.3, we compare the account presented in this paper with Edelberg (1992)'s seminal work on cross-attitudinal anaphora. Note that the following theory is meant to be for cases where attitude verbs do not appear in the embedded sentences of attitude reports. Also the following theory says nothing about *de re* ascriptions. To extend the theory to cover such cases is a topic for further investigation.

4.1 Syntax (and an intuitive exposition of how to interpret DRSs with meta discourse referents and the coordination predicate)

Suppose that r is a discourse referent. Then, $[r]$ is a meta discourse referent of r. Meta discourse referents are used in two purposes. First, they are used to register which discourse referents appear in content DRSs: whenever a discourse referent r appears in a content DRS in a matrix DRS, its meta discourse referent $[r]$ is in the universe of the matrix DRS. Secondly, they are used to describe coordination among mental states and to record anaphoric link between NPs in attitude contexts. Let me explain these points by example. Consider again the Hob-Nob sentence.

(1) Hob believes that a witch blighted Bob's mare. Nob believes that she killed Cob's sow.

I propose that whenever a discourse referent appears in the universe of a content DRS, its appearance is registered by introducing its meta discourse referent into the universe of the matrix DRS whose condition contains the content DRS. Thus, when the first sentence of (1) updates the empty DRS, we have the following DRS.

(9)
$$\begin{array}{|l|}\hline h, [x] \\ \hline Hob'(h) \\ Att\left(h, \left\{ \left\langle BEL, \begin{array}{|l|}\hline x \\ \hline witch'(x) \\ blighted\text{-}Bob\text{'}s\text{-}mare'(x) \\ \hline \end{array} \right\rangle \right\}\right) \\ \hline \end{array}$$

x in the content DRS is introduced by the indefinte NP 'a witch' in the embedded sentence of it. $[x]$ in the universe of the matrix DRS of (9) is the meta discourse referent of x, which registers an appearance of the discourse referent x in the universe of the content DRS of (9). Then, what does this DRS represent? Here it is useful to mention the notion of *mental file*. Informally speaking, in (9), $[x]$ represents a mental file whose possessor and contents are specified in the attitude ascription condition of (9). More specifically, (9) is true iff there is a mental file of Hob which contains the information *being a witch* and *blighted Bob's mare* with the attitudinal mode of belief (the model theoretic interpretation of DRSs containing meta discourse referents is defined in the next section). Let us develop this picture in more detail.

It is common to assume mental files to explain object-oriented information processing by cognitive systems (cf. Recanati, 2012). If two or more pieces of information are taken by a subject as being about the same object, then, it is said that they are stored in the same mental file for some object. For example, suppose that by seeing an apple in front of her, Mary thinks that it is red and round. In this case, she has a perceptual mental file for the apple which contains information of *being red* and information of *being round*. If, as Kamp has suggested, DRSs are mental representations hearers construct during interpreting discourses, then, for a DRS $\langle U, Con\rangle$, any pair of a discourse referent in U and the conditions of it in Con can be seen as a temporal mental file constructed and used to interpret the discourse in question by its hearer. These files are temporal ones in the sense that they are sustained only during perceiving something or interpreting of discourses. But some files are sustained for a long period, which would be in subjects' long-term memories. For example, I takes many pieces of information as being about the Empire State Building—being located in Manhattan, having elevators, and so on—and this means that I have a mental file for it containing these pieces of information. Many mental files are properly connected to a particular object, but this is not necessarily the case. Some files fail to have any particular referent (for example, the Vulcan file LeVerrier had); some files contain pieces of information whose sources are different objects (for example, the Madagascar file Marco Polo had). It is a common and implicit assumption that mental files are connected to understanding of referential terms like proper names, indexicals, demonstratives, and definite descriptions. However, this needs not be the case, given that the DRS $\langle\{x\},\{delegate'(x), arrive'(x)\}\rangle$ is a mental file which a hearer constructs when she interprets the sentence 'a delegate arrived'.

Let us take a content DRS within the attitude ascription condition $Att(a, ADS)$ as representing a mental file(s) of the agent a. More precisely, each discourse referent r in the universe of a content DRS represents a mental file whose contents are partially described by the conditions of r specified in the content DRS. For example, $\langle\{x\},\{delegate'(x), arrive'(x)\}\rangle$ appearing inside $Att(a, ADS)$ represents a mental file of the subject a containing information of *being a delegate* and *arrived* (and probably more). Meta discourse referents are the devices to represent such mental files *outside* attitude ascription conditions. If a meta discourse referent $[x]$ is in the universe of a matrix DRS, this means that there is a mental file whose contents are specified by a content DRS with the discourse referent x, and whose subject is specified by the attitude ascription condition containing the content DRS.

Another element in an attitude ascription condition unexplained yet is the modes of mental states. For them, let us assume that each piece of information in a mental file is accompanied with some attitudinal mode. In (9), the conditions of the content DRS is associated with the mode of *belief*. This means that in the mental file represented by the DRS, information of *being a delegate* and information of *arrived* are associated with the mode of *belief*. It may be the case—indeed, it is often the case—that a mental file contains pieces of information with different attitudinal modes. For example, Mary believes that a delegate arrived and doubt that she registered. In this case, she has a mental file containing information associated with the mode of *belief* and information associated with the mode of *doubt*, which is represented by the attitude ascription condition (7).

Let us move on to the second purpose of using meta discourse representations: to describe coordination relation between mental states and to register anaphoric link between NPs in attitude contexts. As are usual discourse referents, meta discourse referents in the universe of a DRS as a context of interpretation are possible antecedents for anaphoric pronouns in subsequent sentences. In addition to this we also assume that: (i) meta discourse referents are available *only* for meta discourse referents introduced by anaphoric pronouns appearing in the embedded sentences of attitude reports; and (ii) cross-attitudinal anaphora is registered not by using the identity symbol and discourse referents, but by using *the coordination predicate* and *meta* discourse referents. Let us see how these ideas work.

(10) is the preliminary DRS corresponding to the second sentence of (1).[5] u in the context DRSs of (10) is a discourse referent introduced by 'she' in the embedded sentence of the second sentence of (1). Again, $[u]$ registers the appearance of the discourse referent u in the universe of the content DRS of (10), and represents a mental file the agent of the attitude in question—in this case, Nob—has.

[5] For the notion of preliminary DRS, see Kamp et al. (2011) sec. 2.3.

(10) $$\begin{array}{|l|}\hline n,\ [u] \\ \hline Nob'(n) \\ Att\left(n, \left\{ \left\langle BEL, \left\langle \left\{ \begin{array}{|l|}\hline u \\ \hline pers(u) \\ female(u) \\ \hline \end{array} \right\}, \begin{array}{|l|}\hline u \\ \hline \textit{killed-Cob's-sow}'(u) \\ \hline \end{array} \right\rangle \right\rangle \right\}\right) \\ \hline \end{array}$$ [6]

Anaphora resolution proceeds as follows. Let us assume that coordination relation which makes cross-attitudinal anaphoric linkage between NPs sense is a *relation between mental files.* To represent this relation, we introduce '$>$', the coordination predicate for meta discourse referents. $[r] > [r']$ represents that the mental file represented by $[r]$ is coordinated with the mental file represented by $[r']$. If $[r]$ takes $[r']$ as its antecedent, this information is recorded as $[r] > [r']$ in the condition of the *matrix* DRS. For example, the anaphoric linkage between 'a witch' in the first sentence of (1) and 'she' in the second sentence of (1) is recorded by using two meta discourse referents $[x]$, introduced by 'a witch', and $[u]$, introduced by 'she', and the coordination predicate $>$. In this case, $[x]$ is the antecedent of $[u]$, and thus this information is registered as $[u] > [x]$ in the condition of (10). So, taking (9), whose universe contains $[x]$, as its context, we obtain (12) as the result of anaphora resolution of (10).

(11) $$\begin{array}{|l|}\hline n,\ [u] \\ \hline Nob'(n) \\ Att\left(n, \left\{ \left\langle BEL, \begin{array}{|l|}\hline u \\ \hline \textit{killed-Cob's-sow}'(u) \\ \hline \end{array} \right\rangle \right\}\right) \\ [u] > [x] \\ \hline \end{array}$$

Finally, merging[7] this with (9) results in the following DRS, which is the DRS translation of the discourse (1).[8]

[6] For simplicity, here I treat proper names as not presuppositional.

[7] K is the result of the merge of K_1 and K_2 ($K_1 \uplus K_2$) iff $K = \langle U_{K_1} \cup U_{K_2}, Con_{K_1} \cup Con_{K_2} \rangle$ (Kamp et al. 2011, sec. 2.3).

[8] We can easily construct DRSs for cases where content DRSs contain two or more discourse referents. Here is an example (I owe this example to an anonymous referee).

(12) a. Hob believes that a witch found a unicorn. Nob believes that she killed it.

b. $$\begin{array}{|l|}\hline h,\ [x],\ [y],\ n,\ [u],\ [v] \\ \hline Hob'(h),\ Nob'(n) \\ Att\left(h, \left\{ \left\langle BEL, \begin{array}{|l|}\hline x,\ y \\ \hline witch'(x) \\ unicorn'(y) \\ found'(x, y) \\ \hline \end{array} \right\rangle \right\}\right), Att\left(n, \left\{ \left\langle BEL, \begin{array}{|l|}\hline u,\ v \\ \hline kill'(u, v) \\ \hline \end{array} \right\rangle \right\}\right) \\ [u] > [x],\ [v] > [y] \\ \hline \end{array}$$

(13)
$$\begin{array}{|l|}\hline h, [x], n, [u] \\ \hline Hob'(h) \\ Nob'(n) \\ Att\left(h, \left\{ \left\langle BEL, \begin{array}{|l|}\hline x \\ \hline witch'(x) \\ blighted\text{-}Bob\text{'}s\text{-}mare'(x) \\ \hline \end{array} \right\rangle \right\}\right) \\ Att\left(n, \left\{ \left\langle BEL, \begin{array}{|l|}\hline u \\ \hline killed\text{-}Cob\text{'}s\text{-}sow'(u) \\ \hline \end{array} \right\rangle \right\}\right) \\ [u] > [x] \\ \hline \end{array}$$

This DRS, and thus the reading of (1) in question, is true iff (i) there are two subjects, Nob and Hob; (ii) there are two mental files X (represented by $[x]$) and U (represented by $[u]$) such that X is Hob's file containing information that ... *is a witch* and that ... *blighted Bob's mare* associated with the mode of belief and U is Nob's file containing information that ... *killed Cob's sow* associated with the mode of belief; and (iii) Nob's mental file U is coordinated with Hob's mental file X.

4.2 Semantics

In this section, I define intensional semantics for DRS involving meta-discourse referents and $>$.

A model M is any seven-tuple $\langle D, A, W, MF, C, \triangleright, V\rangle$ which satisfies the following conditions.

D is a set of individual objects. A is a set of agents (bearers of propositional attitudes), which is a subset of D. W is a set of worlds. For simplicity, we assume that the domain is constant across worlds.

MF is a non-empty set taken as the set of mental files. For each w, MF_w is the set of mental files agents have in w. MF must be disjoint with the set of discourse referents constituting the basic vocabulary of DRSs.

C is a function which specifies the contents of ones' mental files. To explain this, first we need to introduce the notion of *information state* (Kamp et al., 2011, pp. 157-158). Standardly, an intentional semantics for a language L assigns to each formula of L a set of possible world, that is, the set of all possible worlds where the formula is true. Instead of a set of possible worlds, intentional semantics for DRSs assigns to each (proper) DRS an information state. In Kamp et al. (2011, p. 157), an information state assigned to K relative to an intensional model $M = \langle D, W, V\rangle$, in symbols $[\![K]\!]^s_M$, is a set of pairs of a possible world w and an embedding function $f : U_K \to D$ such that every condition in Con_K is verified by f in w with respect to M.

(14) $[\![K]\!]^s_M := \{\langle w, f\rangle \mid \langle \Lambda, f\rangle \models_{M,w} K\}$

All embedding functions appearing in $[\![K]\!]^s_M$ have the same domain, that is, U_K. So the information state assigned to K registers information about the universe of K and thus about candidates of antecedents of anaphoric pronouns.

Turning back to C, the range of C is the set of information states, but here we understand an information state I as a set of pairs of a world and a function which maps a *mental file* to a member of the domain D. Partly following Kamp et al. (2011), C assigns an information state in this sense to each ordered triple of an agent $a \in A$, a world w, and an attitudinal mode Φ. For example, suppose that m is one of a's mental files and that m contains the condition P with the mode BEL. In this situation, if $\langle w, f\rangle \in C(\langle a, w, BEL\rangle)$, then $f(m)$ satisfies P in w. We require that all functions in the members of the information state $C(\langle a, w, \Phi\rangle)$ share the same domain, which is a subset of MF_w. Let us call this domain the base of $C(\langle a, w, \Phi\rangle)$. We also require that for any attitudinal modes Φ and Ψ, the base of $C(\langle a, w, \Phi\rangle)$ is identical to the base of $C(\langle a, w, \Psi\rangle)$. This shared base is the set of all mental files the agent a has in w. Let us call it MF^a_w. The last requirement is that for any agents a and b, MF^a_w is disjoint from MF^b_w. This reflects the intuition that no two agents can share the same mental file.

$\rhd : W \to \mathcal{P}(MF \times MF)$ determines coordination among mental files in each world. For each world w, $\rhd_w$ is a binary relation on MF_w. Note that $\rhd_w$ is both intrapersonal and interpersonal, that is, coordination is both intrapersonal and interpersonal. Semantic/inferential behaviors of cross-attitudinal anaphora force us to put some constraints on $\rhd$. Due to a space constraint we can not discuss details, but let me very quickly mention some structural features of coordination. First, to treat intrapersonal cases properly, we require $\rhd_w$ to be reflexive. Second, given Edelberg (1986)'s well-known discussion about non-symmetricity of intentional identity, $\rhd_w$ needs not be symmetric. Putting further constraints on $\rhd_w$, of course, will lead to different 'logics' of cross-attitudinal anaphora.

V is an interpretation function assigning a model theoretic meaning to each non-logical vocabulary, which is defined as usual.

An embedding function is a function from the universe of some DRS K to the domain of the model. Now the universe of DRS contains not only discourse referents but also meta discourse referents. For any discourse referent x we require that any embedding function must assign x to $[x]$.

In addition to this, we need functions which map each discourse referent appearing the universe of a content DRS to a member of MF. Let us call such a function an m-embedding function, since it maps a discourse referent to a mental file.

Verification depends not only on embedding functions but also m-embedding functions. First let us define g^h_m, the m-counterpart of an embedding function g with respect to an m-embedding function h, where for all x and y if $h(x) = h(y)$ then $g(x) = g(y)$. g^h_m is the function which satisfies the following conditions.

(15) a. $\mathfrak{D}(g^h_m)$ (the domain of g^h_m) is $h[\mathfrak{D}(g)]$ (the image of the domain of g under h), which is a subset of MF.
b. For all $x \in h[\mathfrak{D}(g)]$, $g^h_m(x) = g(x')$, where $x' \in h^{-1}[x]$

Verification conditions are defined as follow. Suppose that g is an embedding function and h is an m-embedding function, then

(16) a. $g, h \models_{M,w} [x] > [y]$ iff $h(g([x])) \rhd_w h(g([y]))$
b. $g, h \models_{M,w} Att(x, ADS)$ iff
- $g(x) \in A$, and
- For each $\langle \Phi, K \rangle$ in ADS, $\forall w \forall f(\langle w, f \rangle \in [\![K]\!]^s_M \rightarrow \exists f'(\langle w, f' \rangle \in C(\langle g(x), w, \Phi \rangle) \wedge f^h_m \subseteq f'))$ [9]

c. For other types of conditions, $g, h \models_{M,w}$ is defined as $g \models_{M,w}$

Finally, truth condition. A DRS K is true with respect to the model M in w iff there are an embedding function g and an m-embedding function h which verify all conditions in Con_K with respect to M in w.

4.3 Edelberg (1992)

The account presented in this paper has a great similarity with Edelberg (1992)'s seminal work on intentional identity, according to which (1) is semantically analyzed as (17).

(17) $\exists \alpha \exists \beta (BEL(\text{Hob}, [\![\alpha \text{ blighted Bob's mare}]\!] \wedge BEL(\text{Nob}, [\![\beta \text{ killed Cob's sow}]\!] \wedge \alpha \approx \beta))$,

where 'α' and 'β' are variables ranging over 'person-bound' menta images and '$\approx$' represents the counterpart relation between ideas. Both our account and Edelberg's account appeal to intrapersonal mental representations and some relation between them.

So, the presented analysis of cross-attitudinal anaphora might be taken as a DRT formulation of Edelberg's semantics of cross-attitude anaphora. But even if this were true,[10] this would make difference. Since our account is formulated within the framework of DRT, it inherits advantages of DRT, which is not available in Edelberg's account. For example, it gives a bottom-up account of how to get the truth conditional analysis of cases of cross-attitudinal anaphora, by showing the corresponding DRSs construction procedures. Moreover, we can appeal to machineries in DRT, in particular, the notion of accessibility, to predict distribution of possible anaphoric links. For example, in (18), 'he' in the second sentence can not take 'someone' in the first sentence as its antecedent.

(18) *Mary does not believe that someone$_i$ murdered Jones. Bob believes that he$_i$ murdered Smith.

The first sentence of (18) is translated as (19a) and the second one is as (19b).

[9] The relation between $[\![K]\!]^s_M$ and $C(\langle g(x), w, \Phi \rangle)$ specified here is a modified version of $\preceq$ in Kamp, et al. (2011, p. 160)

[10] Actually, this is not true, at least since the coordination relation needs not be symmetric in our account, but the counterpart relation is symmetric in Edelberg's theory. (Edelberg, 1992, sec. 9 and 10)

(19) a.

$$\begin{array}{|l|}\hline m \\ \hline Mary'(m) \\ \neg \begin{array}{|l|}\hline [x] \\ \hline Att\left(m, \left\{\left\langle BEL, \begin{array}{|l|}\hline x \\ \hline murdered\text{-}Jones'(x) \\ \hline\end{array} \right\rangle\right\}\right) \\ \hline \end{array} \\ \hline \end{array}$$

b.

$$\begin{array}{|l|}\hline b, [u] \\ \hline Bob'(b) \\ Att\left(n, \left\{\left\langle BEL, \left\langle \left\{ \begin{array}{|l|}\hline u \\ \hline pers(u) \\ male(u) \\ \hline\end{array} \right\}, \begin{array}{|l|}\hline u \\ \hline murdered\text{-}Smith'(u) \\ \hline\end{array} \right\rangle \right\rangle\right\}\right) \\ \hline \end{array}$$

Since $[x]$ appears within the scope of negation and is not in the universe of the matrix DRS (19a), $[x]$ is not accessible for $[u]$. This immediately explains why (18) is not acceptable.

5 Conclusion

In this paper, we offered a DRT account of cross-attitudinal anaphora, based on the idea that coordination between subjects' mental states plays a crucial role for cross-attitudinal anaphora. We extended DRT language by adding meta discourse referents and the coordination predicate—in the model theoretic semantics, they correspond to mental files and the coordination relation between them, which are taken as primitive in the semantics—, and used them to record anaphoric linkage between NPs in attitudinal contexts.

References

[1987] Asher, N. (1987). 'A Typology for Attitude Verbs and Their Anaphoric Properties', *Linguistics and Philosophy*, 10, 125-197.

[1986] Edelberg, W. (1986). 'A New Puzzle about Intentional Identity', *Journal of Philosophical Logic*, 15(1), 1-25.

[1992] Edelberg, W. (1992). 'Intentional Identity and The Attitudes', *Linguistics and Philosophy*, 15(6), 561-596.

[1967] Geach, P. T. (1967). 'Intentional Identity', *Journal of Philosophy*, 64: 20, 627-632.

[1981] Kamp, H. (1981). 'A Theory of Truth and Semantic Representation', in J. Groenendijk et al. (eds), *Formal Methods in the Study of Language*. Amsterdam: Mathematisch Centrum

[2011] Kamp, H., J. van Genabith, and U. Reyle (2011). 'Discourse Representation Theory', in D. Gabbay and F. Guenthner (eds.) *Handbook of Philosophical Logic, Volume 15*, Springer, pp. 125-394.

[2012] Recanati, F. (2012). *Mental Files*, Oxford: Oxford University Press.

Truth-conditionals and Use-conditionals
an expressive modal analysis

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Abstract. I propose that on some non-standard interpretations of conditionals, the antecedent influences not the truth-conditional, but the use-conditional evaluation of the consequent by restricting the modal base of a necessity operator introduced by the conditional form. On this view, conditionals can be grouped into *truth-conditionals* and *use-conditionals*, depending on interpretation. I argue that such an analysis allows to predict properties of hypothetical conditionals, biscuit conditionals, and conditional hedges within a unified account.

1 Interpretations of conditionals

The three examples of conditionals given in (1) through (3) below each have distinct salient interpretations, for which I seek to account for in a unified analysis.

(1) If John remembered to go shopping, there's beer in the fridge.

(2) If you are thirsty, there's beer in the fridge.

(3) If I'm not mistaken, there's beer in the fridge.

The salient interpretation of (1) is what I take to be the standard interpretation, also labeled the **hypothetical conditional** interpretation, on which the truth of the consequent (=proposition of the main clause) is evaluated under the assumption that the antecedent (=proposition of the if-clause) holds (variants of analyses on these lines have been prominent in the formal literature, starting with Stalnaker 1968 [17]). The salient interpretation of (2) is that of a **biscuit conditional** (term due to Austin's [1] original example "There are biscuits on the sideboard if you want some"), on which the information the consequent provides is relevant for the addressee only in case the antecedent holds. Finally, (3) is most plausibly interpreted as a **conditional hedge**, a kind of disclaimer, on which interpretation the conditional form indicates possible error on part of the speaker, that is the possibility of the consequent not holding, in order to avoid the consequences of providing potentially false information.

There are differences in how the conditional antecedent and consequent relate to each other on the three interpretations, as summarized below. The analysis aims at accounting for these differences while deriving the interpretations the three examples intuitively receive.

Conditional (in)dependence, conditional perfection

The hypothetical conditional in (1) conveys (truth-)conditional dependence between the antecedent and the consequent in the sense that the truth of the consequent is to be evaluated only under the assumption that the antecedent holds. The intuition underlying truth-conditional analyses of such conditionals is that (1) says that the world is such that there's beer only if John went shopping. In other words, antecedent and consequent are *truth-conditionally dependent*.

In the biscuit conditional in (2), on the other hand, there is no such dependence, as the addressee's thirst is irrelevant for the truth of whether or not there is beer in the fridge. Franke (2007) [9] defines this as *conditional independence*, formally implemented as the property of a conditional that it is not sufficient to learn the truth value of the antecedent in order to find out that of the consequent. The intuition I build on is that what biscuit conditionals like (2) assert is the consequent only, which is only relevant (essentially in a Gricean sense, *i.e.* relevant to the conversational goals of the participants) in case the antecedent holds. Following this intuition, previous analyses in the philosophical literature have assumed that what is conditioned on is assertion itself, *i.e.* the consequent is only asserted when the antecedent holds (DeRose and Grandy 1999 [5]), or that biscuit conditionals indicate the existence of a "potential literal act" (such as the assertion that there is beer) in case the antecedent holds (Siegel 2006 [16]). The analysis I propose is also one where assertion is conditioned on, but in a sense closer to extant analysis of conditionals within formal semantics.

The intuitions regarding what the conditional hedge in (3) conditions on are similar to that on the biscuit conditional in that the antecedent targets felicity rather than truth conditions, but different in that the conditional hedge targets Quality (of the speaker's belief that the consequent holds), rater than Relevance (both in a Gricean sense). However, in contrast to both (1) and (2), *conditional perfection* arises from (3): the truth of consequent and antecedent is in a material biconditional (or perfect conditional), relation $\leftrightarrow$ (see Van der Auwera 1997 [2] for discussion): "If there's beer in the fridge, I am not mistaken", while a somewhat odd thing to say, intuitively has the same truth conditions as (3). Below, I develop a formal analysis which is capable of predicting this property of conditional hedges while also being applicable to biscuit and hypothetical conditionals.

2 Conditionals as modals

I take conditionals to be modal constructions, in which the antecedent restricts the modal base of the consequent, following a possible-world semantics analysis of modality in conditionals (*cf.* Kratzer 2012 [13] for an overview), thus accounting for the intuitions on hypothetical conditionals as described in regard to (1) above. I further assume that when there is no overt modal in the consequent, the conditional introduces a covert "*human necessity*" (Kaufmann and Schwager 2009 [12]) modal, the conversational backgrounds being a modal base reflecting the relevant circumstances and a stereotypical ordering source to control for issues like those raised by strengthening of the antecedent.

I take the relevant circumstances to be those relevant to the connection between antecedent and consequent conveyed by the conditional, which in the case of a hypothetical conditional is truth-conditional independence, in the case of non-standard interpretations a connection mediated by Gricean maxims. I further assume the modal base to be the propositions compatible with the speaker's beliefs regarding the relevant circumstances, rather than the relevant circumstances as such, making the modal base doxastic as well as circumstantial.[1] To illustrate the standard interpretation of (truth-)conditional on this view, a paraphrase for the meaning of a hypothetical conditional is given in (4).

(4) A truth-conditional "If Ψ then Φ" is true iff in all worlds (stereotypical, compatible with the speaker's beliefs) in which Ψ holds, Φ also holds.

It is worth noting that the details of what kind of modal base and ordering source are chosen are not central to my analysis of truth- and use-conditionals, as long as the modal force is necessity, and the proposition in the conditional antecedent is used to restrict the modal base, thus excluding worlds in which the antecedent does not hold. This sets the stage on which the conditional consequent is evaluated. Also, my analysis is fully compatible with a double-modal analysis of conditionals (Frank 1996 [8]), on which the modal the consequent introduces a modal base and ordering source potentially different from that introduced by the conditional form.

2.1 Use conditions

The view of use conditions I base my analysis on connects them to truth conditions in the following way. Taking assertion as an example, the meaning of an utterance is split into the descriptive, or truth-conditional, and the expressive, or use conditional, dimension. Truth conditions of a proposition are determined by valuation against worlds. When the proposition (usually one, if possibly complex) in the descriptive, truth-conditional dimension of utterance meaning is true in this sense, this is to say that something true is asserted. When the propositions (typically many) in the expressive, use-conditional meaning dimension are true, this means that the assertion is felicitous. Thus, when the source of the proposition in the expressive meaning dimensions are lexical conventional implicatures (*dog* vs. *cur*, see Gutzmann 2015 [11]) and parentheticals, the utterance's use conditions are straightforwardly determined by the truth or falsity of these propositions, *i.e.* their truth conditions have become use conditions.

In addition to such use-conditional propositions derived from the lexical content, which are valuated against worlds in the same way as truth conditions are, I propose that there are types of propositions in the expressive dimension, which are not valuated against worlds, but evaluated in regard to Gricean

[1] In the paraphrases for the modal base, I will use "the speaker's beliefs" to meand "the speaker's beliefs regarding the relevant circumstances" for brevity.

conversational maxims (Grice 1975 [10]). Concretely, I propose that the entire propositional content in the descriptive dimension is evaluated in such a way in the expressive dimension, and that it is this part of the expressive dimension where conditioning on relevance, quality, etc., in non standard-interpretations of conditionals occurs. This essentially amounts to propositions based on Gricean maxims being valuated in the usual way, and the utterance being felicitous if they hold. According paraphrases for the truth- and use conditions of an assertion with the prejacent proposition Φ are shown in (5) and (6).

(5) Truth conditions: $\textsc{Assert}(\Phi)$ is true w.r.t w iff Φ holds in w.

(6) Use conditions: $\textsc{Assert}(\Phi)$ is felicitous w.r.t w iff Φ is relevant to the participants' goals, as informative as required, backed by adequate evidence,... in w.

Notice that (6) makes no mention of expressive content originating in the lexical content of the utterance. This is for ease of exposition — for the same reason, I will only consider examples where no expressive content arises from the lexical material. Also, no mention is made of the truth of Φ in the use conditions, as I assume that asserting a false proposition is not necessarily infelicitous, provided that the speaker is not aware of its falsity. I will return to this latter point in section 4.1. In this section, I implement the view of use conditions sketched above in a model of indicative conditionals, starting with the standard interpretation.

2.2 Truth-conditionals

The descriptive, truth-conditional meaning of a hypothetical conditional "if Ψ, (then) Φ" in the formalization I assume is shown in (7), where $\|\mathcal{A}(\Phi)\|^t$ stands for the truth-conditional denotation of an utterance where a speech act $\mathcal{A}$ based on the prejacent proposition Φ. As for the notation representing the conditional, $\Box^H$ stands for a human necessity modal as outlined above, w for the actual world, $f(w)$ for the (doxastic / circumstantial) modal base (worlds compatible with the speaker's beliefs regarding the relevant circumstances in w), g for the (stereotypical) ordering source.

The subscript $[\Psi]$ on the modal $\Box^H$ with the conditional consequent Φ in its nuclear scope indicates restriction of the modal base $f(w)$ by the conditional antecedent Ψ, yielding a restricted modal base f^+ in which non-Ψ worlds have been discarded.

(7) $\|\textsc{Assert}(\Box^H_{[\Psi]}[\Phi])\|^t = \|\textsc{Assert}(\Box^H[\Phi])\|^t$ w.r.t w, f^+, g

(where $f^+ = \lambda w.f(w) \cup \|\Psi\|^t$)

The truth-conditional meaning of an assertion of (8) thus comes out as (9)

(8) If John remembered to go shopping, there's beer in the fridge.

(9) (8) is true iff in all worlds (stereotypical, compatible with the speaker's beliefs) in which John remembered to go shopping, there's beer in the fridge.

Next, in order to derive the use conditions of truth-conditionals, I adopt the general framework of Potts (2005) [15], distinguishing between descriptive (=at-issue) or truth-conditional (types marked with superscript a for "at-issue"), and expressive or use-conditional (types marked with superscript c for "conventional implicature") levels of utterance meaning. I use the types t^a and t^c for propositions within the descriptive and expressive meaning dimensions, t^a being the type for truth-conditional content, t^c for lexical CIs and parentheticals.[2]

In addition to these types, I introduce the use-conditional meaning type u^c (utterance) following McCready (2015) [14]. This is the type which I propose is evaluated in terms of Gricean maxims, rather than valuated against worlds in the usual manner for propositions. Also following McCready, elements of type u^c arise from the type-shifting operation *utterance lifting* (UL), by which propositional content, once asserted, is moved into the expressive domain, *i.e.* undergoes type shift from type t^a to type u^c. I use a simplified version of UL as a function from (at-issue) propositions to (expressive) utterances in the present proposal.[3]

(10) $\textsc{ul}^{\mathcal{A}} = \lambda\Phi.\mathcal{A}(\Phi) : < t^a, u^c >$

(where $\mathcal{A}(\Phi)$ is a speech act based on proposition Φ)

With this rule in place, performing a speech act based on a proposition containing no lexical expressives or parentheticals can be represented as in (11), where (a.) shows the two dimensions of meaning before assertion, containing propositional (descriptive, truth-conditional) content only, and (b.) shows utterance meaning[4] after assertion. The representation follows the convention $\langle\tau^a, \tau^c\rangle$ where all truth-conditional elements τ^a which are part of the utterance's meaning are shown on the left, all use-conditional elements τ^c on the right. A parallel representation for a speech act based on a conditional proposition (*i.e.* a truth-conditional) is given in (12).

(11) a. $\langle\Phi, \varnothing\rangle$

b. $\langle\Phi, \mathcal{A}(\Phi)\rangle$

(12) a. $\langle\Box^H_{[\Psi]}[\Phi], \varnothing\rangle$

b. $\langle\Box^H_{[\Psi]}[\Phi], \mathcal{A}(\Box^H_{[\Psi]}[\Phi])\rangle$

In order to derive the use conditions which $\mathcal{A}(\Phi)$ and $\mathcal{A}(\Box^H_{[\Psi]}[\Phi])$ respectively contribute in the expressive dimension, $\mathcal{A}$ needs to be resolved to a specific speech act. I discuss the case of assertions of conditionals below, aiming to arrive at a formalization of use-conditions as paraphrased in (6) above, and to account for the differences between indicative use- and truth-conditionals.

[2] As mentioned, however, the examples will not contain any of the latter, and I remain agnostic in regard to the question of whether or not content of type t^c gets evaluated for felicity in terms of Gricean maxims.

[3] McCready's assumption that u is of a resource-sensitive shunting type u^s, making an additional operation of assertion-to-content necessary to reintroduce at-issue meaning, is ignored here for ease of exposition.

[4] Note that I use the label "utterance meaning" to refer to the meaning of proposition used in a speech act, rather than just for the parts of its meaning of type u^c.

2.3 Use-conditionals

In the case of truth-conditionals, the modal base of the covert modal introduced by the conditional form on truth-conditional level is restricted by the conditional antecedent, reflecting (truth-)conditional dependence. In the case of use-conditionals, not the truth, but the felicity of the consequent depends on the truth of the antecedent. I explain this as restriction of the modal base on the expressive rather than the descriptive level, as paraphrased in (14) for a use-conditional, parallel to the paraphrase for a truth-conditional in (13), repeated from (4).

(13) A truth conditional "If Ψ then Φ" is true iff in all worlds (stereotypical, compatible with the speaker's beliefs) in which Ψ holds, Φ also holds.

(14) A use conditional "If Ψ then Φ" is felicitous iff in all worlds (stereotypical, compatible with the speaker's beliefs) in which Ψ holds, $\mathcal{A}(\Phi)$ is felicitous.

The next question to be addressed is how to evaluate the felicity of $\mathcal{A}(\Phi)$ (which in the examples considered so far is assertion), and where in the derivation of utterance meaning to introduce conditional restriction in order for it to operate on the expressive level of meaning in use-conditionals. I assume that what has to happen compositionally to get the interpretations we are after is what (15) schematically shows for a truth-conditional, (16) for a use-conditional. (17) shows the assumption I make for what happens on the use-conditional level in the case of the speech-act $\mathcal{A}$ being an assertion, namely restriction of the modal base of a non-asserted human necessity modal on the use-conditional level.

(15) $[\mathcal{A}\ [\text{if}\ \Psi\ [\Box^H\ \Phi]]]$ (16) $[\text{if}\ \Psi\ [\text{M}\ [\mathcal{A}\ \Phi]]]$ (17) $[\text{if}\ \Psi\ [\Box^H\ [\mathcal{A}\ \Phi]]]$

Now, the crucial question can be put as follows: what does Ψ restrict when modifying $\mathcal{A}(\Phi)$? In (16), the placeholder is labeled M for modal, anticipating the analysis I propose, but there are other options — one could follow Siegel's proposal and have the conditional antecedent somehow quantify over potential speech acts, or DeRose and Grandy's proposal, making the assertion of φ depend on the truth of the antecedent. I propose an approach I consider more straightforward in light of extant formal theories of conditionals, in which speech acts are neither quantified over nor suspended in this sense. Rather, the modal operator is introduced not on the descriptive, but on the expressive level. In the case of $\mathcal{A}$ being an assertion, only a human necessity modal, as the one familiar from propositions of truth-conditionals, is introduced (other modals could be introduced in the case of speech-acts like imperatives).

The denotation of the two dimensions of meaning of a truth-conditional under standard, hypothetical, interpretation, corresponding to the structure in (15), is shown in (18), that of a use-conditional, corresponding to the structure in (17), is shown in (19) below, alongside the denotation of a plain, non-conditional assertion of Φ in (20). Each representation is shown before (a.) and after (b.) assertion with utterance lifting.

(18) a. $\langle \Box^H_{[\Psi]}[\Phi], \varnothing \rangle$

b. $\langle \Box^H_{[\Psi]}[\Phi], \Box^H_{[\Psi]}[\Phi] \rangle$

(19) a. $\langle \Phi, \varnothing \rangle$

b. $\langle \Phi, \Box^H_{[\Psi]}[\Phi] \rangle$

(20) a. $\langle \Phi, \varnothing \rangle$

b. $\langle \Phi, \Phi \rangle$

The denotation of the use-conditional in (19) is derived as follows. The prejacent Φ of type t^a is asserted and undergoes utterance lifting to u^c. At this point, the human necessity modal introduced by the conditional, with the modal base restricted by the antecedent Ψ, enters the derivation, taking Φ in its nuclear scope. The resulting representation of utterance meaning in (19)b. contains the proposition Φ on the descriptive side, the modal expression $\Box^H_{[\Psi]}[\Phi]$ of type u^c on the expressive side. On the view of use-conditions sketched above, elements of type u^c are evaluated according to Gricean maxims to determine the utterance's use-value, or felicity, here against the modal base restricted by Ψ.[5]

3 Explaining interpretatrions

Note that the felicity conditions of the standard, truth-conditional interpretation shown in (18)b. are the same as those of the non-standard, use-conditional interpretation as shown in (19)b., but the truth-conditions differ for each interpretation. This reflects that basis of evaluation regarding relevance, quality, etc. should be the entire conditional, not just the consequent, in the case of a truth-conditional. In the case of the use-conditional, conditioning only occurs on the expressive, but not on the descriptive level, as the modal introduced by conditional form enters the derivation only after assertion, *i.e.* after utterance lifting has occurred.

In the case of the biscuit conditional in (2), repeated here as (21), this means that the truth conditions of the entire constructions are the same as those of the conditional consequent, and the felicity conditions, where modal base restriction by the antecedent occurs, are as paraphrased in (22).

(21) If you are thirsty, there's beer in the fridge.

(22) Asserting "there is beer in the fridge" is felicitous (w.r.t to w,f^+,g) iff in all stereotypical worlds compatible with the speaker's beliefs *where the addressee is thirsty*, it is relevant to the participants' goals, as informative as required, [...] in w that there is beer in the fridge.

Next, the felicity conditions of the conditional hedge (23), repeated from (3), as predicted in the current proposal are paraphrased in (24).

(23) If I'm not mistaken, there's beer in the fridge.

(24) Asserting "there's beer in the fridge" is felicitous iff in all stereotypical worlds compatible with the speaker's beliefs *where the speaker is not mistaken*,...

[5] See section 4.1 for discussion of the alternative idea that propositions in elements of type u^c are valuated like those of type t^c, but with respect to speaker belief.

This is a special case in that, given a belief of the speaker that there is beer in the fridge, eliminating worlds in which the speaker is mistaken (*i.e.* only considering worlds in which the speaker's beliefs are true) only leaves worlds in which the consequent holds.[6] Thus, when (23) receives a truth-conditional interpretation, its truth depends solely on the existence of a speaker belief that Φ. From this, the intuitive conditional perfection of this example can be predicted: given a speaker belief that Φ, modal base restriction to worlds where Φ holds leaves only worlds in which the speaker is not mistaken. Conversely, given a speaker belief that Φ, restriction to worlds where the speaker is not mistaken leaves only Φ-worlds, as mentioned. Thus, the conditional in (23) is a material biconditional $\leftrightarrow$ on the truth-conditional level. As this case shows, the truth-conditional and the use-conditional interpretations can happily coexist (even though only the use-conditional interpretation seems to be informative).

Choosing an interpretation

As for the question of how it is decided which interpretation a conditional receives, I suggest that the use-conditional interpretation is be preferred when restricting the modal base is uninformative on the at-issue level due to conditional independence. It should be noted here that I assume that both a use- and a truth-conditional reading are in principle available for all conditionals, but that one will usually be more salient. For illustration, consider the following ambiguous example.

(25) If you are interested in art, we will go to the museum.

This conditional has a reading on which the information that the speaker and some third party will go to the museum is relevant if the addressee is interested in art, and a reading on which the speaker intends to take the addressee to the museum only if the addressee is interested in art. The former is a use-conditional, the latter a truth-conditional reading. Different contexts will bring out the salient interpretation. Once we have settled on a use-conditional interpretation, we need to decide which part of felicity, or which Gricean maxim, is targeted by modal base restriction with the antecedent. In the case of a use-conditional interpretation of (25), for instance, relevance is a salient option, if the consequent is intended as an invitation to join.

Examples for ambiguity arising from the possibility of different parts of felicity being targeted include the contrast between discourse-structuring and problem-solving conditionals Csipak (2015) [4] observes. A case where it is not immediately clear which part of felicity is targeted are appropriateness hedges like "If I may be frank…", which target the same part of felicity as parenthetical disclaimers (*cf.* McCready 2015 [14]) which is not easy to grasp with Gricean maxims, but could be subsumed under Manner (a more thorough survey of which aspects of felicity use-conditionals can target has to be left for further research).

[6] On a side note, this is potentially an argument against making the doxastic / circumstantial modal base realistic.

4 Conclusion & outlook

Summing up, I have proposed that the properties of conditionals with salient non-standard interpretations such as biscuit conditionals and conditional hedges can be accounted for in a unified analysis with hypothetical conditionals when assuming that a covert modal, the modal base of which is restricted by the conditional antecedent, can not only be introduced on the descriptive (truth-conditional), but also on the expressive (use-conditional) level. The latter reading is triggered by conditional independence, that is when modal base restriction on the descriptive level does not result in truth conditions that differ from that of the bare consequent, thus rendering restriction uninformative.

In the remainder of this section, I discuss two alternative approaches within the current proposal, followed by possibilities for integration with other analyses and expansion of data coverage.

4.1 An assertion modal

There is an additional possibility for the introduction of a modal which can be modified by the conditional antecedent on the use-conditional level, namely the assumption that assertion introduces a modal like human necessity, with a doxastic modal base. The argument for this goes as follows. Assuming that elements of type u^c, introduced by utterance-lifting, are evaluated by Gricean maxims to determine felicity of the utterance, how does truth factor in? It would be welcome to have a way of including elements of type u^c in the expressive part of utterance meaning in propositional valuation to determine their use-values, in parallel to elements of type t^c. The intuition this is based on is that a proposition is felicitously asserted if the speaker *believes* it is true, whether or not this is actually the case. This can easily be reflected in utterance meaning by assuming that utterance lifting introduces a covert modal with a doxastic modal base and a stereotypical (or possibly empty) ordering source, similar to the human necessity operator introduced by the conditional form. An according representation of a plain assertion before and after utterance meaning is shown in (26), modified from (20).

(26) a. $\langle \Phi, \varnothing \rangle$

b. $\langle \Phi, \Box^H[\Phi] \rangle$

On this analysis of assertion, the paraphrase for use-conditions of assertions given in (6) can be revised as in (27), for simplicity assuming that Gricean quality requires the speaker to believe Φ, so that Φ is required to hold in all worlds compatible with the speaker's beliefs for felicity.

(27) Use conditions (revised): ASSERT(Φ) is felicitous w.r.t. w, f, g iff Φ is *true in all stereotypical worlds compatible with the speaker's beliefs*, relevant to the participants' goals, as informative as required, backed by adequate evidence,...

This modified view of use conditions has another possibly welcome effect on the analysis presented so far: there is no need to assume that the conditional form introduces a human necessity modal on the non-propositional part (assuming that elements of type u^c are no propositions) use-conditional level, just as it does on the truth-conditional, propositional level. Rather, the conditional antecedent, now not restricting a modal base on propositional level, restricts the modal base of the conditional introduced by the speech act assertion.

4.2 Symmetric use- and truth-conditionals

There is a potentially more natural view of use-conditionals (as well as truth conditionals) on which their meanings do not differ from truth-conditionals, only their interpretations do. Above, I proposed that because of what amounts to uninformativity of restriction of the conditional modal base on the truth-conditional, the conditional is interpreted on the use-conditional level in the case of biscuit conditionals and conditional hedges. There seems no harm, however, in assuming that the standard truth-conditional interpretation goes for cases of conditional independence as well, it just happens to have the same truth-value as the consequent. On such a view, truth- and use-conditionals to not only share the same expressive meaning, but also the same descriptive meaning. Just as modal base restriction on the expressive level does not change the use-conditions of hypothetical conditionals, modal base restriction on the descriptive level does not change the truth-conditions of biscuit conditionals and conditional hedges. The differences in interpretation arise as modal base restriction only has an effect on one level of meaning.

There is one possible problem with this view, in form of a difference between the use conditions of, for instance, a biscuit conditional, and those of a hypothetical conditional, in which conditioning occurs on the descriptive level. In the case of the biscuit conditional, conditioning occurs on the expressive level, that is the propositions based on the Gricean paraphrases are valuated against the restricted modal base, yielding the desired interpretation. When conditioning has already happened on the descriptive level, as in hypothecial There is one possible difference between the use conditions paraphrased here and those arising from a hypothetical conditional in which conditioning occurs on the descriptive level. In the case of the biscuit conditional, conditioning occurs on the expressive level, *i.e.* propositions based on the Gricean paraphrases are valuated against the restricted modal base, yielding the desired interpretation. When conditioning has already happened on the descriptive level, as in hypothetical conditionals, the restriction of the modal base potentially happens within the proposition based on Gricean maxims, thus not influencing the evaluation of felicity in the desired way anymore.

If this issue can be resolved, a symmetric approach to use- and truth-conditionals would be a direction to consider for further research as a variant of the present proposal.

4.3 Connecting other analyses, broadening data coverage

An obvious starting point for integration of the present proposal with other analyses of related phenomena, is McCready's (2015) [14] analysis of such parenthetical hedges and disclaimers within the same framework. The proposal also naturally integrates with analyses of conditionals within a framework of modal semantics, and can be readily applied to double-modal analyses of conditionals. I have built on Kaufmann and Schwager's (2009) analysis of conditionals, so that expansion of the scope to the conditional imperatives they analyze is straightforwardly possible. Such an expansion could also shed light on the potential of the alternative paths sketched in the two previous points. Analyses of other non-standard conditionals in a similar framework such as Condoravdi and Lauer's (2015) analysis of (near-)ananakastic conditionals are also possible targets for expansion, especially considering that there are variants of anankastic conditionals which share properties with biscuit conditionals, as discussed in Francez 2015 [7] under the label "chimerical conditionals". Finally, in order to test the limits of which kinds of conditionals can be accounted for on the present analyses, expansion to the non-standard conditionals with properties differing from better-studied ones, as well as sentences "expressing conditional thoughts", but without conditional form, as discussed in Elder and Jaszczolt (2016) [6], is an interesting perspective.

References

1. Austin, J.: Ifs and Cans. In: Philosophical Papers. Oxford University Press (1979)
2. Van der Auwera, J.: Pragmatics in the last quarter century: The case of conditional perfection. Journal of Pragmatics 27(3), 261–274 (1997)
3. Condoravdi, C., Lauer, S.: Anankastic conditionals are just conditionals. Semantics and Pragmatics (2015)
4. Csipak, E.: Free factive subjunctives in German — Ich hätte da eine Analyse. Ph.D. thesis, Georg-August-Universität Göttingen (2015)
5. DeRose, K., Grandy, R.E.: Conditional assertions and "biscuit" conditionals. Noûs 33(3), 405–420 (1999)
6. Elder, C.H., Jaszczolt, K.M.: Towards a pragmatic category of conditionals. Journal of Pragmatics 98, 36–53 (jun 2016)
7. Francez, I.: Chimerical conditionals. Semantics and Pragmatics 8, 1–35 (2015)
8. Frank, A.: Context Dependence in Modal Constructions. Ph.D. thesis, Universität Stuttgart (1996)
9. Franke, M.: The pragmatics of biscuit conditionals. 16th Amsterdam colloquium (2007)
10. Grice, H.P.: Logic and conversation. In: Syntax and Semantics, Vol. 3, Speech Acts, pp. 41–58. Academic Press, New York (1975)
11. Gutzmann, D.: Use-conditional meaning: Studies in multidimensional semantics. Oxford Unviersity Press (2015)
12. Kaufmann, S., Schwager, M.: A unified analysis of conditional imperatives. In: Semantics and Linguistic Theory. vol. 19, pp. 239–256 (2009)
13. Kratzer, A.: Modals and Conditionals. Oxford University Press (2012)
14. McCready, E.: Reliability in pragmatics. Oxford University Press (2015)
15. Potts, C.: The Logic of Conversational Implicatures. Oxford University Press (2005)
16. Siegel, M.E.A.: Biscuit conditionals: Quantification over potential literal acts. Linguistics and Philosophy 29(2), 167–203 (apr 2006)
17. Stalnaker, R.C.: A theory of conditionals. In: Rescher, N. (ed.) Studies in Logical Theory, pp. 41–55. Blackwell (1968)

Negotiating Epistemic Authority*

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Abstract. Why do we trust what other people say, and form beliefs on the basis of their speech? One answer: they are taken to have *epistemic authority*. Intuitively this means that the other person (or institution, or group) is taken to be authoritative in what they say, at least with respect to a particular domain. Here, we want to claim that there are (at least) two varieties of epistemic authority, one based on reliability and one on assuming (nonepistemic) authority. We claim that both are subject to linguistic negotiation. This paper begins by reviewing McCready's (2015) theory of reliability, and then turns to strategies for attempting to assume epistemic authority, focusing on those involving the use of not-at-issue content. We then show the results of two experiments which test the interaction of stereotypes about gender with epistemic authority, and how this is mediated by language use, focusing on the case of gendered pronouns. Finally, the results are explored for Bayesian views of argumentation and analyzed within McCready's Reliability Dynamic Logic.

1 Introduction

Why do we trust what other people say, and form beliefs on the basis of their speech? One answer: they are taken to have *epistemic authority*. Intuitively this means that the other person (or institution, or group) is taken to be authoritative in what they say, at least with respect to a particular domain. Here, we want to claim that there are (at least) two varieties of epistemic authority, one based on reliability and one on assuming (nonepistemic) authority. We claim that both are subject to linguistic negotiation. This paper begins by reviewing McCready's (2015) theory of reliability, and then turns to strategies for attempting to assume epistemic authority, focusing on those involving the use of not-at-issue content. We then show the results of two experiments which test the interaction of stereotypes about gender with epistemic authority, and how this is mediated by language use, focusing on the case of gendered pronouns. The first experiment concerns English and the second Cantonese. Finally, the results are explored

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for Bayesian views of argumentation and an analysis is sketched within McCready's Reliability Dynamic Logic.

2 Passive assumption of authority

One way to be authoritative, in the sense of having one's speech consistently believed, is to be a speaker who is judged reliable with respect to speaking truth. If one is judged reliable, one is likely to have a kind of epistemic authority, in the sense that the things one says are likely to be believed. Here, reputation is key given that belief is a form of cooperation; it is known, for the game-theoretic case. that the use of reputation in strategizing in repeated Prisoner's Dilemma (18; 19) yields extremely good results, and is therefore likely to be evolutionarily stable.

One way to model reputation with respect to reliability is given by (13), which we will briefly summarize. On this theory, reputations can be derived in part from *histories*, defined as sequences of objects $act \in A, A$ the set of possible actions for a given agent in a given (repeated) game. These objects are records of an agent's actions in past repetitions of the game. Game histories are n-tuples of sequences of records representing the history of the agent's actions at each decision point. For the case of communication, these are of course histories of speech acts. A player's reputation in a game is derived from his history in that game. A player's reputation with respect to some choice as his propensity, based on past performance, to make a particular move at that point in the game. Such propensities are computed from frequencies of this or that move in the history. Specifically, the propensity of player a to play a move m in a game g at move i is: the proportion of the total number of game repetitions that the player chose the action m at choice point i.

$$F_{H_a^{g,n}}(move) = \frac{card(\{act \in H_a^{g,n} | act = move\})}{card(H_a^{g,n})}$$

Always, $0 \leq F_{H_a^{g,n}}(move) \leq 1$, so the above number can be viewed as a probability: in effect, the information that the game participants have about a's likelihood of choosing move m.

An agent's propensity to play a strategy is a real number in $[0, 1]$. This fact supports a scalar view of propensities, and indeed of cooperation itself: an agent has a propensity for using strategy σ iff i.e. the contextual standard for having that propensity (9). Thus,

$$Prop(a, \sigma) \text{ iff } F_{H_a^g}\sigma > s,$$

where s is the contextual standard for propensity-having. These propensities can also be used to decide whether to assign someone epistemic authority with respect to some claim. In the context of the repeated PD, (18; 19) make use of reputations and find that there are optimal strategies, given an index of reliability (here in [0,1], but for them in the range 1–5), is to trust if, for example, a has a propensity for reliability (where this for them amounts to setting some arbitrary number above which cooperation is dictated), or if $\sum_{Coop(\sigma)} F_{H_a^g}\sigma$ is above some threshold (not necessarily s) (for the sum of frequencies of all a's cooperative strategies), or if the other agent's reliabilty index is higher than yours. Since such strategies are public, the other agent has an incentive to maintain her R-rating high: i.e. to genuinely be reliable. Any of the above seem reasonable bases to choose to accept someone's epistemic authority, or not.

The above must be combined with other information about reliability. This is so because of the need to decide whether to give someone epistemic authority even in the first communication, before any kind of history is available. This decision corresponds closely to the distinction between Humean and Reidian views on trust in testimony (13; 15). One way to model the Reidian view, on which decisions about trust aren't made automatically but rather on the basis of some metric, is that of (4), who takes speakers to make judgements about people's epistemic authority based on stereotypical information about factors like their gender, race, occupation, and personal grooming. This seems sensible: one might be more likely to believe a clean-shaven man in a suit about his having had his wallet stolen and needing money for the train than the same statement made by a homeless woman carrying a bottle in a brown paper bag (depending of course on one's other beliefs). This heuristic gives a first guess about reliability which can then be modified by interaction.

All this can be embedded in a more general model of information change; (13) proposes a new flavor of dynamic semantics for this purpose (6). The basic idea is to virtually always update with content acquired from any source, but only 'conditionally.' To make this work, information states σ are complex and consist of possibly many substates. Each IS is a set of worlds (simplification), ordered with a 'plausibility ranking' reflecting epistemic preferences on states. Each substate is indexed by an index $j \in \mathsf{Source} \cup \mathcal{A}$. Here Source is the set of evidence sources and $\mathcal{A}$ the set of agents, which is constrained to only hold indices which the epistemic agent has had experience with. This set is ordered by a total ordering $\leq_a$, where $i \prec j$ iff $P(Rel(i)) < P(Rel(j))$, when $P(Rel(i))$ is the probability that source i yields reliable information.

Updates are of the form $E_i\varphi$, for E_i an operator indicating source in i-type evidence. A sentence $E_i\varphi$ always induces update of state σ_i. Some cases are indeterminate cases, such as the use of direct evidentials in some languages that have them, where it may not be clear what the source is: visual, auditory, ... In such cases, all possible substates are updated. But in the testimonial case, states indexed with agentive sources a are updated. So, at the level of substates, update with φ always takes place when φ is observed — but this is *not* the same as coming to believe φ at a global level. Global beliefs are defined on the global state σ_T resulting from unifying all substates σ_i. This unification is done via a merge operation ($\Cap$): all substate content survives when non-contradictory, but in case of conflict, information from higher-ranked sources trumps lower-ranked source-indexed information. Thus the global state almost never exhibits conflicts; it only will if two sources are precisely equally ranked, which is unlikely given the range of real values, but can be explicitly banned by enforcing a version of Lewis's Limit Assumption, here for sources rather than worlds (10).

More formally, global information states σ: consist of sets of elements (substates) of the form $\sigma_i = \langle X, \leq_a \rangle$ where $X \subseteq W$ (the set of states). The substates are plausibility frames in the sense of (1; 2): multi-agent Kripke frames $\langle X, R_a \rangle_{a \in \mathcal{A}}$, where the accessibility relations R_a are called 'plausibility orders', written $\leq_a$, and assumed to be locally connected preorders. This simplifies a bit: sometimes the substates can be more complex, in particular in the case of testimonial agents, as the substates associated with them also have a similar structure. Total information states are written σ_T, and are of the form $\langle X, \leq_a \rangle$ for $X \in \wp(W)$. They are derived by recursively merging all plausibility relations found in $\sigma_i \in \sigma$ via a lexicographic merge operation, which respects priority ordering; so an agent's beliefs thus are derived on the basis of the most reliable source, and so on down the source hierarchy. From this, we get resolution in cases of conflicting sources.

Update in this system follows the $[.]_{\Uparrow}$ of (1; 2), defined as follows.

– $\sigma[\varphi]_{\Uparrow} = \sigma'$, where $S' = S$ and $s \leq'_a t$ iff either (i) $s \notin \varphi$ and $t \in s(a) \cap \varphi$, or (ii) $s \leq_a t$.

This definition thus leaves the set of states the same, but upgrades those states which satisfy φ above those which don't, otherwise leaving the relative plausibilities untouched. Using this operation ensures that substates will be comparable without recourse to revision.

Support and entailment are defined as follows. A total information state $\langle X, \leq_a \rangle$ is said to *support* a proposition φ, $\sigma \models \varphi$, iff $\{s \in X | s \in best_a(s(a))\} \subseteq \phi$, where $best_a\phi := \{s \in \phi | t \leq_a s \text{ for all } t \in \phi\}$.[3] The definition of entailment is the standard fixed-point dynamic one modulo the use of $[.]_{\Uparrow}$, as defined above (with ';' dynamic conjunction as usual):

$$\phi_1, \ldots, \phi_n \models_\sigma \psi \text{ iff } \sigma[\phi_1]; \ldots ; [\phi_n] = \sigma[\phi_1]; \ldots ; [\phi_n]; [\psi].$$

Evidential update is defined via the following clause, which ensures that only the substate corresponding to the information source is updated, and all others are left alone.

1. $\sigma[\mathsf{E}_i\varphi] = \sigma'$ where, for all $\sigma_j \in \sigma$, $\begin{cases} \sigma'_j = \sigma_j[\varphi] & \text{if } i = j \\ \sigma'_j = \sigma_j & \text{if } i \neq j \end{cases}$

For an example, suppose agent a learns $\varphi =$'It is raining' from evidence source b (agent b). Then: $\sigma' = \sigma$ except that $\sigma'_b \in \sigma' = \sigma_b[\varphi]$, by the definition of evidential update.

Thus: in all cases, the result of evidential update with φ is belief in φ. But this belief may just be belief relative to the source, i.e. within σ_i for source i. 'Genuine' belief requires global belief wrt the global state. Essentially: $B_a\varphi$ iff $\{s \in \sigma_T | s \in best_a(s(a))\} \subseteq \phi$, where $best_a\phi := \{s \in \phi | t \leq_a s \text{ for all } t \in \phi\}$. The total belief state is derived by lexicographic merge, so the content of our examples will be believed unless some higher-ranked source disagrees. What happens when a conflict arises? Consider a case of conflicting agents. Agent a claims ϕ and agent b claims $\neg\phi$. a, let's suppose, is pretty trustworthy. b is unknown; let's suppose that he looks somewhat untrustworthy. The result is that $a > b$ in the priority ordering for lexicographic merge. Thus the merge of σ_a and σ_b verifies ϕ.

So far: update of substates, substates unified via merge, merge priority determined by ordering. But what's the source of the ordering? Without a substantive theory of how the ordering is derived, the theory seems to have little empirical content. The claim of

[3] Note that this is essentially identical to the definition of belief in (**??**).

(13) is that the ordering is probability-based. The probabilities in question are probabilities of *reliability*. They indicate the (perceived) likelihood that information derived from the source is correct.

These probabilities arise from two factors. The first factor is experience with reliability of the source, as derived from histories; the second is the initial probabilities of reliability. These come in two types: prior beliefs about the reliability of different evidence sources, and beliefs about the reliability of the providers of testimony based on various aspects of their presentation. For an example of the first, one generally can take direct evidence to be more reliable than hearsay: if I see that it's raining outside, I am likely to discount the fact that this morning's weather report said it would be sunny. For the second, as mentioned above, judgements about the reliability of individuals are often made on the basis of stereotypical factors about their appearance and how they are categorized (4). One might judge the kempt to be more reliable than the unkempt, the professional to be more reliable than the amateur, or someone from the same social group as you to be more reliable than someone from an outgroup. As we'll see in the next section, these kinds of judgements can be manipulated, yielding effects on the attribution of epistemic authority.

The two factors are taken to interact as follows: given an initial probability and a sequence of events of information acquisition, conditionalize on the initial probability for each new acquisition event, with respect to truth-tracking. The idea is to modify the probability that the source is reliable based on whether the new information is correct or not:

$$\frac{P_I(R \cap C)}{P_I(C)}$$

The whole notion of authoritativeness analyzed here is (in a sense) a passive one. One becomes authoritative by speaking the truth and by looking reasonably trustworthy. This is a kind of authority acquired by being a good citizen in the testimonial sense, essentially that of (5). But is there a more active way to acquire epistemic authority by linguistic means? We think yes: by use of argumentative and other linguistic devices. Some of these will be explored in the next section.

3 Using expressive content for authority negotiation

How can one actively try to acquire epistemic authority (or deny it to others), as opposed to simply acquiring it by living a virtuous testimonial life? One way, of course, is just to assert one's authority:

(1) (You should believe me because) . . .
  a. I know all about this topic.
  b. I'm your teacher.
  c. I'm your dad.

This strategy will be effective to precisely the degree that the speaker already has epistemic authority, because in the absence of epistemic authority, either the hearer won't accept what is said (1a), or the speaker's external authority is already rejected (1b,c). Consequently, a less direct strategy (or set of strategies) is needed. In the remainder of this section, we examine the use of expressive content (21) in the assumption of epistemic authority, considering several cases.

We are choosing to focus on expressive content for two reasons. Expressive content is often talked about as 'inflicted' on the hearer (21; 16), which means (if correct) that the content of the expressive cannot easily be contested. This is an important feature when it comes to manipulations of epistemic authority (and in argumentation in general), as it removes the need to have epistemic authority already in order to have one's claims accepted, as with (1) above. This feature is not universally present in not-at-issue content either; (26) notes that presuppositions for example can be challenged in discourse, meaning that their content lacks the key feature of expressives we are interested in here. The second reason is the close connection of many expressives to social meanings, which are obviously relevant for epistemic authority. This point will be detailed as we proceed.

In this section, we will briefly consider the cases of particles, honorifics, and, finally, our main concern, those expressives which serve to indicate membership in various social groups.

First, particles like the Japanese *yo* (with falling intonation) work to try to 'force' the hearer to accept the content of the sentence (11; 3). Indeed, (17) presents an analysis of this particle in terms of epistemic authority. His idea is that *yo* indicates that the speaker has at least as much epistemic authority as anyone else present with respect to the content of the sentence. This implies that the particle can be used strategically to try to claim such epistemic authority for the speaker; use of the particle (if unchallenged) indicates that the speaker already has epistemic authority.

This view has some empirical effects. In the following example, the speaker requests belief via the claim of teacherhood.

(2) watashi-wa anata-no sensei desu yo
1P.Formal-Top 2P.Formal-Gen teacher Cop.Hon PT
'I am your teacher, don't forget.'

However, the use of strengthening *yo* implicates that the speaker doesn't have authority already, which further implies that the speaker takes his epistemic authority qua teacher to be insufficient, resulting in a failed authority grab. Compare here the observation of (25) that falling *yo* infelicitous in e.g. instructions from a commanding officer in the army, because the attempt at claiming authority represented by *yo* (in the terms of this paper) is not compatible with the presence of absolute authority.

The second case is honorifics, which, although they on a separate dimension from epistemic claims (at least according to (8; 22; 12; 14), and others), to the extent that one's social status influences her epistemic authority the use of (anti-)honorifics should count as a strategy for assuming it, or taking it from others. Notably: 'raising' the addressee could cede some epistemic authority to them. In terms of examples, while the following are both grammatical and felicitous, there is a sad mismatch between content, honorific tone and particle: it's as if the speaker is desperately trying to assert himself. This is unlikely to yield genuine epistemic authority.

(3) watashi-no itteiru koto-o shinjite kudasai yo
1P.Formal-Gen saying thing believe please.Pol PT

'Believe what I'm saying, please.'

vs the pure authority grab:

(4) ore-no itteru koto-o shinjiro
1P.Inf-Gen saying thing believe-Imp
'Believe what I'm saying!'

Finally, many expressives tag aspects of character which can be relevant to determinations of epistemic authority via social status; we can call these *social expressives*. This strategy is less direct than the above in that it is entirely a side effect. The main method here is to ascribe other individuals membership in groups which are or are not privileged in a social sense, and use that (lack of) privilege to implicate something about their epistemic authority. The same is true for slurs: by placing the addressee or other individual in a subordinate group, explicitly or implicitly (cf. (24)), it becomes possible to emphasize one's own epistemic authority over them. It is widely noted in the feminist philosophy literature (and elsewhere on the internet etc.) that the overt or covert primary position of males in society, and their consequent authority, can lead to differences in epistemic authority as well. For instance, the claims of men are often believed over the claims of women, all else being equal. If this is true, the use of e.g. gendered 2P pronouns in situations where other options are available (cf. (23)) could lead to the changes in who is taken to have epistemic authority, meaning that the use of gendered language can be a strategy for its assumption.

Here, we are interested in testimony: the main question in ceding epistemic authority involves how one should assign probabilities of likely reliability to individuals.

As mentioned above, (4) cites one technique, which is to make use of stereotypes about groups, for example that 'women are not logical', 'Asians are well educated', and so on; she presents some compelling examples of such cases, though examples which operate at the level of at-issue claims rather than expressive implications. However, many expressives tag aspects of character which can be relevant to determinations of epistemic authority via social status. We can call these *social expressives*; they are mainly terms which categorize individuals into categories that — at least on a stereotypical or prejudicial level — are relevant to the (non)attribution of epistemic authority. The basic method is to ascribe other individuals membership in groups which are associated with some stereotype, and then use that (lack of) privilege to implicate something about their epistemic authority.

Two examples of social expressives are slurs and gendered language. By definition, slurs are negative and subordinating (cf. (24)), so can be used to emphasize one's own epistemic authority over categorized individual, given that other relevant individuals share the prejudices the slurs express. With gendered language, the situation is more subtle, because gender is not in any sense pejorative in the way of slurs. Still, the deployment of stereotypes about gender to acquire epistemic authority. It is a truism (and a common claim in feminist philosophy as well (4)) that the overt or covert primary position of males in society, and their consequent authority, can lead to differences in epistemic authority as well. For example, it is often said that the claims of men are often believed over the claims of women, all else being equal. If this is true, the use of e.g. gendered 3P pronouns could easily lead to the changes in who is taken to have epistemic authority, meaning that the use of gendered language can be a strategy for its assumption. In order to see whether this is correct, we conducted several experiments, focusing on the use of gender stereotypes in argumentation.

4 Experiments: gender in argumentation

4.1 Experiment 1: English

We ran an experiment to test the relation between gendered speech and epistemic authority in argumentation. We tested two different types of argument which involve the authority of a source: the direct, or abusive, form of the *ad hominem* argument and the argument from authority (or position to know). Schematically these arguments are as follows (27):

– Ad-hominem:
  - Source *a* is a person of bad character / has bad character for veracity
  - *a* argues that α
  - **Conclusion**: α should not be accepted
– Argument from authority (position to know):
  - Source *a* is in a position to know about things in a certain subject domain *S* containing proposition *A*
  - *a* asserts that *A* is true
  - **Conclusion**: *A* is true

In each case, the source *a* is part of one of the premises of the argument.

The goal of the experiment was to test whether manipulating the gender of the source induces a difference in the convincingness of the argument. To test this a protocol similar to the one used by (7) to investigate the argument from authority was used.

First, a preliminary experiment was run to determine three distinct sets of topics according to their gender bias. This was done as a categorization task on Amazon Mechanical Turk. Participants were presented with a topic and asked to choose which category most closely matched that topic: Men, Women or Both. 17 topics in total were tested, out of which 15 were selected, 5 in each group. Each topic had an agreement of 80% or above, meaning that four participants agreed the topic was associated with the relevant category. Participants could categorize multiple topics and were paid 0.05 USD for each categorized topic.

These topics were then used to produce 15 distinct arguments, in two major forms: the ad hominem one, and the argument from authority one. Examples of each form follow (using a male biased topic):

– Authority argument
  - A and B are friends. A wants to buy a power drill and is thinking about which one to buy. A wants a high performance drill to perform heavy duty work.
  - *A:* I wonder if this one is a good choice.
  - *B:* I have a friend who says he knows a lot about power tools, and he says this model is really powerful.
– *Ad hominem*

- A and B are friends. A wants to buy a power drill and is thinking about which one to buy. A wants a high performance drill to perform heavy duty work.
- *A:* I heard from Jamie that this model is really powerful.
- *B:* She doesn't know anything about it.

Besides the argument scheme, two independent variables were tested: the gender of the source (by using *he*, *she* or *that friend/Jamie* to refer to it) and the gender bias of the topic, based on the results of the preliminary experiments. 450 US-based participants were recruited on the Amazon Mechanical Turk and paid 0.2 USD for their participation. They judged the convincingness of 5 different arguments (4 fillers+1 target item) presented in pseudo-random order. Convincingness was rated on a 5 point Likert scale. Linear mixed effect models with maximal random effect structure were fitted to the data using the lmer package in R. Effects of condition and group were confirmed by likelihood-ratio tests.

The results show that generally, authority arguments are judged more convincing than *ad hominem* ($\chi^2 = 145.38, p < 0.01$) and that the gender of the source and the gender bias of the topic have no main effect. Further analyses showed that these variables have no effect in the case of the *ad hominem* argument. However, the results of the argument from authority show that there is a significant interaction between the gender of the source and the gender bias of the topic ($\chi^2 = 11.023, p = 0.026$). It was observed that men are generally more trusted for topics biased towards men (which is expected) but that women are not more trusted than men for topics biased towards women, and that there was no preference for neutral topics.

Discussion To explain why authority arguments are preferred to *ad hominem* ones, we argue that when considering authority arguments the only question is how reliable the source of the argument is. The reliability of the speaker is not directly relevant. This can readily be integrated in an approach like that of **(author?)** (7, 20) who propose a Bayesian treatment of argumentation. In that approach, the convincingness of an argument is proportional to how much the content of the argument affects the audience's prior belief in the conclusion targeted by the argument. The reliability of the source is factored in the likelihood of using an argument a to target a conclusion C. There the speaker's reliability remains constant across possible sources and does not weigh in on the evaluation of the argument.

However in the case of the *ad hominem* argument the speaker's reliability is at odds with that of the source, which might explain why those arguments are generally dispreffered since they put the speaker's credibility against that of the source. As stated above, gender biases can integrated into the Bayesian approach of argumentation. This amounts to modifying the belief that source is reliable by conditionalizing on its gender. However in the Bayesian approach the *ad hominem* and authority arguments are seen as dual to each other: one lowers the reliability of the source while the other increases it. As such, it should be expected that both forms would equally be affected by gender biases, contra the results of our experiment. One way to model that difference would thus be to explicitly distinguish between the reliability of the speaker and that of the source of an information in the way an argument is evaluated, and the fact that they may potentially be at odds. The approach lends itself to such a modification, but further experimentations are needed to validate whether this move is an effective way to account for the data presented here.

4.2 Experiment 2: Chinese

A second experiment, similar to the one presented above, was run using material in Chinese rather than English. Mandarin Chinese does not have gendered pronouns, but in its written form uses characters which distinguish between male and female referents for third person pronouns.Cantonese does not have gendered pronouns either, and the character commonly used to transcribe the Cantonese third person pronoun is not gendered either. The goal of the experiment was to check whether the use of a gendered character or a neutral one affected the way subjects evaluated the convincingness of an argument.

The overall protocol of the experiment was similar to the one described above, except that the participants were speakers of Cantonese and were recruited among the students of the Education University of Hong Kong and participated voluntarily. A first categorization task was run to identify the gender biases of different topics. Nine topics (3 male oriented, 3 female oriented and 3 neutral) were retained to be used in an experiment testing the convincingness of an argument. Speakers of Cantonese are able to read written Chinese fluently and feedback from some of the participants confirmed that they did not realize that some of the experimental material was written using Cantonese specific characters or Mandarin Chinese specific ones.

88 participants judged the convincingness of 9 pseudo-randomized different arguments (6 fillers+3 target items) on a 5 point Likert scale. Since gendered effects were only observed on authority arguments in the English experiments, only authority arguments were used in the Chinese experiment. An online questionnaire was hosted on the `IbexFarm` platform and the link sent to participants. Linear mixed effect models with maximal random effect structure were fitted to the data using the lmer package in R. Effects of condition and group were confirmed by likelihood-ratio tests.

The analysis of the results shows no significant effect, either of the gender bias of the topics, the gender of the source or of their interaction. This suggests that, if participants have gender biases, the use of gendered character did not trigger theses biases when participants evaluated the arguments. Further investigations will be made by changing the expression referring to the source by one that is explicitly gendered (e.g. *my sister/daughter* etc.) in order to see whether such expressions make biases appear in a way that is comparable with the English experiments.

5 Conclusion

This paper has reviewed the analysis of reliability of (13) — a combination of stereotype-based probability ascriptions and examination of communicative histories — and proposed it as one means of acquiring epistemic authority. The other method is more proactive: to manipulate stereotypes and other aspects of the context via the deployment of expressive content. We looked at one such instance in detail via experimental methods:

the use of gendered pronouns to influence judgements about reliability, both in English and Cantonese. The results are intriguing, but still preliminary.

We propose to model these phenomena in the RDL logic proposed by McCready (2015). In that model, individuals are assigned credibilities on the basis of stereotypes (among other things). We propose to extend the model by allowing individuals to have credibilities with regard to specific content, typically individuals of gender g will have a high credibility for g-relevant topics. But before fully formalizing the details, a better understanding of the phenomena is however needed. The case of men being more credible for men-biased topics comes out naturally from the gender-based credibility model proposed above. However, the fact that the converse is not true for women is more problematic. One way to model this is to consider that, by default, men enjoy a higher credibility than women, which then gets cancelled in the case of women-biased topics. That possibility will be investigated in the near future via an additional experiment, and the basic data will be verified using a larger population of subjects as well.

Bibliography

[1] Baltag, A., Smets, S.: A qualitative theory of dynamic belief revision. In: Bonanno, G., van der Hoek, W., Wooldridge, M. (eds.) Logic and the Foundations of Game and Decision Theory, pp. 13–60. No. 3 in Texts in Logic and Games, Amsterdam University Press (2008)
[2] Baltag, A., Smets, S.: Talking your way into agreement: Belief merge by persuasive communication. In: Baldoni, M., Baroglio, C., Bentahar, J., Boella, G., Cossentino, M., Dastani, M., Dunin-Keplicz, B., Fortino, G., Gleizes, M.P., Leite, J., Mascardi, V., Padget, J.A., Pavón, J., Polleres, A., Fallah-Seghrouchni, A.E., Torroni, P., Verbrugge, R. (eds.) MALLOW. CEUR Workshop Proceedings, vol. 494. CEUR-WS.org (2009)
[3] Davis, C.: Decisions, dynamics and the Japanese particle *yo*. Journal of Semantics 26, 329–366 (2009)
[4] Fricker, M.: Epistemic Injustice. Oxford University Press (2007)
[5] Grice, H.: Logic and conversation. In: Cole, P., Morgan, J. (eds.) Syntax and Semantics III: Speech Acts, pp. 41–58. Academic Press, New York (1975)
[6] Groenendijk, J., Stokhof, M.: Dynamic predicate logic. Linguistics and Philosophy 14, 39–100 (1991)
[7] Hahn, U., Harris, A.J., Corner, A.: Argument content and argument source: An exploration. Informal Logic 29(4), 337–367 (2009)
[8] Harada, S.: Honorifics. In: Shibatani, M. (ed.) Japanese generative grammar, pp. 499–561. Academic Press, New York (1976)
[9] Kennedy, C.: Vagueness and gradability: The semantics of relative and absolute gradable predicates. Linguistics and Philosophy 30(1), 1–45 (2007)
[10] Lewis, D.: Counterfactuals. Basil Blackwell, Oxford (1973)
[11] McCready, E.: What man does. Linguistics and Philosophy 31, 671–724 (2008)
[12] McCready, E.: A semantics for honorifics with reference to Thai. In: Aroonmanakun, W., Boonkwan, P., Supnithi, T. (eds.) Proceedings of PACLIC 28. pp. 513–521. Chulalongkorn University (2014)
[13] McCready, E.: Reliability in Pragmatics. Oxford University Press (2015)
[14] McCready, E.: The semantics and pragmatics of honorification (2015), manuscript, AGU
[15] McCready, E.: Rational belief and evidence-based update. In: Hung, T.W., Lane, T.J. (eds.) Rationality: Constraints and Contexts, pp. 243–258. Elsevier (2016)
[16] Murray, S.: Varieties of update. Semantics and Pragmatics 7(2), 1–53 (2015)
[17] Northrup, O.: Grounds for Commitment. Ph.D. thesis, UCSC (2014)
[18] Nowak, M., Sigmund, K.: The dynamics of indirect reciprocity. Journal of Theoretical Biology 194, 561–574 (1998)
[19] Nowak, M., Sigmund, K.: Evolution of indirect reciprocity by image scoring. Nature 393, 573–577 (1998)
[20] Oaksford, M., Hahn, U.: Why are we convinced by the ad hominem argument?: Bayesian source reliability and pragma-dialectical discussion rules. In: Zenker, F. (ed.) Bayesian Argumentation, pp. 39–58. Springer, NL (2013)

[21] Potts, C.: The expressive dimension. Theoretical Linguistics 33, 165–198 (2007)
[22] Potts, C., Kawahara, S.: Japanese honorifics as emotive definite descriptions. In: Proceedings of SALT XIV (2004)
[23] Schlenker, P.: Maximize presupposition and Gricean reasoning. Natural Language Semantics 20, 391–429 (2012)
[24] Stanley, J.: How Propaganda Works. Princeton University Press (2015)
[25] Suzuki Kose, Y.: Japanese Sentence-Final Particles: A Pragmatic Principle Approach. Ph.D. thesis, University of Illinois at Urbana-Champaign (1997)
[26] von Fintel, K.: Would you believe it? the King of France is back! In: Reimer, M., Bezuidenhout, A. (eds.) Descriptions and Beyond. Oxford (2004)
[27] Walton, D.N., Reed, C., Macagno, F.: Argumentation Schemes. Cambridge University Press, Cambridge (2008)

Lessons Learned from a Prototype Implementation of Montagovian Lexical Semantics

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Abstract. We present a general-purpose implementation of the process of lexical semantics analysis theorised in the Montagovian Generative Lexicon ΛTY_n (hereafter MGL). The prototype software itself serves as a proof of concept of the MGL theory. The implementation process, the data structures and algorithms, also provide valuable results as to the expressive power required by MGL. While the implementation of terms and types for the purpose of meaning assembly assumed by MGL is in itself straightforward, some lexical phenomena imply additional mechanisms in order to process the logical representation using implicit knowledge. We therefore also present a minimal architecture for knowledge representation, and how it can be applied to different phenomena.

Keywords: Lexical Semantics, Prototype Software, Montagovian Generative Lexicon, Knowledge Representation for Natural Language Semantics.

1 Theories and Implementations of Lexical Semantics

Formal lexical semantics theories aim to integrate to the toolbox of compositional analysis of natural language developed since Montague considerations of (logical) polysemy. Based on original studies such as [4,8], then on a theory thoroughly developed in [22], there have been many formulations that build upon powerful type-theoretic foundations, with a generative, dynamic account of the lexicon at their heart. Such recent type-theoretic accounts of lexical meaning include Type Composition Logic (TCL) presented in [1], Dynamic Type Semantics (DTS) presented in [3], Type Theory with Records (TTR) presented in [7], Unified Type Theory (UTT) presented in [12], and the Montagovian Generative Lexicon (MGL) presented in [24].

Several partial or complete implementations of those theories have been provided for demonstration purposes, using logical or functional programming, or theorem provers such as Coq ([6] is an example among many). Concerning MGL, however, one of the stated goals was (paraphrasing slightly [24]) to provide an integrated treatment from syntax to semantics extending existing analysers based on Montagovian semantics such as [19] with mechanisms for lexical semantics that *are easily implemented in a typed functional programming language like Haskell*. Our goal in this publication is to present an actual prototype implementation (using functional and object programming in Scala) of the lexical semantics of that framework.

We detail some of the necessary data structures and algorithms used, what we learned from this implementation on the underlying logic properties of MGL, and sketch an architecture for simple knowledge representation that is necessary for the representation of certain lexical phenomena. The demonstrably functioning prototype illustrates both the validity of type-theoretic formulations of lexical meaning, and the deep interaction of lexical meaning with at least some sort of knowledge representation already evoked in [5].

2 A MGL Prototype

2.1 The Montagovian Generative Lexicon

MGL makes use of ΛTY_n (an adaptation of the many-sorted logic TY_n proposed in [21] in second-order λ-calculus, given in the syntax of System-F). The idea is to perform an usual Montague analysis (performing syntax analysis via proof-search and substituting semantic main λ-terms to syntactic categories). Lexical mechanisms are then implemented in the meaning-assembly phase via a rich system of types based on ontologically different sorts and optional λ-terms that model lexical adaptation. The mechanisms, given in Fig. 1, can be summarised as follows:

- First, the input utterance is super-tagged and analysed using categorial-grammar mechanisms, which is the only step of proof-search of the process, yielding a syntactic term whose components are syntactic categories. The lexicon is then used in standard Montagovian fashion to substitute λ-terms, yielding a main semantic term, typed with many sorts and the type **t** for propositions.
- Second, as a many-sorted logic is used, some type mismatches might (and should) occur, allowing mechanisms of lexical semantics to disambiguate between terms. The lexicon provides *optional* λ-terms that are used as *lexical transformations*. These optional terms are inserted depending on their typing and yield a λ-term with no type mismatches.
- Finally, β-reduction yields a normal, η-long λ-term of type **t** (the type of propositions), i. e. a logical formula that can be used in any usual semantics, such as model-theoretic or game-theoretic semantics.

As the first step is already well-studied and implemented, the object of concern is the second step: given a term reflecting the syntactic structure of an utterance, to construct a semantic λ-term in a many-sorted logic, making use of available transformations, and yielding a suitable formula. This is the object of our prototype implementation.

2.2 Modelling Types and Terms

The data structures and algorithms responsible for implementing the terms and types of ΛTY_n are the core mechanisms of the software. They are given as two Scala sealed abstract classes, `TermW` and `TypeW`, with a flat hierarchy of case classes implementing the various possible terms and types; this simple categorisation allows us to easily construct and detect patterns of objects.

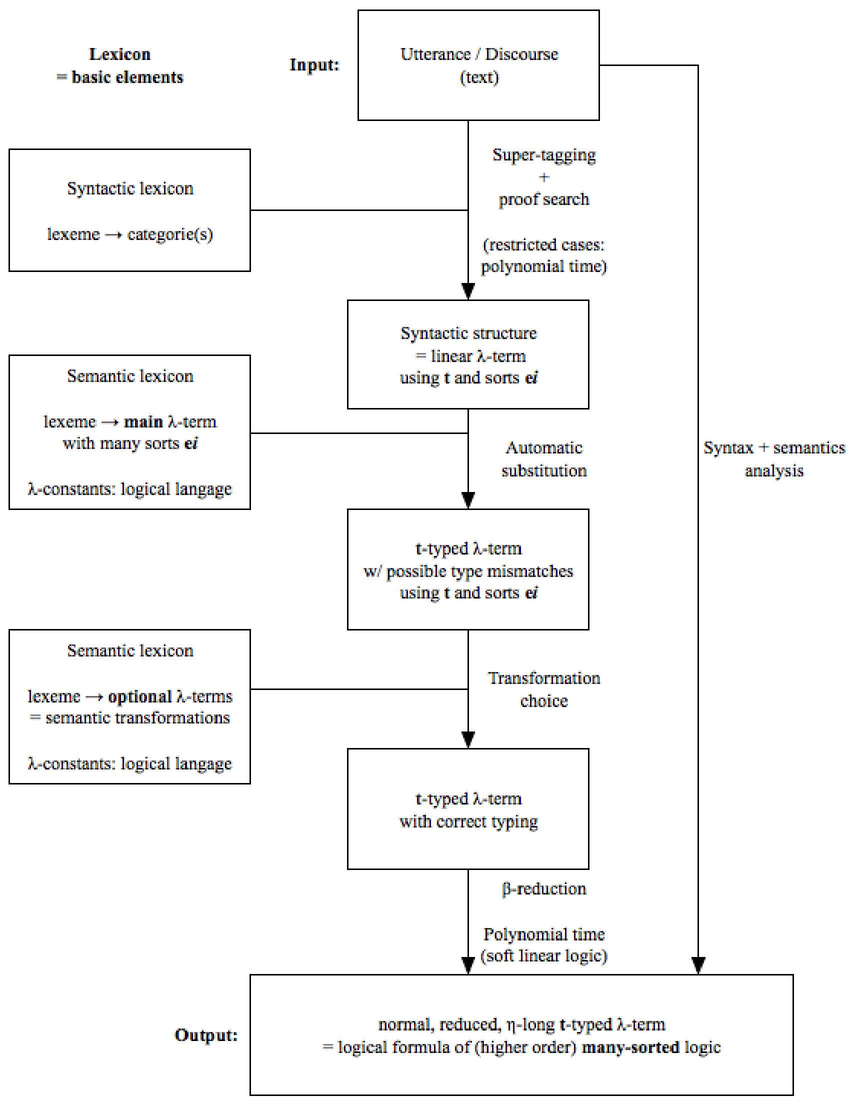

Fig. 1. MGL Process Summary

Terms and types are constructed as binary trees (abstractions and applications of more than one argument to a given term/type can be easily curryied).

For terms, leaves are `AtomicTerms` (constants), `TermVarIds` (variables) with an identifier and type, or specific `Transformations` and `Slots`, while inner nodes are `TermBindings` (λ-abstracted terms), or `TermApplications` of a predicate and argument. For types, leaves are constant `Sorts`, pre-defined objects such as `PropositionType` for **t**, or second-order variable identifiers `TypeVarIds`, while nodes are `TypeFunctions` between two types A and B modelling $A \to B$, or `TypeApplications` modelling $A\ \{B\}$.

A simplified UML class diagram presents this straightforward architecture in Fig. 2.

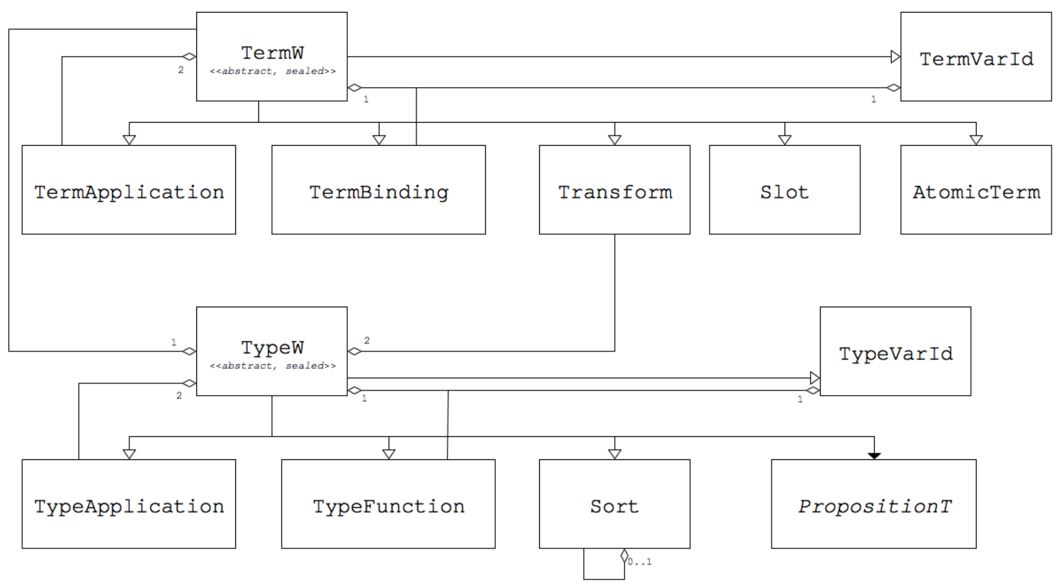

Fig. 2. Class diagram of the core package for terms and types.

Several algorithms are provided in order to work with types and terms; they are mostly simple recursive tree-walking algorithms, making the most of memoisation when possible (e. g., lists of available ressources are incrementally built as terms and types are constructed in order to minimise computations). Algorithms include the type-checking of applications, comparison between types, automated α-conversion of variables in order to prevent issues of scope, replacement of term and type variables, β-reduction, and the automated specialisation of types for polymorphic terms (i. e., a predicate with a type containing one or several type variables will be specialised to the correct types if applied to an argument with a compatible but specified type).

Most are linear in complexity (with exceptions in the adaptative mechanisms below); all algorithms are at most polynomial in time.

2.3 Explicit Adaptation

The core of MGL is to provide *transformations* as optional terms, on top of the main λ-term associated to each lexeme. A canonical example is *the book is heavy and interesting*. Supposing three basic sorts:

- R for readable materials,
- φ for physical objects,
- I for informational contents;

the book can be modelled as the_book^R, *heavy* as $\text{heavy}^{\varphi\to\mathbf{t}}$, *interesting* as $\text{interesting}^{I\to\mathbf{t}}$. The example utterance is a case of *co-predication*, as two predicates are simultaneously asserted on two different *facets* (with different, incompatible sorts) of a same object, and MGL will resolve this by having the lexicon provide two optional terms associated with *book* in order to access these two facets: $f_{phys}^{R\to\varphi}$ and $f_{info}^{R\to I}$. A polymorphic conjunction $\&^{\Pi} = \Lambda\alpha\Lambda\beta\lambda P^{\alpha\to\mathbf{t}}\lambda Q^{\beta\to\mathbf{t}}\Lambda\xi\lambda x^{\xi}\lambda f^{\xi\to\alpha}\lambda g^{\xi\to\beta}.(\text{and}^{\mathbf{t}\to\mathbf{t}\to\mathbf{t}}\ (P\ (f\ x))(Q\ (g\ x)))$ is needed for the co-predication. This yields, after suitable substitutions, application, and reduction, the term (and (heavy(f_{phys} book)) (interesting (f_{info} book))), which is normal and of type $\mathbf{t}$.

In our implementation, there are several important differences with the theory outlined above. As we distinguish between type constants and variables, there is no need to explicitly abstract types. This is because the only second-order operation ever used in ΛTY_n is specialisation (i. e., the replacement – or instantiation – of type variables). Moreover, second-order (type) variables are all introduced by λ-bound first-order (term) variables.

We also distinguish between term variables that are necessary for the definition of an abstracted term (such as P, Q and x, the two predicates and the argument of the conjonction above) and *adaptation slots*, the places where optional λ-terms (such as f and g above) can be inserted. This is because the optional terms can be provided by various different mechanisms, and might not be provided at all if the term is well-formed (there is no lexical adaptation taking place in utterances such as *heavy and black rock*; in that case, MGL provides an useful, if slightly redundant, *id* optional polymorphic term that can be inserted in order to get the identity on any type).

We provide optional terms as `Transforms`, which are distinguished from other terms. Each term has a list of available transformations, constructed recursively from the leaves (the transformations available to atomic terms should be given in the lexicon). We also distinguish `Slots` for explicit adaptations; the list of slots is maintained during the construction of the terms. Our polymorphic *and* conjunction then becomes:

```
lambda p^(B->t).lambda q^(G->t).lambda x^A.((And^{(t->(t->t))}
    (p^(B->t) (f^{(A->B)} x^A))) (q^(G->t) (g^{(A->G)} x^A)))
```

During the attempted resolution of the application of the conjunction to terms for *heavy* and *interesting*, the polymorphic *and* is specialised to sorts representing φ and I, and cannot be reduced further with the application of the argument *book*. A further algorithm is provided in order to model the choice of transformations, trying to match all available transformations to the adaptation slots. As all permutations are considered, this is potentially the most costly computation taking place. The result is a *list* of possible interpretations (given as term applications with slots filled by transformations): there might be zero, one, or finitely many. A further check on the list of terms obtained will filter those, if any, with a suitable typing, that will form the desired result(s). In the tests conduced with the input of the example, four interpretations where produced, with only the correct one of a resolvable type (**t**):

```
((And^{(t->(t->t))} (heavy^{(P->t)} (morph_R->Phy^{(R->P)}
                                                   book^{R})))
    (interesting^{(I->t)} (morph_R->I^{(R->I)} book^{R})))
```

2.4 Implicit Adaptation

Polymorphic operators such as *and*, with explicit adaptation slots, are needed for co-predications. However, most lexical adaptations can take place implicitly, simply by reacting to a type mismatch such as $(p^{A\to B}\ a^C)$ and applying any suitable transformation to resolve the type mismatch. In order to do this automatically, there are two possibilities to resolve such type mismatches: by adapting the predicate, yielding $((f^{(A\to B)\to(C\to B)})\ p)\ a)$, or the argument, resulting in $(p\ (f^{C\to A}\ a))$. There is also a third situation to consider, that of a partial application $(\lambda x^A.\tau\ a^C)$, in which the argument can be adapted as above, but the typing of the predicate might be as not be determined at the moment of the adaptation.

A procedure analyses such applications with type mismatches and no explicit adaptation slots, and inserts suitable, automatically generated adaptation slots, then proceeds as with explicit adaptations. For example, a simple term application such as `(P^{(e->t)} a^{A})`, with a transformation `f_{A->e}` available to the atomic term `a` yields the straightforward (and only felicitous) interpretation `(lambda x^A.(P^{(e->t)} (f_{A->e}^{(A->e)} x^A)) a^{A})`, that reduces to `(P^{(e->t)} (f_{A->e}^{(A->e)} a^{A})`.

Implicit adaptations are necessarily reduced to those simple cases. Trying to account automatically for co-predications would imply to try any possible permutation of types and transformations at all nodes of a term, which would be exponential in complexity; thus, the need for explicit operators such as the polymorphic *and*.

2.5 Lexicalisation

In addition to the core mechanisms, a `tecto` package provides support for a tectogrammatical/syntactic structure in the form of an unannotated binary tree of lexemes ; this serves as a factory for the input of already analysed text, and as a more streamlined form of output for adapted terms.

A `lexicon` package enables the storage of lexical entries that associate lexemes (as strings) to terms, complete with typing, transformations and ambiguities. Lexica can be merged, in order to have combine the treatment of different phenomena, treated as standalone modules, for complex sentences. Lexica also provide automated translations from a syntactic structure (a `tecto` term) to a semantic one (a `TermW` term, initially not adapted, reduced or even type-checked). Semantic terms can be presented either by a straightforward translation to syntactic terms, or printed to a string in the usual fully-parenthesed prefix notation with apparent typing (as in the examples of this article).

2.6 Phenomena Coverage

Many lexical phenomena discussed in [22,13] can be modelled using the simple mechanisms of ΛTY_n in their prototypal implementation given above; some others require additional mechanisms.

Lexical adaptations, including alternations, meaning transfers, grinding, qualia-exploitation and "Dot-type"-exploitation are all supported by the adaptation mechanisms, as given previously. *Simple predications* only require to have suitable transformations available, and to use the implicit adaptation mechanisms ; *co-predications* require explicit adaptation using polymorphic operators. Theoretical grounds have been laid in [2,24].

Constraints of application are required in order to perform co-predications correctly. As explained in [17], the simultaneous reference to different facets of a same entity can be infelicitous in some circumstances, such as the use of destructive transformations (grinding, packing) or metaphorical use of some words. Thus, the following co-predications are infelicitous to some degree : **The salmon was lighting-fast and delicious*, *? Birmingham won the championship and was split in the Brexit vote*. In order to block such co-predications, we have proposed to place constraints on transformations in order to block their usage depending on the other transformations that have been used on the same term. The first version of this system given in, e. g., [2], distinguishes between *flexible* (allowing all other facets) and *rigid* (blocking all other facets) transformations. The latest version, given in [14], proposes a $\Lambda^{\multimap}TY_n$, a system with terms of the linear intuitionistic logic as types, that (among other things) allow any arbitrary type-driven predicate to act as a constraint on the use of transformations.

In this prototype implementation, all transformations come with a member function that can be defined as a constraint, and a compatibility check of all transformations can be performed using every constraint, the default constraint being the boolean constant `true` (that simply models flexible transformations). As the constraint can effectively be any function, the precision is the same as in [14].

Ontological inclusion, called *type accommodation* in [22] and modelling the lexical relation of hyponymy, can be supported by tweaking the system of sorts. The theoretical and empirical basis for doing so are discussed in [18], in which we argue that *coercive sub-typing* is an accurate and helpful mechanism for resolving ontological inclusion, but no other lexical phenomena.

In order to support sub-typing, each sort can be defined with an optional `parent` sort. A careful review of the typing comparison mechanism will then be enough, together with a rewriting of the equality method for sorts, in order to support subtyping. This is not implemented yet, but does not require (much) additional processing power.

Performative lexical adaptations, such as quantification, Hilbert operators for determiners, and the alternate readings of plurals and mass nouns, are supported as far as the meaning assembly phase is concerned. However, in order to be useful, this category of lexical phenomena (as well as hypostasis and several others) require additional mechanisms in order to incorporate the knowledge gathered from the analysis of the sentence into the logical representation. The basic architecture is supported, but mechanisms of resolution remain preliminary and will be discussed next, especially in Section 3.3.

3 Layers of Lexica and Knowledge Representation

3.1 The Additional Layers

Theories of semantics deriving from [22] generally encompass some degree of common sense world knowledge: it is considered known that a *committee* (and other such group nouns) is made of several people and is a felicitous argument of predicates requiring a plural argument, and that *engine*s are part of *car*s and thus that predicates such as *powerful* or *fuel-guzzling* can apply to *car*s via their constitutive quale. It has been argued (e. g. in [9]) that such complex knowledge does not belong in a semantic lexicon; we will paraphrase Im and Lee from [10], defining semantics to be the meaning conveyed by an utterance to a competent speaker of the language in itself, excluding, for instance, the specific situation in which the utterance is made, but including any previous discourse. Thus, the full contents of a given fairy tales should be able to be described within semantics, while a political essay will probably require additional knowledge about the position of the author and the specifics of the period of writing.

From our point of view, designing a complete tool for type-theoretic lexical semantics imply the careful definition of various lexica that can convey the necessary, elementary world-knowledge for each word. A lexicon for general use will associate to all relevant lexemes their semantics (in the form of main and optional λ-terms) as can be given in a dictionary of a language. However, there are two common cases in which the general lexicon is not sufficient.

First are the specific lexica: vocabularies relevant only to a community (professional jargons, local dialects, and other linguistic constructs specific to small groups of people), and/or to a specific literary universe (fairy tales, space opera, mythology, politic speeches, etc.). Such lexica are activated on an as-needed basis, switched often, and are more specific than the general-use lexicon.

Lexical semantics also requires a lexicon used for the current enunciation. A competent speaker of any language is able to use generative mechanisms in order to introduce new lexical concepts, either by hypostasis (the use of a new word, the meaning of which can be inferred from context and morphology), or by creative use (giving a new, contextually evident meaning to an existing word).

In our view, the lexicon of the enunciation starts empty and can be augmented when the analysis of the discourse encounters words that are not present in the current active lexica. We think that such mechanisms can enable the *learning* of lexical semantic data. In addition to these lexical layers of meaning, we tend to implement different lexical phenomena using different lexica for simplicity's sake, and create a merged lexicon from every relevant one when processing text.

3.2 Individuals, Facts and Contexts

To summarise our argument in Section 3.1 above, in addition to mostly static lexical data, some sort of knowledge representation is needed to process even simple lexical phenomena such as collective and distributive readings for plurals. Namely, we need to keep track of the *individuals* mentioned in a given discourse, and of the *facts* asserted of those individuals. To be complete, we would also need to keep track of *agents*, in order to model dialogues or multiple points of view in which certain agents assert certain facts. Our implementation prototype currently supports individuals, as atomic terms of type A (for named entities: human agents, towns...) or $A \to \mathbf{t}$ (for common nouns, that can be resolved to a specific individual of type A by the means of an Hilbert-based determiner) for any sort A. We also account for facts, as predicates (`TermBindings` or atomic terms) of type $\alpha \to \mathbf{t}$ for any arbitrarily complex type α, that are used in a term application, and apply to an individual. In the analysis of a term, individuals and types are extracted and added to the context of enunciation. The hierarchy of lexical layers given above can be implemented as a hierarchy of *contexts*, some containing initial individuals and facts relevant to each lexicon; in a such complete system, the context of the real world would, to resolve the paradox mentioned in [25], include the fact that there is no King of France (and therefore that *The king of France is bald*, while grammatical, is not felicitous because there are no qualifying referents for the entities described, and thus cannot be assigned a truth value). Such contexts are specific objects (aggregating individuals, facts and a related lexicon) in our implementation.

3.3 The Parsing-Knowledge Loop

We use a specific lexicon to list some common semantic terms for quantifiers, counting terms, logical and Hilbert operators (detailed in e. g. [23], more recently in [16]). Other lexica can make use of these terms in order to construct, for instance, Link-based semantics for plurals (originally given in [11]), using lexical transformations as suggested by [20] and detailed in [15]. Some functions associated to the logical lexicon then resolve the operators, given a term and a context. This updated process of analysis is given in Fig. 3.

To explain what the analysis of plural readings in MGL entail, consider the following example from [15] : *Jimi and Dusty met* is analysed as $|\lambda y^e.(y=j) \vee (y=d)| > 1 \wedge meet(\lambda y^e.(y=j) \vee (y=d))$. One elementary issue is that the predicate *met* applies to group individuals (such as *a committee*) and constructions made of more than one individuals (such as *Jimi and Dusty*) but not to singular individuals (such as *a student*). Thus, the lexical entry for the predicate is $\lambda P^{e \to t}.|P| > 1 \wedge meet(P)$ – a logical conjunction with a cardinality operator.

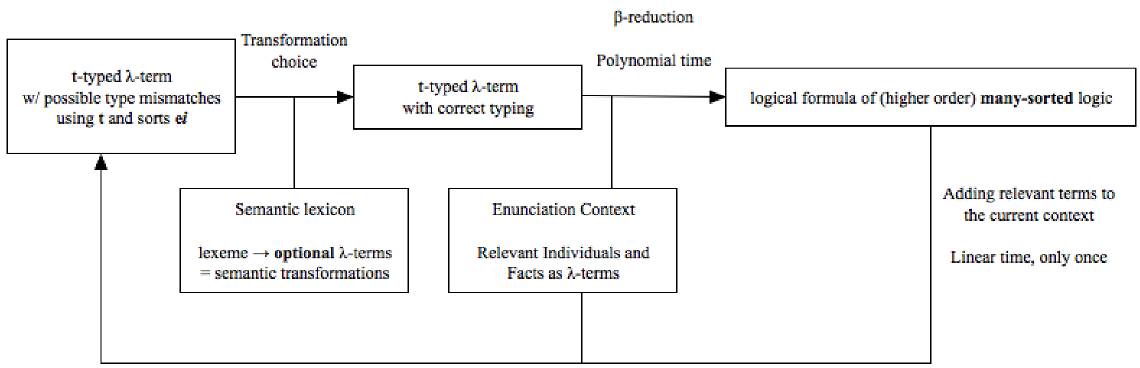

Fig. 3. Parsing-Knowledge Representation Feedback

Those two simple elements can be defined in System-F (the calculus in which ΛTY_n, the logic of MGL, is implemented). The issue is that, in order for our system to infer correctly that Jimi and Dusty are two different individuals, and thus that the above term resolves to $meet(\lambda y^e.(y = j) \vee (y = d))$, we must use processing power beyond the simple construction and reduction of terms: a minimal system of knowledge representation and logical inference. Within our architecture encompassing individuals and facts, and with a functional lexicon for logical connectives (including the logical *and* operator of that example), as well as quantification and counting (including the cardinality operator), this example can be treated.

However, this requires a given term to be parsed at least twice: the first time, the syntactic structure is converted into a semantic term and lexical transformations are applied, the second, facts that emerge from the transformations are added to the lexicon, and the logical lexicon can be used in order to process the operators that have been introduced. Our prototype implementation does not incorporate such feedback yet, as the first step can result in several different interpretations; this remains a work in progress. As a result, straightforward composition for plurals are tentatively supported (such as in the previous example), but ambiguous covering readings for plurals are not yet available.

3.4 Hypostasis and Quantificational Puzzles

An enunciation-context lexicon that is filled with individuals and facts inferred from the primary semantic analysis can serve, in a limited way, to account for *hypostasis*. Words absent from the lexicon, but syntactically placed in the position occupied by individuals, will be added as primary entities to the lexicon, and their precise typing inferred from the predicates they are applied to. An elementary mechanism should be enough to have a correct representation from Lewis Carroll's *Jabberwocky*. Of course, most competent human speakers also use morphosyntactic inference to attach at least some degree of connotative meaning to the words being proposed (e. g., *Star Wars*'s *plasteel* can be inferred as a fictive material somehow combining the characteristics of plastics and steel by any English speaker).

This is completely beyond the power of our early software. Rather, we can have the process of meaning assembly outline which lexemes are not in the lexicon, and use human input for correcting the precise types and terms associated.

The process of counting, quantifying and selecting entities using Hilbert operators can also shed some light on the quantificational puzzles mentioned in [1] and several other related works. The issue with having universal quantification used together with co-predication on multi-faceted entities can be seen in examples such as *There are five copies of* War and Peace *and a copy of an anthology of Tolstoï's complete works on the shelf* (what is the answer to questions such as *How many books… ?*, and what exactly is the type of *book* in such questions ?), or *I read, then burnt, every book in the attic* (the entities being predicated form two different sets). In order to resolve such quantificational puzzles satisfactorily, the methods for counting and quantifying must be adapted to each predicate, and only apply to individuals of the appropriate type. For our purpose, this implies a close monitoring of the entities introduced by lexical transformations and their context of appearance. This is also a work in progress.

4 Results

4.1 A Fragment of Second-Order

We have proven that MGL can actually be computationally implemented. This was not really in doubt, but the way that the combination of types and terms are implemented illustrates that the time and space complexity of most of the process is limited: the algorithms used are mostly linear tree walks, with a few quadratic worst-case operations. The most complex step is the choice of optional terms for adaptation slots, of complexity $|t| \times |s| \times n$ at worst (the product of the number of optional terms available, adaptation slots, and length of the term); the hypothesis behind MGL is that the number of available optional terms at any point remains manageable. Thus, the step not actually implemented in this prototype (but for which many implementations exist), syntactic analysis, is the costliest of the process detailed in Fig. 1 and the complete process of parsing is polynomial in time.

MGL accounts such as [24] point out that the whole expressive power of second-order λ-calculus is not used, and that all could be implemented using first-order terms if all possible adaptations were listed at each step (which is syntactically much longer to write). Indeed, our implementation only supports the single second-order operation of type specialisation (by distinguishing type variables from other types and using pattern matching to recognise and rewrite types), which is required for having polymorphic terms. There are no features of ΛTY_n that require additional power: sub-typing can be implemented by an optional `parent` field in `Sorts`, arbitrary complex on co-predications are supported by including a check on transformations that can be any arbitrary function, quantification, counting and Hilbert operators can be included…

4.2 Minimal Processing Architecture

Our prototype implementation includes the skeleton of an architecture that represents the individuals, facts and agents appearing during the semantic analysis.

This goes beyond the straightforward process of producing a logical representation for an utterance, as some of the terms of that logical representation might be analysed differently depending on the context; we argue that that process is still part of a semantic analysis. The individuals, facts and agents are stored in objects called *contexts*, organised in a hierarchy that includes the most specific context (modelling the analysis of the current discourse), universe-specific contexts (describing whether the discourse is part of a fictional, historical or activity-specific setting), dialect- and language-specific contexts, each associated to an appropriate lexicon. A complete analysis would minimally involve the construction of the logical representation of an utterance, the update of the enunciation context with individuals and facts introduced by that utterance, and a re-interpretation of the logical representation in the active contexts. This minimal processing architecture can be completed with no difficulties; our implementation includes relevant data structures and algorithms, but requires significant work on examples of performative lexica in order to be thoroughly tested.

4.3 Perspectives

This prototype implementation has already served its primary purpose: to illustrate that MGL can be computationally implemented, and that the examples usually given with the theory actually work. As it is, however, this implementation is more of a proof of concept than useful software.

To be actively used by the community, more work would be required to give it an helpful interface, both for the user and for existing analysers; we also would like to convert from and to representations of the other most active type-theoretic accounts of lexical semantics. The knowledge-representation architecture remains a work in progress, and requires solid efforts in order to correspond to our ambitions. However, what MGL really requires in order to be useful is a large-cover library of types and terms; our hope is that this prototype will help to build software that can learn those features from corpora.

References

1. Nicholas Asher. *Lexical Meaning in Context: A Web of Words*. Cambridge University Press, March 2011.
2. Christian Bassac, Bruno Mery, and Christian Retoré. Towards a Type-Theoretical Account of Lexical Semantics. *Journal of Language, Logic, and Information*, 19(2), 2010.
3. Daisuke Bekki. Dependent Type Semantics: An Introduction. In Zoé Christoff, Paolo Galeazzi, Nina Gierasimczuk, Alexandru Marcoci, and Sonja Smet, editors, *Logic and Interactive RAtionality (LIRa) Yearbook 2012*, volume I, pages 277–300. University of Amsterdam, 2014.
4. M. Bierwisch. Wördliche Bedeutung - eine pragmatische Gretchenfrag. In G. Grewendorf, editor, *Sprechakttheorie und Semantik*, pages 119–148. Surkamp, Frankfurt, 1979.
5. Peter Bosch. The bermuda triangle: Natural language semantics between linguistics, knowledge representation, and knowledge processing. In *Text Understanding in LILOG, Integrating Computational Linguistics and Artificial Intelligence, Final Report on the IBM Germany LILOG-Project*, pages 243–258, London, UK, UK, 1991. Springer-Verlag.

6. Stergios Chatzikyriakidis and Zhaohui Luo. Natural language inference in coq. *J. of Logic, Lang. and Inf.*, 23(4):441–480, December 2014.
7. Robin Cooper. Copredication, dynamic generalized quantification and lexical innovation by coercion. In *Fourth International Workshop on Generative Approaches to the Lexicon*, 2007.
8. D. A. Cruse. *Lexical Semantics*. Cambridge, New York, 1986.
9. J. A. Fodor and E. Lepore. The emptiness of the lexicon : Reflections on James Pustejovsky's *The Generative Lexicon*. *Liguistic Inquiry*, 29(2), 1998.
10. Seohyun Im and Chunngmin Lee. A developed analysis of type coercion based on type theory and conventionality. In Robin Cooper and Christian Retoré, editors, *ESSLLI proceedings of the TYTLES workshop on TYpe Theory and LExical Semantics*, Barcelona, August 2015.
11. Godehard Link. The logical analysis of plurals and mass terms: A lattice-theoretic approach. In P. Portner and B. H. Partee, editors, *Formal Semantics - the Essential Readings*, pages 127–147. Blackwell, 1983.
12. Zhaohui Luo. Contextual analysis of word meanings in type-theoretical semantics. In *Pogodalla and Prost*, pages 159–174. 2011.
13. Bruno Mery. *Modélisation de la Sémantique Lexicale dans le cadre de la Théorie des Types*. PhD thesis, Université de Bordeaux, July 2011.
14. Bruno Mery. Lexical Semantics with Linear Types. In *NLCS '15, the Third Workshop on Natural Language and Computer Science*, Kyoto, Japan, July 2015.
15. Bruno Mery, Richard Moot, and Christian Retoré. Computing the Semantics of Plurals and Massive Entities using Many-Sorted Types. In Tsuyoshi Murata, Koji Mineshima, and Daisuke Bekki, editors, *New Frontiers in Artificial Intelligence JSAI-isAI 2014 Workshops, LENLS, JURISIN, and GABA, Kanagawa, Japan, October 27-28, 2014, Revised Selected Papers*, volume 9067 of *Lecture Notes in Artificial Intelligence*, page 357. Springer-Verlag Berlin Heidelberg, 2015.
16. Bruno Mery, Richard Moot, and Christian Retoré. Typed Hilbert Operators for the Lexical Semantics of Singular and Plural Determiner Phrases. In *Epsilon 2015 – Hilbert's Epsilon and Tau in Logic, Informatics and Linguistics*, Montpellier, France, June 2015.
17. Bruno Mery and Christian Retoré. Recent advances in the logical representation of lexical semantics. In *NCLS – Workshop on Natural Language and Computer Science, LiCS 2013*, Tulane University, New Orleans, June 2013.
18. Bruno Mery and Christian Retoré. Are books events ? Ontological Inclusions as Coercive Sub-Typing, Lexical Transfers as Entailment. In *LENLS '12, in jSAI 2015*, Kanagawa, Japan, November 2015.
19. Richard Moot. Wide-coverage French syntax and semantics using Grail. In *Proceedings of Traitement Automatique des Langues Naturelles (TALN)*, Montreal, 2010.
20. Richard Moot and Christian Retoré. Second order lambda calculus for meaning assembly: on the logical syntax of plurals. In *Coconat*, Tilburg, Netherlands, 2011.
21. Reinhard Muskens. Meaning and Partiality. In Robin Cooper and Maarten de Rijke, editors, *Studies in Logic, Langage and Information*. CSLI, 1996.
22. James Pustejovsky. *The Generative Lexicon*. MIT Press, 1995.
23. Christian Retoré. Sémantique des déterminants dans un cadre richement typé. *CoRR*, abs/1302.1422, 2013.
24. Christian Retoré. The Montagovian Generative Lexicon Lambda Ty_n: a Type Theoretical Framework for Natural Language Semantics. In Ralph Matthes and Aleksy Schubert, editors, *19th International Conference on Types for Proofs and Programs (TYPES 2013)*, volume 26 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 202–229, Dagstuhl, Germany, 2014. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.
25. Bertrand Russell. On denoting. *Mind*, 14(56):479–493, 1905.

On the interpretation of dependent plural anaphora in a dependently-typed setting

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Abstract. Anaphora resolution may involve a reference to a dependency relation between objects. One typical example is a dependent interpretation of pronoun *it* in the mini-discourse *Every boy received a present. They each opened it*, which is a well-known phenomenon in the literature of plural anaphora. The standard way to account for dependent interpretation is to record dependency relations by sets of assignment functions (van den Berg, 1996a,b; Nouwen, 2003; Brasoveanu, 2008). This approach, however, has to make substantial changes to the central notion of context in a way that is specialized for the treatment of dependent interpretation. In this paper, we provide an alternative account from the perspective of dependent type theory. We handle dependency relations in terms of dependent function types (Π-types), which are independently motivated objects provided in dependent type theory (Martin-Löf, 1984). We will adopt dependent type semantics (Bekki, 2014) as a semantic framework and illustrate how dependent function types encode dependency relations and naturally provide a resource for dependent interpretation.

1 Introduction

Interpretation of pronouns can be sensitive to linguistically introduced dependency relations between objects. Consider the following examples (van den Berg, 1996a,b; Krifka, 1996; Nouwen, 2003; Brasoveanu, 2008)[4]:

(1) a. If every^1 boy received a^2 present, they$_1$ opened it$_1$.
  b. Every^1 boy received a^2 present. They$_1$ opened it$_2$.

In (1a), given a reading where *every boy* receives a wide scope over *a present* (henceforth, $\forall$–$\exists$ reading), the whole sentence can mean that if every boy received

[4] An anaphor is subscripted by index, while its antecedent is superscripted by the same index.

a present, each boy opened the present which he received. Similarly, the second sentence in (1b) can be understood to mean that each boy opened the present he received. In both cases, $\forall$–$\exists$ reading induces a dependency relation between boys and presents. This quantificational dependency plays a crucial role in interpreting the singular pronoun *it* in the consequent of (1a) and the second sentence of (1b).

More generally, the reference to dependency relations is possible when a semantic link between the restrictor of the universal quantifier and the subject of a subsequent discourse can be established.

(2) a. If every boy receives a^1 present, some boy will open it_1.
    b. Every boy will receive a^1 present. Some boy will open it_1.

(3) a. If every boy receives a^1 present, every young boy will open it_1.
    b. Every boy will receive a^1 present. Every young boy will open it_1.

(4) a. If every boy receives a^1 present, John will open it_1.
    b. Every boy will receive a^1 present. John will open it_1.

In (2), the subject of the subsequent discourse, *(some) boy*, is the same noun phrase as the restrictor of the universal quantifier. Thus, it can mean that some boy will open a present where he receives (attributed to Lauri Karttunen in Hintikka and Carlson, 1979; Ranta, 1994). In (3), *young boy* is a subset of *boy*. Again, a similar interpretation is allowed (van den Berg, 1996b). There is no explicit link in the case of (4), but if we have the background information that John is a boy, i.e., the information that links *John* to *boy*, it can mean that John will open the present which he received. Conversely, when it is difficult to establish such a link, the dependent interpretation of pronoun in question seems to be inadequate. See the contrast between (5a) and (5b).

(5) a. Every man will receive a^1 present. Some wife will open it_1.
    b. Every man will receive a^1 present. *Some woman will open it_1.

In (5a), since it is relatively easy to find a relation between *man* and *wife*, the second sentence can be understood to mean that some man's wife will open the present he received. On the other hand, the similar reading is not possible in (5b) unless a strong relation between *man* and *woman* is provided in the context.

Another similar example is an instance of so-called *quantificational subordination*, which is originally discussed by Karttunen (1976).

(6) a. Harvey courts a^1 girl at every convention. She_1 is very pretty.
    b. Harvey courts a^1 girl at every convention. She_1 always comes to the banquet with him. The_1 girl is usually also very pretty.

Although this example is much more complicated than what we had so far, the similar structure seems to be involved here. (6a) can only mean that there is one specific girl such that Harvey courts her at every convention and she is very pretty. If we discard this reading and force ourselves to keep $\forall$–$\exists$ reading, there is no way to establish an anaphoric link between *a girl* and the singular pronoun *she*. However, *she* can refer to a girl in each convention in (6b), since the subsequent discourse contains quantificational adverbs such as *always* and *usually*, which provide links to *every convention*.

The observation above suggests that a dependency relation between objects should be tracked through a discourse as an anaphoric resource, in order to be referred to later on to interpret pronouns. Since the reference to dependency relations is crucially involved in phenomena of plural anaphora in general, constructing a formal mechanism to account for dependencies is one of the central issues in the dynamic semantics literature. The standard way is to model it as sets of assignments (van den Berg, 1996a,b; Nouwen, 2003; Brasoveanu, 2008). Another proposed approach is to model it by a more complex notion of assignment function called *parametrized sum individuals* (Krifka, 1996). However, since it is not straightforward to integrate functional relations directly into the underlying semantics, both approaches have to make substantial changes to the central notion of context in a way that is specialized for the treatment of plural anaphora.

In this paper, we propose an alternative account. We handle dependency relations in terms of *dependent function types* in dependent type theory. In contrast to the mechanisms introduced in previous model-theoretic approaches, dependent function types are independently motivated objects that are provided in dependent type theory from the beginning. We will adopt dependent type semantics (Bekki, 2014, henceforth DTS) as a semantic framework and illustrate how dependent function types encode dependency relations in question and be naturally provided as anaphoric resources in discourse. In the following section, we will first provide an overview of DTS. In section 3, we will provide a central idea of handling the reference to dependency relations and show how it can be applied to those examples mentioned above. Comparison of our approach with previous works will be provided in section 4.

2 Dependent type semantics

2.1 Dependent types and natural language sentences

DTS (Bekki, 2014) is a proof-theoretic natural language semantics based on dependent type theory. Dependent type theory (Martin-Löf, 1984) is a formal system that extends simple type theory with a notion of *types depending on terms*. There are two type constructors Π (dependent function type) and Σ (dependent product type) to construct dependent types and this rich type structure provides a foundation for handling context dependence in natural language. One of the distinctive features of DTS, as compared to other frameworks based on dependent type theory, is that it is augmented with underspecified terms called @-terms, so as to provide a unified analysis of entailment, anaphora, and presupposition from inferential and computational perspective. DTS also gives a fully compositional account of inferences involving anaphora (Bekki, 2014).

The type constructor Π is a generalized form of the functional type. A term of type $(\Pi x : A)B(x)$ is a function f which takes any element a of A and returns

	Π-type	Σ-type
Martin-Löf (1984)	$(\Pi x : A)B(x)$	$(\Sigma x : A)B(x)$
DTS	$(x : A) \to B(x)$	$\begin{bmatrix} x : A \\ B(x) \end{bmatrix}$

Fig. 1. Notation of Π-type and Σ-type.

a term $f(a)$ of type $B(a)$ dependent on the choice of the argument a. In other words, a dependent function is a function whose codomain is dependent on the given argument. The type constructor Σ is a generalized form of the product type. A term of type $(\Sigma x : A)B(x)$ is a pair (m, n) which consist of a term m of type A and a term n of type $B(m)$, where the type of the second element n depends on the choice of the first element m.

According to Curry-Howard correspondence, a type can be regarded as a proposition and a term of the type can be regarded as a proof of the proposition. From this point of view, A Π-type corresponds to a universal quantifier and a proof term of universal sentence is a function. If $x \notin fv(B)$, i.e., there is no dependencies involved, $(\Pi x : A)B$ corresponds to implication. A Σ-type corresponds to an existential quantifier, and a proof term of existential sentence is a pair. If $x \notin fv(B)$, $(\Sigma x : A)B$ corresponds to conjunction. See, e.g., Martin-Löf (1984) for more details and inference rules for Σ and Π constructors. Figure 1 shows the notation of Σ-type and Π-type adopted in DTS.

Since Π-types correspond to propositions with the universal quantifier, the sentence *every boy entered* can be represented as follows.

(7) $$\left(u : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix}\right) \to \mathbf{enter}(\pi_1 u)$$

Here, **entity** is a basic type for all entities. Σ-types are associated with projection functions π_1 and π_2, which enables one to access to each part of the pair: from a pair $t : \begin{bmatrix} x : A \\ B(x) \end{bmatrix}$, one can derive $\pi_1 t : A$ and $\pi_2 t : B(\pi_1 t)$. Since u in (7) is a pair of type $\begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix}$, $\pi_1 u$ corresponds to an entity which is a boy. Thus, (7) corresponds to the proposition that for all entity that is a boy, that entity entered.

A sentence with an existential quantifier such as *a boy entered* is represented in terms of Σ-types. Again, $\pi_1 u$ corresponds to an entity which is a boy, and thus, (8) corresponds to the proposition that there exists an entity which is a boy and entered.

(8) $$\begin{bmatrix} u : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix} \\ \mathbf{enter}(\pi_1 u) \end{bmatrix}$$

One advantage of having Σ-types is that it can capture an *externally dynamic* property (Groenendijk and Stokhof, 1991) of existential quantifier and conjunction. For instance, a discourse such as (9a) is known to be problematic in a sense that its syntactically-corresponding formula in predicate logic, (9b), fails to represent an anaphoric link between *a boy* and *he*.

(9) a. A boy entered. He whistled.
 b. $\exists x(\mathbf{boy}(x) \wedge \mathbf{enter}(x)) \wedge \mathbf{whistle}(x)$

On the other hand, Σ-type can straightforwardly provide the semantic representation of this discourse as follows.

$$(10) \quad \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix} \\ \mathbf{enter}(\pi_1 u) \end{bmatrix} \\ \mathbf{whistle}(\pi_1 \pi_1 v) \end{bmatrix}$$

Although a term u is no longer accessible from the argument position of **whistle**, one can still pick up the term via a newly introduced term v, since v is a pair and each of its parts is accessible by applying (a sequence of) projection function. In this way, Σ-type can pass a variable binding relation to a subsequent discourse.

2.2 DTS and anaphora resolution

The remaining question is how one can obtain the term $\pi_1\pi_1 v$ in (10) for the representation of the pronoun *he*. In DTS, anaphoric expressions are represented in terms of underspecified terms called @-terms. Anaphora resolution in DTS is then defined as a process that replaces the @-term with the specific term which is constructed via type checking and proof construction (Bekki and Sato, 2015). For instance, the pronoun *he* is assigned the semantic representation in (11), where the type annotation to the @-term represents the requirement that *he* refers to some entity being male.

$$(11) \quad \pi_1 \left(@_i \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{male}(x) \end{bmatrix} \right)$$

Dynamic conjunction between sentences is defined in terms of Σ-type. Thus, the semantic representation of the whole discourse in (9a) is given as follows.

$$(12) \quad \begin{bmatrix} v : \begin{bmatrix} u : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix} \\ \mathbf{enter}(\pi_1 u) \end{bmatrix} \\ \mathbf{whistle}\left(\pi_1 \left(@_i \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{male}(x) \end{bmatrix} \right) \right) \end{bmatrix}$$

This underspecified representation is required to have sort **type**. This condition (*felicity condition* of a sentence) invokes type checking and leads to proof construction associated with the @-term. In the current example, one needs to find a proof term that satisfies the following inference.

(13) $\Gamma,\ v : \begin{bmatrix} u : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix} \\ \mathbf{enter}(\pi_1(u)) \end{bmatrix} \vdash\ ? : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{male}(x) \end{bmatrix}$

Here, Γ is a global context that represents background knowledge, and v is a term accessible from the position of the @-term, which corresponds to the information provided up to this point of the mini-discourse. From these premises Γ and v, one needs to construct a proof term of $\begin{bmatrix} x : \mathbf{entity} \\ \mathbf{male}(x) \end{bmatrix}$. Now, suppose that the global context contains the proof term in (14), which corresponds to the knowledge that every boy is male.

(14) $k_b : \left(u : \begin{bmatrix} y : \mathbf{entity} \\ \mathbf{boy}(y) \end{bmatrix} \right) \to \mathbf{male}(\pi_1(u))$

By using this knowledge k_b together with v, one can eventually construct a proof term of the required type that can replace the underspecified term in (12), yielding the fully-specified representation in (10).

Note that this anaphora resolution procedure in DTS can account for the following *externally static* property of universal quantifiers.

(15) a. Every^1 boy received a present. *He$_1$ looks happy.
b. Every boy received a^1 present. *It$_1$ was a toy car.

In these cases, since the first sentences are universal sentences, proof terms provided to the subsequent discourse are functions. Thus, neither an entity being a boy embedded in the domain of the function (i.e., an entity in the restrictor), nor an entity being a present embedded in the codomain of the function (i.e., an entity in the nuclear scope) can be picked up by the same operation as the previous example of existential sentences.

In this way, rich type structures of Σ-type and Π-type, together with the anaphora resolution process in DTS, provide a proof-theoretic account of dynamic properties of the existential and universal quantifiers.

3 Dependency relations and dependent interpretation

3.1 Basic example

As we mentioned above, the structure of Π-type corresponds to the externally static property of the universal quantifier, and it can block an anaphoric link exemplified in (15b). However, since DTS is a proof-theoretic semantics and the anaphora resolution process is associated with inference, there is one possibility to pick up an entity embedded in the codomain of the function. That is to apply an appropriate argument to the function and obtain the return value. This is one of the key ideas that underlies our analysis of the interpretation of pronouns involving the reference to dependency relation.

We claim that that Π-type directly captures quantificational dependency between objects and that its proof term can be used for inference during the anaphora resolution process in DTS. To illustrate this idea, let us consider the simplest example in (2b), which is repeated below as (16).

(16) Every boy will receive a[1] present. Some boy will open it$_1$.

Since a universal quantifier corresponds to a Π-type, the first sentence can be represented as follows.

$$(17)\quad \left(u : \begin{bmatrix} x : \textbf{entity} \\ \textbf{boy}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \textbf{entity} \\ \textbf{present}(y) \end{bmatrix} \\ \textbf{receive}(\pi_1 u, \pi_1 v) \end{bmatrix}$$

The terms $\pi_1 u$ and $\pi_1 v$ pick up the entity being a boy and the entity being a present, respectively. The type as a whole represents the proposition that, for every boy, there exists a present such that the boy received it. This representation corresponds to the distributive reading in question. Thus, a term of this type is a function that receives a pair consists of an entity and a proof of that entity being a boy, and then returns a tuple that consists of an entity, a proof of that entity being a present, and a proof of the boy and the present being in the receiving relation. This means that the representation of the first sentence introduces a function that corresponds to the dependency relation between boys and presents.

The second sentence is represented by Σ-type and the pronoun *it* can be defined as an underspecified term of type **entity**. Thus, by combining the semantic representation of the two sentences in terms of dynamic conjunction, (16) is represented as the following Σ-type.

$$(18)\quad \begin{bmatrix} f : \left(u : \begin{bmatrix} x : \textbf{entity} \\ \textbf{boy}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \textbf{entity} \\ \textbf{present}(y) \end{bmatrix} \\ \textbf{receive}(\pi_1 u, \pi_1 v) \end{bmatrix} \\ \begin{bmatrix} z : \begin{bmatrix} x : \textbf{entity} \\ \textbf{boy}(x) \end{bmatrix} \\ \textbf{open}(\pi_1 z, @_1 \textbf{entity}) \end{bmatrix} \end{bmatrix}$$

In this way, the proof term f of the first sentence which corresponds to a dependency relation between boys and presents serves as anaphoric resources. In the current case, anaphora resolution of the pronoun *it* yields the following inference.

$$(19)\quad \Gamma,\ f : \left(u : \begin{bmatrix} x : \textbf{entity} \\ \textbf{boy}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \textbf{entity} \\ \textbf{present}(y) \end{bmatrix} \\ \textbf{receive}(\pi_1 u, \pi_1 v) \end{bmatrix},\ z : \begin{bmatrix} x : \textbf{entity} \\ \textbf{boy}(x) \end{bmatrix} \vdash\ ? : \textbf{entity}$$

There are two proof terms which are accessible from the position of @-term: the term f, which is a proof term of the first sentence, and z, which is a term corresponding to the subject of the second sentence. The proof construction goes as follows. First, by applying z to the function f, one obtains the proof term fz that is a pair corresponding to the present which is received by the boy, $\pi_1 z$. Second, by taking the first projection of the first projection of fz, one obtains a term $\pi_1\pi_1(fz)$ of type **entity**. Therefore, by replacing the @-term with the obtained term $\pi_1\pi_1(fz)$, the second argument of **open** will be filled with an entity which depends on the term z, namely, an entity which depends on the subject of the second sentence.

Some reader may think that proof terms have something in common with discourse referents in Discourse Representation Theory (Kamp and Reyle 1993; Kamp et al. 2011, henceforth DRT) in that both are objects introduced by sentences and referred to afterward in order to resolve anaphora. We should say, however, that there are at least two crucial differences. Firstly, while discourse referents are limited to individuals, proof terms can have any types, which allows richer inner structure (Ranta, 1994). Thus, DTS can directly handle the information of dependency relations as a proof term of dependent function type. Secondly, together with the anaphora resolution mechanism provided in DTS, proof terms can contribute to the logical inference, which yields a new proof term serving as an antecedent.

Note that it is Ranta (1994) who first pointed out the possibility of using a dependent function to interpret an anaphoric pronoun *it* in a sentence like "if you give every child a present, some child will open it" (Hintikka and Carlson, 1979). However, the brief discussion there was confined to this specific example and did not generalize to other examples. It is also not clear how to obtain a fully-specified representation from the given sentence in a compositional way. The point we make here is that the idea that a proof term of a dependent function type can serve as a discourse referent can be applied to the well-known phenomena of dependent interpretation involved in plural anaphora in general.

3.2 More examples

We have observed that a similar anaphoric link can be established even though the subject of the subsequent discourse does not exactly match the restrictor of the universal quantifier of the first sentence. Those examples are repeated below as (20) and (21).

(20) Every boy will receive a^1 present. Every young boy will open it$_1$.
(21) Every boy will receive a^1 present. John will open it$_1$.

Both the first and the second sentences in (20) can be represented in terms of Π-types. Thus, the whole sentence receives the following semantic representation.

(22)
$$\begin{bmatrix} f : \left(u : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix} \right) \to \begin{bmatrix} v : \begin{bmatrix} y : \mathbf{entity} \\ \mathbf{present}(y) \end{bmatrix} \\ \mathbf{receive}(\pi_1 u, \pi_1 v) \end{bmatrix} \\ \left(t : \begin{bmatrix} z : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy}(x) \end{bmatrix} \\ \mathbf{young}(\pi_1 z) \end{bmatrix} \right) \to \mathbf{open}(\pi_1\pi_1 t, @_1\mathbf{entity}) \end{bmatrix}$$

The premises of the inference associated with the resolution of $@_1$ here are terms f and t. Since $\pi_1 t : \begin{bmatrix} x : \mathbf{entity} \\ \mathbf{boy} \end{bmatrix}$ can be derived from the given t and can be applied to f, one eventually obtains a term $\pi_1\pi_1(f(\pi_1 t))$, which corresponds to a present dependent on each young boy, $\pi_1\pi_1 t$. Similarly, (21) is represented as follows.

(23) $$\left[\begin{array}{l} f : \left(u : \begin{bmatrix} x : \textbf{entity} \\ \textbf{boy}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \textbf{entity} \\ \textbf{present}(y) \end{bmatrix} \\ \textbf{receive}(\pi_1 u, \pi_1 v) \end{bmatrix} \\ \textbf{open}(john, @_1\textbf{entity}) \end{array}\right]$$

To find a semantic link between *John* and *boy*, one needs the background knowledge that John is a boy. If the global context Γ supplies the background knowledge $k_j : \textbf{boy}(john)$ which corresponds to the proposition John is a boy, one can construct the term $(john, k_j) : \begin{bmatrix} x : \textbf{entity} \\ \textbf{boy}(x) \end{bmatrix}$. Again, the function f can be supplied an argument.

When a relation between the restrictor of the universal quantifier and the subject of the subsequent discourse is not clear, then it simply fails to search a proof. For instance, in the case of (5b), repeated here as (24a), there exists neither an explicit link nor an implicit link between men and women.

(24) a. Every man will receive a^1 present. *Some woman will open it$_1$.

b. $$\left[\begin{array}{l} f : \left(u : \begin{bmatrix} x : \textbf{entity} \\ \textbf{man}(x) \end{bmatrix}\right) \to \begin{bmatrix} v : \begin{bmatrix} y : \textbf{entity} \\ \textbf{present}(y) \end{bmatrix} \\ \textbf{receive}(\pi_1 u, \pi_1 v) \end{bmatrix} \\ \begin{bmatrix} z : \begin{bmatrix} x : \textbf{entity} \\ \textbf{woman}(x) \end{bmatrix} \\ \textbf{open}(\pi_1(z), @_1\textbf{entity}) \end{bmatrix} \end{array}\right]$$

In this case, one needs an argument applying to the function f in order to construct a proof of the present received by some man. Thus, unless some relation which bridges men and women is available in the global context, there is no way to obtain the required proof term from z and f.

Let us now turn back to our first example (1) involving plural anaphora. The example is repeated below as (25). (26) is a related example with adjectival quantifiers (Krifka, 1996).

(25) Every^1 boy received a^2 present. They$_1$ opened it$_2$.
(26) Three^1 students each wrote an^2 article. They$_1$ each sent it$_2$ to L&P.

Although providing a comprehensive analysis of plural anaphora including an analysis of the so-called collective reading is not the main target of this paper, let us briefly sketch the main idea of handling the reference to dependency relation involved in plural anaphora. In our analysis explained above, there are two things that play an essential role to account for the reference to dependency relation. Firstly, the initial sentence must have the $\forall$–$\exists$ reading which induces a dependency relation between objects in terms of a dependent function. Secondly, a singular pronoun in the subsequent discourse can be interpreted anaphorically, by supplying an adequate argument to the dependent function introduced by the initial sentence. These two points are maintained in our analysis of plural anaphora as in (25) and (26).

As for the first point, we follow an analysis of generalized quantifiers and adjectival quantifiers in DTS (Tanaka et al., 2014; Tanaka, 2014), which provides semantic representation of those quantificational expressions by using dependent function. According to their analysis, generalized quantifiers such as *most*[5] and adjectival quantifiers such as *three* are uniformly represented as involving existential quantification over dependent functions whose domain is restricted by cardinality condition[6]. Thus, this dependent function can be used for anaphora resolution as the cases we have seen so far.

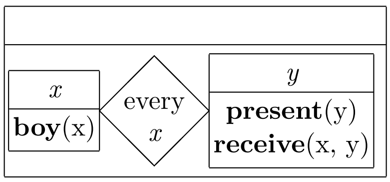
The essential role of the plural pronoun *they* is, then, to supply terms that are adequate for the arguments of the dependent function. Semantic representation of *they* are also given in terms of the @-term. In contrast to the singular pronoun such as *it*, the requirement of type annotation of the @-term associated with *they* is to find a predicate and a proof term of the cardinality condition. This is because the domain of dependent function provided by quantificational expression is restricted by the predicate and the cardinality condition. Therefore, the term replacing the @-term can supply adequate arguments to the function, which enables the dependent interpretation of singular pronoun which comes after.

4 Previous approaches

In this section, we provide a brief overview of some of the existing solutions in dynamic semantics to handle the reference to dependency relation.

In classical DRT (Kamp and Reyle, 1993), the reference to dependency relation is handled by using copy mechanism. First, the sentence *every boy received a present* yields the following Discourse Representation Structure (DRS).

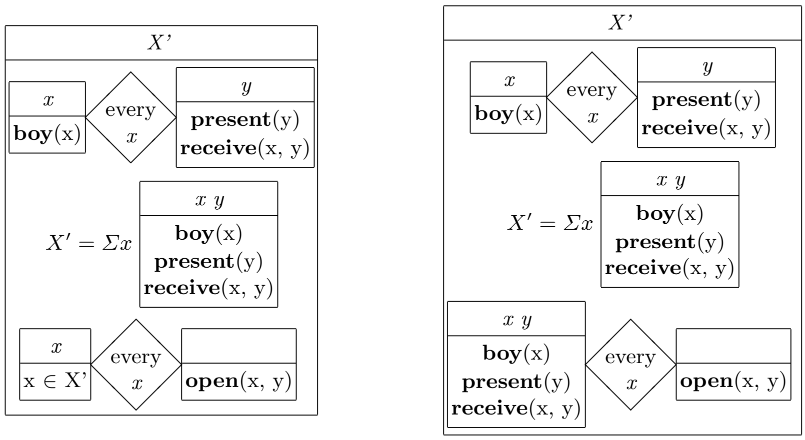
(27)



[5] In the case of *every*, we can provide its semantic representation in two ways: one possibility is to treat it simply as Π-type as we have seen above, and another possibility is to represent it in the same way as other generalized quantifiers such as *most*. Since these two formulas are mutually deducible, the story about generalized quantifiers presented here can be applied to the case of *every* as well.

[6] As dependent function types correspond to $\forall$–$\exists$ reading (or distributive reading), the semantic representation of *three* provided by Tanaka (2014) should correspond to the semantic representation of *three ... each*. To obtain the semantic representation of *three ... each* in a compositional way, we can integrate the existing analysis of plural objects into our framework. Since this is beyond the scope of this paper, we leave it for another occasion.

The construction of this DRS triggers the operation called *abstraction*, which constructs a new plural discourse referent X' consisting of object that satisfies the condition of x. The pronoun *they* refers to this X' and yields the DRS in Figure 2.a, where universal quantification over X' takes place. In this DRS, however, there is no discourse referent which can be associated with singular y in $\mathbf{open}(x, y)$. In such a case, there is an option to apply copy operation which copies the conditions of x constituting X' to the restrictor part of the duplex condition. The corresponding DRS is in Figure 2.b. In this way, the singular variable y in $\mathbf{open}(x, y)$ can refer to each present associated with each boy.



a. DRS for (25) before applying copy operation. b. DRS for (25) after applying copy operation.

Fig. 2. DRS associated with (25).

Krifka (1996) criticizes this solution of performing representation-based copying operation and proposes the analysis based on the enriched assignment function called *parametrized sum individuals*. Parametrized sum individuals are sets of pairs of an individual and a variable assignment associated with the individual. A possible instance of parametrized individuals for *every boy received a present* looks as follows.

$$\langle x, \{\langle b_1, \{\langle y, p_1\rangle\}\rangle, \langle b_2, \{\langle y, p_2\rangle\}\rangle, \langle b_3, \{\langle y, p_3\rangle\}\rangle, ...\}\rangle$$

The individuals can be either singular or plural. Since individuals are followed by assignments associated with them, this structure captures dependency relations between objects. In the case of the distributive interpretation, each parametrized individual is independently evaluated against predicates. Thus, singular pronoun can be interpreted along each parametrized individual, which produces an effect of interpretation sensitive to dependency relation.

The standard way to encode dependency relations is to adopt *information states for plurals* proposed by van den Berg (1996a,b) in Dynamic Plural Logic.

In this approach, formulas are interpreted relative to information states, which are sets of assignments, instead of to assignments. A possible information state for *every boy received a present* looks as follows.

$$\{\{\langle x, b_1\rangle, \langle y, p_1\rangle\}, \{\langle x, b_2\rangle, \langle y, p_2\rangle\}, \{\langle x, b_3\rangle, \langle y, p_3\rangle\}, ...\}$$

When distribution over x is involved, predicates are evaluated against each assignment of information states. Since the assignment of new values takes place independently of each assignment function, the variables introduced may be dependent on x. This is the source of dependency.

Our intuition about the $\forall$–$\exists$ reading of *every boy received a present* is that it introduces a quantificational dependency, that is, a function f such that x is a boy receiving a present $f(x)$. However, there is no natural place in the standard dynamic semantics theory to store such a function for subsequent anaphora. Therefore, those formalisms mentioned above need to capture dependency relations in an indirect way, which requires integrating a special mechanism or structure into the underlying framework.

There are several other issues which can be raised from a more empirical side. Firstly, the copy mechanism in DRT is triggered by the resolution of plural pronoun *they*. However, we have seen that there are cases such as (16), (20), and (21), where a plural pronoun does not appear but still the reference to dependency relation takes place. It seems that the further stipulation or operation is needed in DRT to handle more general cases including these examples. Secondly, there exists no proof theory for both of the frameworks proposed by Krifka and van den Berg. Van den Berg's analysis can account for a case such as (20), where a subset relation allows the reference to a dependent relation. In general, however, a semantic link between the restrictor of the universal quantifier and the subject of the subsequent discourse is not limited to subset relation, as we can observe in the example such as (5a): rather, it seems that resolving dependent plural anaphora involves a more general kind of inference, of which a semantic link in terms of subset relations is a special instance.

An advantage of proposed analysis in DTS is that dependent function type is an independently motivated object provided in dependent type theory, and thus, we do not need to extend our framework to account for dependency relations. By following the standard operation of dynamic conjunction and anaphora resolution procedure in DTS, it can naturally provide a function as discourse referent, which straightforwardly leads to the dependent interpretation of singular pronouns. In addition, since DTS is a proof-theoretic semantics and anaphora resolution process involves proof search, it can provide more general and uniform account of semantic links between the restrictor of the universal quantifier and the subject of the subsequent discourse.

5 Conclusion

In this article, we have argued for a new account of dependency relations between objects as dependent functions in dependent type theory. This contrasts with approaches in dynamic semantics tradition, where a function does not serve as a discourse referent, and the enriched notion of assignment functions plays an essential role in handling dependencies. We have seen that the proposed account is capable of explaining the dependent interpretation of pronouns by integrating with anaphora resolution mechanism of DTS. This new account may offer a basis for proof-theoretic analysis of plural anaphora.

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Bibliography

Daisuke Bekki. *Representing Anaphora with Dependent Types*, pages 14–29. Springer Berlin Heidelberg, Berlin, Heidelberg, 2014.

Daisuke Bekki and Miho Sato. Calculating projections via type checking. In *Proceedings of TYpe Theory and LExical Semantics (TYTLES) in ESSLLI2015*, Barcelona, Spain, 2015.

Adrian Brasoveanu. Donkey pluralities: plural information states versus non-atomic individuals. *Linguistics and Philosophy*, 31(2):129–209, 2008.

Jeroen Groenendijk and Martin Stokhof. Dynamic Predicate Logic. *Linguistics and Philosophy*, 14(1):39–100, 1991.

Jaakko Hintikka and Lauri Carlson. Conditionals, Generic Quantifiers, and Other Applications of Subgames. In Esa Saarinen, editor, *Game-Theoretical Semantics*, volume 5 of *Studies in Linguistics and Philosophy*. Springer Netherlands, 1979.

Hans Kamp and Uwe Reyle. *From Discourse to Logic: Introduction to Modeltheoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory*. Studies in Linguistics and Philosophy. Kluwer Academic Publisher, 1993.

Hans Kamp, Josef Van Genabith, and Uwe Reyle. Discourse Representation Theory. In Dov M. Gabbay and Franz Guenthner, editors, *Handbook of Philosophical Logic*, volume 15, pages 125–394. Springer Netherlands, 2011.

Lauri Karttunen. Discourse referents. In J. D. McCawley, editor, *Syntax and Semantics Vol. 7*, pages 363–386. Academic Press, 1976.

Manfred Krifka. Parametrized sum individuals for plural anaphora. *Linguistics and Philosophy*, 19(6):555–598, 1996.

Per Martin-Löf. *Intuitionistic Type Theory: Notes by Giovanni Sambin of a series of lectures given in Padua, June 1980*. Bibliopolis Naples, 1984.

Rick Nouwen. *Plural Pronominal Anaphora in Context : Dynamic Aspects of Quantification*. PhD thesis, Utrecht Institute for Linguistics OTS, 2003.

Aarne Ranta. *Type-theoretical Grammar*. Oxford University Press, 1994.

Ribeka Tanaka. A proof-theoretic approach to generalized quantifiers in dependent type semantics. In Ronald de Haan, editor, *Proceedings of the ESSLLI2014 Student Session*, pages 140–151, Tübingen, Germany, 2014.

Ribeka Tanaka, Yuki Nakano, and Daisuke Bekki. *Constructive Generalized Quantifiers Revisited*, volume 8417 of *Lecture Notes in Computer Science*, pages 115–124. Springer International Publishing, 2014.

Martin van den Berg. Dynamic generalized quantifiers. In Jaap Van Der Does and Jan Van Eijck, editors, *Quantifiers, Logic, and Language*, pages 63–94. CSLI publications, 1996a.

Martin van den Berg. *Some aspects of the Internal Structure of Discourse. The Dynamics of Nominal Anaphora*. PhD thesis, University of Amsterdam, 1996b.

An Analysis of Selectional Restrictions with Dependent Type Semantics

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Abstract. Predicates in natural languages impose selectional restrictions on their arguments. In this paper, we analyze selectional restrictions of predicates within the framework of Dependent Type Semantics (DTS), a framework of natural language semantics based on dependent type theory. We also analyze two phenomena, coercion and copredication for logical polysemous nouns, which present challenges to a simple analysis of selectional restrictions, in terms of operators that shift the meanings of predicates.

1 Introduction

Predicates in natural languages impose selectional restrictions on their arguments. For example, the transitive verb *marry* expects its subject and object to be expressions denoting human. Thus, from the utterance of (1), we can infer that Bob and Ann are both human.

(1) Bob married Ann.

One potential way to explain this inference is to treat selectional restrictions of predicates as entailment. According to this analysis, the verb *marry* is assigned the meaning in (2a) and the whole sentence in (1) has the interpretation in (2b).

(2) a. $\lambda y \lambda x.\mathbf{marry}(x, y) \wedge \mathbf{human}(x) \wedge \mathbf{human}(y)$
    b. $\mathbf{marry}(bob, ann) \wedge \mathbf{human}(bob) \wedge \mathbf{human}(ann)$

A problem with this analysis is that it cannot handle the inference in (3).

(3) Bob didn't marry Ann. $\Rightarrow$ Bob and Ann are human.

From the negation of (1), one can also infer that Bob and Ann are human. If selectional restrictions of predicates were part of entailment, we would assign the interpretation (4) to the negative sentence in (3). This does not account for the inference in (3).

(4) $\neg(\mathbf{marry}(bob, ann) \wedge \mathbf{human}(bob) \wedge \mathbf{human}(ann))$

In general, the contents of selectional restrictions project out of the scope of negation, modals, and conditionals (Asher [2], Magidor [7]). This is a common feature of inferences known as *presupposition projection* (see e.g., Beaver [3] for an overview).

The goal of this paper is to propose an analysis that treats selectional restrictions as presupposition within the framework of Dependent Type Semantics (Bekki [4]). Using this framework, we also present a formal analysis of two lexical phenomena related to selectional restriction, namely, coercion and copredication for logical polysemies.

2 Selectional restriction: types vs. predicates

Although the presuppositional analysis of selectional restriction goes back at least to McCawley [9], it seems fair to say that its precise formulation has been mostly neglected in the simply typed setting of standard formal semantics where only e (entity) and t (truth-value) are taken as base types.

Recently, there are proposals in the literature that set out to handle selectional restrictions with extended type-theoretic frameworks (Asher [1], Luo [6], Retoré [11]). There are two possible approaches here. One is to represent selectional restrictions as *types*; for instance, using **animate** and **human** as base types, one can assign a type $\mathbf{animate} \to \mathbf{prop}$ to the predicate **cry** and $\mathbf{human} \to \mathbf{human} \to \mathbf{prop}$ to the predicate **marry**, and so on. According to this approach, the violation of a selectional restriction is to be treated as a type mismatch. One problem with this approach is the problem of *subtyping*. Thus, to combine the predicate **cry** of type $\mathbf{animate} \to \mathbf{prop}$ with the term **john** of type **human**, one needs a subtyping relation $\mathbf{human} < \mathbf{animate}$ and extra subtyping rules (cf. Luo [6], Retoré [11]). One drawback is that with additional subtyping rules, the resulting compositional semantics becomes fairly complicated.

Alternatively, one can preserve the base type for entities and represent selectional restrictions as *predicates* over entities. This view seems to be underdeveloped; but it has an advantage in that it can dispense with subtyping and preserve the clear, well-understood conception of syntax-semantics mapping. Our theory is based on this second approach.

3 Dependent Type Semantics

The main challenge here is how to provide a presuppositional analysis of selectional restrictions combined with the selectional-restriction-as-predicate view. We use Dependent Type Semantics (DTS) (Bekki [4]) as a theoretical framework, which provides two crucial tools: *dependent types* (which are a generalization of simple types) and *underspecified terms*. DTS is a proof-theoretic semantics of natural language based on dependent type theory (Martin-Löf [8]). It characterizes the meaning of a sentence from the perspective of *inferences*.

DTS uses two kinds of dependent types.

(i) Π-type (dependent function type), written as $(x : A) \to B$, is a generalized form of a function type $A \to B$; a term of type $(x : A) \to B$ is a function f that takes a term a of A and returns a term $f(a)$ of type $B(a)$.

(ii) Σ-type (dependent product type), written as $(x : A) \times B$ or $\begin{bmatrix} x : A \\ B \end{bmatrix}$, is a generalized form of a product type $A \times B$; a term of type $(x : A) \times B$ is a pair (t, u) such that t is of type A and u is of type $B(t)$. The projection operators π_1 and π_2 are defined in such a way that $\pi_1(t, u) = t$ and $\pi_2(t, u) = u$.

Under the so-called propositions-as-types principle (Martin-Löf [8]), types and propositions are identified; a term t having type A (i.e., $t : A$) serves as a *proof term* for the proposition A.

In this dependently typed setting, Π-type and Σ-type correspond to universal and existential quantifiers, respectively. For example, in DTS, the sentence in (5a) is given the semantic representation (SR) in (5b):

(5) a. Every man entered.

b. $\left(u : \begin{bmatrix} x : \mathtt{entity} \\ \mathbf{man}(x) \end{bmatrix} \right) \to \mathbf{enter}(\pi_1(u))$

The term u here has a Σ-type: it consists of a term (let it be x) having type $\mathtt{entity}$ and some proof term having type $\mathbf{man}(x)$ that depends on x. The term $\pi_1(u)$ in $\mathbf{enter}(\pi_1(u))$ picks up the entity that is the first component of u. In DTS, common nouns such as *man* are treated as a predicate rather than as a type. In other words, that a term x has a property *man* is represented as a proposition $\mathbf{man}(x)$ rather than as a judgement $x : \mathbf{man}$. See Bekki and Mineshima [5] for more discussions on the interpretation of common nouns in our framework.

For Π-types and Σ-types, we use the following formation rules (ΠF, ΣF), introduction rules (ΠI, ΣI), and elimination rules (ΠE, ΣE).

$$\frac{A : \mathsf{s}_1 \quad \begin{array}{c}\overline{x : A}^{\,(i)} \\ \vdots \\ B : \mathsf{s}_2\end{array}}{(x : A) \to B : \mathsf{s}_2}\ (\Pi F), i \qquad \frac{A : \mathsf{type} \quad \begin{array}{c}\overline{x : A}^{\,(i)} \\ \vdots \\ B : \mathsf{s}\end{array}}{\begin{bmatrix} x : A \\ B(x) \end{bmatrix} : \mathsf{s}}\ (\Sigma F), i$$

$$\frac{(x : A) \to B : \mathsf{s} \quad \begin{array}{c}\overline{x : A}^{\,(i)} \\ \vdots \\ M : B\end{array}}{\lambda x.M : (x : A) \to B}\ (\Pi I), i \qquad \frac{M : A \quad N : B[M/x]}{(M, N) : \begin{bmatrix} x : A \\ B(x) \end{bmatrix}}\ (\Sigma I)$$

$$\frac{M : (x : A) \to B \quad N : A}{MN : B[N/x]}\ (\Pi E) \qquad \frac{M : \begin{bmatrix} x : A \\ B(x) \end{bmatrix}}{\pi_1(M) : A}\ (\Sigma E) \qquad \frac{M : \begin{bmatrix} x : A \\ B(x) \end{bmatrix}}{\pi_2(M) : B[\pi_1(M)/x]}\ (\Sigma E)$$

where s, s_1 and s_2 are kind or type (see Bekki and Mineshima [5] for more details).

DTS has an underspecified term @ to handle anaphora and presupposition. We use type annotation for underspecified terms; we write $@ : A$, where the underspecified term @ is annotated with its type A. By using underspecified terms, we can uniformly handle semantic phenomena that depend on the preceding contexts.

Presupposition and anaphora are resolved by constructing a proof term for $@ : A$ with type checking and then replacing $@ : A$ by the constructed term. Type checking ensures that an SR is well-formed (i.e., having type type). For underspecified terms, we use the rule

$$\dfrac{A : \mathsf{s} \quad A\ \mathit{true}}{(@ : A) : A}\ (@)$$

where $\mathsf{s} \in \{\mathsf{kind}, \mathsf{type}\}$. The judgement $A\ \mathit{true}$ triggers a proof search to construct a term having the type A in a given context. The constructed term is to be replaced with @ in the final representation. The annotated type A may contain another underspecified term, for which the type checking is triggered by the judgement $A : \mathsf{s}$ (e.g., $A : \mathsf{type}$) in the @-rule.

As an illustration, consider the sentence in (6a). For this sentence, one can compositionally derive the SR in (6b).[1]

(6) a. He whistled.

b. $\mathbf{whistle}\left(\pi_1\left(@ : \begin{bmatrix} x : \mathtt{entity} \\ \mathbf{man}(x) \end{bmatrix}\right)\right)$

The SR (6b) contains an underspecified term @ annotated with the Σ-type that corresponds to the proposition that there is an entity x such that x is a man. Fot the SR in (6b), the type checking runs as follows.

$$
\dfrac{
\dfrac{}{\mathbf{whistle} : \mathsf{entity} \to \mathsf{type}}\,(CON)
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{}{\mathsf{entity} : \mathsf{type}}\,(CON)
\quad
\dfrac{\dfrac{}{\mathbf{man} : \mathsf{entity} \to \mathsf{type}}\,(CON) \quad \dfrac{}{x : \mathsf{entity}}\,(1)}{\mathbf{man}(x) : \mathsf{type}}\,(\mathit{\Pi E})
}{\begin{bmatrix} x : \mathsf{entity} \\ \mathbf{man}(x) \end{bmatrix} : \mathsf{type}}\,(\mathit{\Sigma F}), 1
\quad
\begin{matrix} \vdots \\ \begin{bmatrix} x : \mathsf{entity} \\ \mathbf{man}(x) \end{bmatrix}\ \mathit{true} \end{matrix}
}{\left(@ : \begin{bmatrix} x : \mathsf{entity} \\ \mathbf{man}(x) \end{bmatrix}\right) : \begin{bmatrix} x : \mathsf{entity} \\ \mathbf{man}(x) \end{bmatrix}}\,(@)
}{\pi_1\left(@ : \begin{bmatrix} x : \mathsf{entity} \\ \mathbf{man}(x) \end{bmatrix}\right) : \mathsf{entity}}\,(\mathit{\Sigma E})
}{\mathbf{whistle}\left(\pi_1\left(@ : \begin{bmatrix} x : \mathsf{entity} \\ \mathbf{man}(x) \end{bmatrix}\right)\right) : \mathsf{type}}\,(\mathit{\Pi E})
$$

The application of the @-rule in this derivation triggers a proof search for the judgement:

$$\begin{bmatrix} x : \mathtt{entity} \\ \mathbf{man}(x) \end{bmatrix}\ \mathit{true}.$$

[1] See Bekki [4] for details on the compositional derivations of SRs in DTS.

Assuming that we have $john$: $\texttt{entity}$ and t : $\mathbf{man}(john)$ in the background global context, we can construct a term $(john, t)$ having the Σ-type in question, i.e., a type annotated for the underspecified term @. This term serves as an antecedent of the pronoun *he*. Replacing @ with the specific term $(john, t)$, the semantic representation in (6b) ends up with $\mathbf{whistle}(\pi_1(john, t))$, which reduces to $\mathbf{whistle}(john)$. In this way, we can derive the interpretation for the sentence containing a pronoun in (6a).

4 Selectional restriction in DTS

To handle selectional restrictions of predicates as presuppositions, we need to calculate whether selectional restrictions are satisfied at the stage of type checking. We propose that selectional restrictions of predicates are specified in the lexicon. For instance, we can define lexical entries of intransitive and transitive verbs as follows.

	syntax	**semantic representation**
cry	$S\backslash NP$	$\lambda x.\mathbf{cry}(x, @ : \mathbf{animate}(x))$
marry	$(S\backslash NP)/NP$	$\lambda y.\lambda x.\mathbf{marry}(y, @_i : \mathbf{human}(y))(x, @_j : \mathbf{human}(x))$

To be concrete, we use CCG (Steedman [12]) as a syntactic framework. The types of predicates **cry** and **marry** in the above SRs are defined as follows:

$$\mathbf{cry} : \begin{bmatrix} x : \texttt{entity} \\ \mathbf{animate}(x) \end{bmatrix} \rightarrow \mathsf{type}$$
$$\mathbf{marry} : \begin{bmatrix} y : \texttt{entity} \\ \mathbf{human}(y) \end{bmatrix} \rightarrow \begin{bmatrix} x : \texttt{entity} \\ \mathbf{human}(x) \end{bmatrix} \rightarrow \mathsf{type}$$

For example, the predicate **cry** takes a pair consisting of an entity x and a proof term for the proposition $\mathbf{animate}(x)$ as an argument and returns a type (as a proposition). In the lexical entry for the intransitive verb *cry*, the proof term for the proposition $\mathbf{animate}(x)$ is underspecified; given that there is an underspecified term $@ : \mathbf{animate}(x)$ in the SR, we have to prove $\mathbf{animate}(x)$ during the stage of type checking, in order to ensure that the subject of *cry* is animate.

As an illustration, consider the sentence in (1). For this sentence, we can derive the following SR in a compositional way.

(7) $\mathbf{marry}(ann, @_1 : \mathbf{human}(ann))(bob, @_2 : \mathbf{human}(bob))$.

The following is the compositional derivation of this SR.

$$\dfrac{\dfrac{\text{Bob}}{NP : bob} \qquad \dfrac{\dfrac{\text{married}}{(S\backslash NP)/NP : \lambda y.\lambda x.\mathbf{marry}(x, @_1 : \mathbf{human}(x))(y, @_2 : \mathbf{human}(y))} \quad \dfrac{\text{Ann}}{NP : ann}}{S\backslash NP : \lambda x.\mathbf{marry}(ann, @_1 : \mathbf{human}(ann))(x, @_2 : \mathbf{human}(x))}>}{\mathbf{marry}(ann, @_1 : \mathbf{human}(ann))(bob, @_2 : \mathbf{human}(bob))}<$$

Now a type checking to ensure that the SR in (7) is well-formed runs as follows:

$$\dfrac{\dfrac{\dfrac{}{\mathbf{marry} : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{h}(x) \end{bmatrix} \to \begin{bmatrix} y : \mathsf{e} \\ \mathbf{h}(y) \end{bmatrix} \to \mathsf{type}}(Con) \quad \dfrac{\dfrac{}{a : \mathsf{e}}(Con) \quad \begin{matrix} \vdots \\ (@_1 : \mathbf{h}(a)) : \mathbf{h}(a) \end{matrix}}{(a, @_1 : \mathbf{h}(a)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{h}(x) \end{bmatrix}}(\Sigma I)}{\mathbf{marry}(a, @_1 : \mathbf{h}(a)) : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{h}(y) \end{bmatrix} \to \mathsf{type}}(\Pi E) \quad \dfrac{\dfrac{}{b : \mathsf{e}}(Con) \quad \begin{matrix} \vdots \\ (@_2 : \mathbf{h}(b)) : \mathbf{h}(b) \end{matrix}}{(b, @_2 : \mathbf{h}(b)) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{h}(x) \end{bmatrix}}(\Sigma I)}{\mathbf{marry}(a, @_1 : \mathbf{h}(a))(b, @_2 : \mathbf{h}(b)) : \mathsf{type}}(\Pi E)$$

Here we abbreviates `entity` as e, **human** as $\mathbf{h}$, *ann* as a, and *bob* as b. There are two open branches containing underspecified terms, $@_1$ and $@_2$, which show that we have to search the preceding context to construct a proof term of $\mathbf{human}(bob)$ and $\mathbf{human}(ann)$. That is to say, for the semantic representation to be well-formed, it is presupposed that x and y, which are respectively the subject and the object of the verb *married*, are both human. In this way, the selectional restriction of a predicate is derived as a presupposition.

Similarly, the SR of the negative sentence in (3) is given as follows.

(8) $\neg\mathbf{marry}(ann, @_2 : \mathbf{human}(ann))(bob, @_1 : \mathbf{human}(bob))$

According to the formation rule of negation, A and $\neg A$ have the same well-formedness condition:

$$\dfrac{A : \mathsf{type}}{\neg A : \mathsf{type}}(\neg F)$$

That is, if we have $A : \mathsf{type}$, then we have $\neg A : \mathsf{type}$ as well. Therefore, the type checking for the negative SR in (8) ends up with the derivation that triggers a proof search in the same way as the type checking for the SR in (6b) given in Section 3. In this way, one can derive the inference pattern of presupposition projection out of the scope of negation. A similar explanation applies to the case of modals and conditionals.

Interestingly, a negative sentence like (9) has two readings (cf. McCawley [9]).

(9) The chair does not cry.

First, this sentence has a reading in which the selectional restriction projects out of the scope of negation, hence resulting in a violation of selectional restriction. In our terms, after composing the meaning of (9), one obtains the SR $\neg\mathbf{cry}(chair, @ : \mathbf{animate}(chair)$; according to the formation rule of negation, the content of selectional restriction, i.e., $\mathbf{animate}(chair)$, projects out of the scope of negation. Thus, for the SR to be well-formed, one needs to construct a proof term of **animate**(*chair*), which is not available in the standard context. Hence, it is predicted that under this reading, a violation of selectional restriction occurs in the sense that the derived SR is not well-formed.

Secondly, and more interestingly, (9) can have a reading in which the selectional restriction does not project and hence is interpreted inside the scope of negation. The presuppositional analysis correctly predicts this reading; with the process of local accommodation, we can derive the SR $\neg(\mathbf{animate}(chair) \wedge \mathbf{cry}(chair))$ for (9). In this case, one does not have to construct a proof of **animate**(*chair*); hence, it is correctly predicted that under this reading, the utterance of (9) is meaningful and can be true. We leave a detailed explanation of local accommodation in the framework of DTS for another occasion.

5 Coercion and copredication for logical polysemies

5.1 Coercion

There are two phenomena that are not explained by a simple analysis of selectional restrictions of predicates. The first one is coercion (Nunberg [10]). For example, if we have a context in which there is a man who ate the omelet in a cafe, we can understand the meaning of (10a) as (10b).

(10) a. The omelet escaped.
b. The man who ate the omelet escaped.

To account for this phenomena, we define an operator called *argument operator* that transforms one predicate into another. The argument operator arg_1 for a one-place predicate and arg_2 for a two-places predicate are defined as follows:

$$arg_1 \equiv \lambda P.\lambda x.P\left(\pi_1\pi_1\left(@_5 : \begin{bmatrix} z : \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \\ (@_4 : \begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{type})(x, (@_3^{pr}(x)))(z) \end{bmatrix}\right)\right)$$

$$arg_2 \equiv$$

$$\lambda P.\lambda y.\lambda x.P\left(\pi_1\pi_1\left(@_7 : \begin{bmatrix} z : \begin{bmatrix} x : \mathsf{e} \\ @_3^{pr}(x) \end{bmatrix} \\ (@_6 : \begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_3^{pr}(x) \end{bmatrix} \to \mathsf{type})(y, (@_4 : @_1^{pr}(y)))(x, (@_5 : @_2^{pr}(x)))(z) \end{bmatrix}\right)\right)(x)$$

Here an underspecified term $@_i^{pr}$ is an abbreviation for $@_i : \mathsf{e} \to \mathsf{type}$.

Let us first focus on the definition of the argument operator arg_1 for one-place predicates. In the definition of arg_1, the underspecified terms $@_1$ and $@_2$ in $@_1^{pr}$ and $@_2^{pr}$ are annotated with type $\mathsf{e} \to \mathsf{type}$; these are underspecified terms for properties. Intuitively, given a one-place predicate P and its argument x of type e, the argument operator arg_1 produces a new predicate P' that existentially introduces a new entity z having some relation R to x.

When one underspecified term appears inside the type annotated with another underspecified term, the inside term has to be resolved first. More specifically, the underspecified terms contained in the argument operator arg_1 are resolved in the following way.

1. First, given the entity x (e.g., *the omelet* in (10a)), one needs to find a suitable property F (e.g., *edible*) holding of x. This property F replaces $@_1^{pr}$.
2. Second, if there is a proof term for the proposition that x has the property F (e.g., *the omelet is edible*), it replaces $@_3^{pr}$.
3. Also, one needs to find a property G that is substituted for $@_2^{pr}$. The property G (e.g., *animate*) has to be chosen so that the newly introduced entity (the first element of the term z) satisfies G.
4. Next, one needs to find a relation R that is substituted for $@_4^{pr}$. In our example, one has to find a relation (e.g., *eat*) that have selectional restrictions specified by predicates $\mathbf{edible}(x)$ and $\mathbf{animate}(y)$. This relation R replaces $@_4$.
5. Finally, one needs to construct a term substituted for $@_5$, which is a tuple consisting of an entity z whose first element satisfies the property G and a proof term for the proposition that the relation R holds of x and z.

In this way, arg_1 transforms the predicate **escape** into a predicate whose argument is an animate entity that has the eating-relation to the omelet.

Let us explain the derivation in more details. To begin with, we can derive the SR of the sentence (10a) as follows.

$$
\dfrac{\dfrac{\text{The omelet}}{\begin{array}{c}NP\\ : o\end{array}} \quad \dfrac{\dfrac{\text{escaped}}{\begin{array}{c}S\backslash NP\\ : \lambda x.\mathbf{escape}(x, @_{\mathbf{5}} : \mathbf{animate}(x))\end{array}} \quad \dfrac{\epsilon}{\begin{array}{c}(S\backslash NP)\backslash(S\backslash NP)\\ : arg_1\end{array}}}{\begin{array}{c}S\backslash NP\\ : arg_1(\lambda x.\mathbf{escape}(x, @_6 : \mathbf{animate}(x)))\end{array}}<}{\begin{array}{c}S\\ : arg_1(\lambda x.\mathbf{escape}(x, @_6 : \mathbf{animate}(x)))(o)\end{array}}<
$$

By unfolding the definition of arg_1, the sentence in (10a) is assigned the SR in (11).

(11) $\mathbf{escape}(Z_1, @_6 : \mathbf{animate}(Z_1))$

Here Z_1 abbreviates:

$$
\pi_1\pi_1\left(@_5 : \begin{bmatrix} z : \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \\ (@_4 : \begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{type})(o, (@_3 : @_1^{pr}(o)))(z) \end{bmatrix}\right).
$$

Let us suppose that we have the following information in the global context $\mathcal{K}_1$:

$$\begin{aligned}\mathcal{K}_1 \equiv\; & \mathsf{type} : \mathsf{kind},\ \mathsf{e} : \mathsf{type},\\ & j : \mathsf{e},\ o : \mathsf{e},\\ & \mathbf{animate} : \mathsf{e} \to \mathsf{type},\ \mathbf{edible} : \mathsf{e} \to \mathsf{type},\\ & \mathbf{eat} : \begin{bmatrix} y : \mathsf{e} \\ \mathbf{edible}(y) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{type},\\ & \mathbf{escape} : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{type},\\ & p_1 : \mathbf{animate}(j),\ p_2 : \mathbf{edible}(o),\ p_3 : \mathbf{eat}(o,\ p_2)(j,\ p_1).\end{aligned}$$

Now type checking is triggered to determine whether the SR (10) is well-formed. This is an example of nested presupposition, and underspecified terms are resolved from the most embedded one. Here we focus on the step to find a relation R that is substituted for the following underspecified term:

$$@_4 : \begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{type}.$$

The type checking tree for the relevant part looks as follows:

$$\dfrac{\dfrac{\begin{array}{c}\mathcal{D}_1\\ \begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} : \mathsf{t}\end{array} \qquad \begin{array}{c}\mathcal{D}_2\\ \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{t} : \mathsf{k}\end{array}}{\begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{t} : \mathsf{k}}\,(\Pi F) \qquad \vdots}{\left(@_4 : \begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{t}\right) : \begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{t}}\,(@)$$

where we use abbreviations k for kind and t for type. The type checking for $\mathcal{D}_1$ runs as follows.

$$\dfrac{\dfrac{}{\mathsf{e} : \mathsf{t}}\,(CON) \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathsf{e} : \mathsf{t}}\,(CON) \quad \dfrac{}{\mathsf{t} : \mathsf{k}}\,(CON)}{\mathsf{e} \to \mathsf{t} : \mathsf{k}}\,(\Pi F) \qquad \dfrac{\vdots}{\mathsf{e} \to \mathsf{t}\ true}}{@_1^{pr} : \mathsf{e} \to \mathsf{t}}\,(@) \qquad \dfrac{}{x : \mathsf{e}}\,(1)}{@_1^{pr}(x) : \mathsf{t}}\,(\Pi E)}{\begin{bmatrix} x : \mathsf{e} \\ @_1^{pr}(x) \end{bmatrix} : \mathsf{t}}\,(\Sigma F), 1$$

Similarly, the type checking for $\mathcal{D}_2$ runs as follows.

$$\dfrac{\dfrac{\dfrac{}{\mathsf{e} : \mathsf{t}}\,(CON) \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathsf{e} : \mathsf{t}}\,(CON) \quad \dfrac{}{\mathsf{t} : \mathsf{k}}\,(CON)}{\mathsf{e} \to \mathsf{t} : \mathsf{k}}\,(\Pi F) \qquad \dfrac{\vdots}{\mathsf{e} \to \mathsf{t}\ true}}{@_2^{pr} : \mathsf{e} \to \mathsf{t}}\,(@) \qquad \dfrac{}{x : \mathsf{e}}\,(1)}{@_2^{pr}(x) : \mathsf{t}}\,(\Pi E)}{\begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} : \mathsf{t}}\,(\Sigma F), 1 \qquad \dfrac{}{\mathsf{t} : \mathsf{k}}\,(CON)}{\begin{bmatrix} x : \mathsf{e} \\ @_2^{pr}(x) \end{bmatrix} \to \mathsf{t} : \mathsf{k}}\,(\Pi F)$$

The judgements $\mathsf{e} \to \mathsf{t}\ true$ in $\mathcal{D}_1$ and $\mathcal{D}_2$ trigger a proof search; given a suitable global context, we can find the antecedents **edible** of type $\mathsf{e} \to \mathsf{t}$ for $@_1^{sr}$, and **animate** of type $\mathsf{e} \to \mathsf{t}$ for $@_2^{sr}$, respectively. Replacing each underspecified term with its antecedent predicate, the above type checking tree is transformed as follows.

$$\dfrac{\dfrac{\overset{\mathcal{D}_1}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{edible}(x) \end{bmatrix} : \mathsf{t}} \quad \overset{\mathcal{D}_2}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t} : \mathsf{k}}}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{edible}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t} : \mathsf{k}}(\Pi F) \quad \overset{\vdots}{\begin{bmatrix} x : \mathsf{e} \\ \mathbf{edible}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t}\ true}}{\left(@_4 : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{edible}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t}\right) : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{edible}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t}}(@)$$

Then we can find an antecedent **eat** for $@_4$ that has a type

$$\begin{bmatrix} x : \mathsf{e} \\ \mathbf{edible}(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t}$$

in the context $\mathcal{K}_1$. In a similar way, we can find a proof term for other @-terms: p_2 for $@_3$, $((j,\ p_1),\ p_3)$ for $@_5$, and p_1 for $@_6$. By eliminating each @-term in (11) and reducing β-redexes, we obtain the SR $\mathbf{espace}(j,\ p_1)$ as a fully specified semantic representation for the sentence (10a).

5.2 Copredication for logical polysemies

The second phenomenon we consider is copredication of logically polysemous nouns. There are nouns having multiple meanings in natural language; it can be classified into accidental and logical polysemy (Asher [1]). For example, the noun *bank* in (12a) is accidentally polysemous, and the noun *book* in (12b) is logically polysemous.

(12) a. # The bank is closed and is muddy.
b. Mary memorized and burned the book.

The sentence (12b) shows that the logically polysemous noun *book* allows copredication, despite the fact that *memorized* and *burned* require different objects (i.e., informational objects and physical objects, respectively) as their object argument. To account for this fact, we can apply argument operators to the verbs *memorized* and *burned*, thereby avoiding the violation of selection restrictions.

We introduce the logical polysemies of nouns as functions. For example, we assign the following functions to the noun *book*.

$$\mathbf{book_{infoOf}}\ : (x : \mathsf{e}) \to (\mathbf{book}(x) \to \begin{bmatrix} y : \mathsf{e} \\ \mathbf{infoOf}(x)(y) \end{bmatrix})$$

$$\mathbf{book_{phyObjOf}}\ : (x : \mathsf{e}) \to (\mathbf{book}(x) \to \begin{bmatrix} y : \mathsf{e} \\ \mathbf{phyObjOf}(x)(y) \end{bmatrix})$$

The function $\mathbf{book_{infoOf}}$ (resp. $\mathbf{book_{phyObjOf}}$) takes an entity x and a proof of $\mathbf{book}(x)$ and returns an entity y that is the informational aspect (resp. the physical aspect) of x.

Now we can derive the SR of the sentence (12b) as follows.

$$\dfrac{\dfrac{\text{Mary}}{\begin{array}{c}NP\\ :m\end{array}} \quad \dfrac{\dfrac{\dfrac{\dfrac{\text{memorized}}{\begin{array}{c}S\backslash NP/NP\\ :MEM\end{array}} \quad \dfrac{\epsilon}{\begin{array}{c}(S\backslash NP/NP)\backslash(S\backslash NP/NP)\\ :arg_2\end{array}}}{\begin{array}{c}S\backslash NP/NP\\ :arg_2(MEM)\end{array}}< \quad \dfrac{\text{and}}{\begin{array}{c}CONJ\\ :\lambda p.\lambda q.\lambda y.\lambda x.\begin{bmatrix}p(y)(x)\\ q(y)(x)\end{bmatrix}\end{array}} \quad \dfrac{\dfrac{\text{burned}}{\begin{array}{c}S\backslash NP/NP\\ :BURN\end{array}} \quad \dfrac{\epsilon}{\begin{array}{c}(S\backslash NP/NP)\backslash(S\backslash NP/NP)\\ :arg_2\end{array}}}{\begin{array}{c}S\backslash NP/NP\\ :arg_2(BURN)\end{array}}<}{\begin{array}{c}S\backslash NP/NP\\ :\lambda y.\lambda x.\begin{bmatrix}arg_2(MEM)(y)(x)\\ arg_2(BURN)(y)(x)\end{bmatrix}\end{array}}\langle\Phi\rangle \quad \dfrac{\text{the book}}{\begin{array}{c}NP\\ :b\end{array}}}{\begin{array}{c}S\backslash NP\\ :\lambda x.\begin{bmatrix}arg_2(MEM)(b)(x)\\ arg_2(BURN)(b)(x)\end{bmatrix}\end{array}}>}{\begin{array}{c}S\\ :\begin{bmatrix}arg_2(MEM)(b)(m)\\ arg_2(BURN)(b)(m)\end{bmatrix}\end{array}}<$$

where

$$MEM \equiv \lambda y.\lambda x.\mathbf{memorize}(y, @_i : \begin{bmatrix} w : \mathsf{e} \\ \mathbf{infoOf}(w)(y) \end{bmatrix})(x, @_j : \mathbf{animate}(x)),$$

and

$$BURN \equiv \lambda y.\lambda x.\mathbf{burn}(y, @_i : \begin{bmatrix} w : \mathsf{e} \\ \mathbf{phyObjOf}(w)(y) \end{bmatrix})(x, @_j : \mathbf{animate}(x)).$$

Thus the sentence in (12b) is assigned the following SR.

$$(13) \quad \begin{bmatrix} \mathbf{memorize}(Z_2, @_{15} : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{infoOf}(x)(Z_2) \end{bmatrix})(m, @_{16} : \mathbf{animate}(m)) \\ \mathbf{burn}(Z_3, @_{17} : \begin{bmatrix} x : \mathsf{e} \\ \mathbf{phyObjOf}(x)(Z_3) \end{bmatrix})(m, @_{18} : \mathbf{animate}(m)) \end{bmatrix}$$

where Z_2 abbreviates

$$\pi_1\pi_1\left(@_7 : \begin{bmatrix} z : \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_2(x) \end{bmatrix} \\ (@_6 : \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_1(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_2(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_3(x) \end{bmatrix} \to \mathsf{t})(b, (@_4 : @^{pr}_1(b)))(m, (@_5 : @^{pr}_2(m)))(z) \end{bmatrix}\right),$$

and Z_3 abbreviates

$$\pi_1\pi_1\left(@_{14} : \begin{bmatrix} z : \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_9(x) \end{bmatrix} \\ (@_{13} : \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_8(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_9(x) \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ @^{pr}_{10}(x) \end{bmatrix} \to \mathsf{t})(b, (@_{11} : @^{pr}_8(b)))(m, (@_{12} : @^{pr}_9(m)))(z) \end{bmatrix}\right).$$

Let us suppose that we have the following information in the global context $\mathcal{K}_2$:

$$
\begin{aligned}
\mathcal{K}_2 \ \equiv\ & \mathsf{t} : \mathsf{k},\ \mathsf{e} : \mathsf{t}, \\
& m : \mathsf{e},\ b : \mathsf{e},\ i_b : \mathsf{e},\ p_b : \mathsf{e}, \\
& \mathbf{animate} : \mathsf{e} \to \mathsf{t},\ \mathbf{book} : \mathsf{e} \to \mathsf{t}, \\
& \mathbf{infoOf} : \mathsf{e} \to \mathsf{e} \to \mathsf{t},\ \mathbf{phyObjOf} : \mathsf{e} \to \mathsf{e} \to \mathsf{t}, \\
& \mathbf{memorize} : \begin{bmatrix} y : \mathsf{e} \\ \begin{bmatrix} w : \mathsf{e} \\ \mathbf{infoOf}(w)(y) \end{bmatrix} \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t}, \\
& \mathbf{burn} : \begin{bmatrix} y : \mathsf{e} \\ \begin{bmatrix} w : \mathsf{e} \\ \mathbf{phyObjOf}(w)(y) \end{bmatrix} \end{bmatrix} \to \begin{bmatrix} x : \mathsf{e} \\ \mathbf{animate}(x) \end{bmatrix} \to \mathsf{t}, \\
& \mathbf{book_{infoOf}} : (x : \mathsf{e}) \to (\mathbf{book}(x) \to \begin{bmatrix} y : \mathsf{e} \\ \mathbf{infoOf}(x)(y) \end{bmatrix}), \\
& \mathbf{book_{phyObjOf}} : (x : \mathsf{e}) \to (\mathbf{book}(x) \to \begin{bmatrix} y : \mathsf{e} \\ \mathbf{phyObjOf}(x)(y) \end{bmatrix}), \\
& p_1 : \mathbf{animate}(m),\ p_2 : \mathbf{book}(o), \\
& p_3 : \mathbf{infoOf}(b)(i_b),\ p_4 : \mathbf{phyObjOd}(b)(p_b).
\end{aligned}
$$

Then we can find a proof term for each @-term in SR Z_2 as follows. Here $@_i \longmapsto T$ means that the underspecified term $@_i$ is replaced with a term T.

$@_1 \longmapsto \mathbf{book}$,

$@_2 \longmapsto \mathbf{animate}$,

$@_3 \longmapsto \mathbf{infoOf}(b)$,

$@_4 \longmapsto p_2$,

$@_5 \longmapsto p_1$,

$@_6 \longmapsto \lambda y.\lambda x.\lambda z. \begin{bmatrix} u : \mathbf{book}(y) \\ \mathbf{book_{infoOf}}(y)(u) =_{\mathsf{e}} z \end{bmatrix}$,

$@_7 \longmapsto$
$((i_b,\ p_3), (\lambda y.\lambda x.\lambda z. \begin{bmatrix} u : \mathbf{book}(y) \\ \mathbf{book_{infoOf}}(y)(u) =_{\mathsf{e}} z \end{bmatrix})(b,\ p_2)(m,\ p_1)(i_b,\ p_3))$.

And we can also find a proof term for each @-term in SR Z_3:

$@_8 \longmapsto \mathbf{book}$,

$@_9 \longmapsto \mathbf{animate}$,

$@_{10} \longmapsto \mathbf{phyObjOf}(b)$,

$@_{11} \longmapsto p_2$,

$@_{12} \longmapsto p_1,$

$@_{13} \longmapsto \lambda y.\lambda x.\lambda z. \begin{bmatrix} u : \mathbf{book}(y) \\ \mathbf{book_{phyObjOf}}(y)(u) =_{\mathsf{e}} z \end{bmatrix},$

$@_{14} \longmapsto$

$((p_b,\ p_3), (\lambda y.\lambda x.\lambda z. \begin{bmatrix} u : \mathbf{book}(y) \\ \mathbf{book_{phyObjOf}}(y)(u) =_{\mathsf{e}} z \end{bmatrix})(b,\ p_2)(m,\ p_1)(p_b,\ p_4)).$

The rest of the underspecified terms can also be replaced with specific terms as follows.

$@_{15} \longmapsto p_3$

$@_{16} \longmapsto p_1$

$@_{17} \longmapsto p_4$

$@_{18} \longmapsto p_1$

By eliminating each @-term in (13) and reducing β-redexes, we obtain the following as a fully specified semantic representation for the sentence (12b).

(14) $\begin{bmatrix} \mathbf{memorize}(i_b,\ p_3)(m,\ p_1) \\ \mathbf{burn}(p_b,\ p_4)(m,\ p_1) \end{bmatrix}$

6 Conclusion

In this paper, we proposed an analysis that treats the selectional restrictions of predicates as presuppositions. In addition, using argument operators, we gave a unified analysis of lexical phenomena that are not accounted for by simple analyses of selectional restrictions. Future work includes extending our analysis to such phenomena as metaphors, which Asher [1] opened up a way to analyze in type theoretical settings.

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References

[1] Asher, N.: Lexical Meaning in Context: A Web of Words. Cambridge University Press (2011)

[2] Asher, N.: Selectional restrictions, types and categories. Journal of Applied Logic 12(1), 75–87 (2014)

[3] Beaver, D.I.: Presupposition and Assertion in Dynamic Semantics. Studies in Logic, Language and Information, CSLI Publications & FoLLI (2001)

[4] Bekki, D.: Representing anaphora with dependent types. In: Logical Aspects of Computational Linguistics, pp. 14–29. Springer (2014)

[5] Bekki, D., Mineshima, K.: Context-passing and underspecification in dependent type semantics. In: Chatzikyriakidis, S., Luo, Z. (eds.) Modern Perspectives in Type Theoretical Semantics. Studies of Linguistics and Philosophy, Springer (2016)

[6] Luo, Z.: Formal semantics in modern type theories with coercive subtyping. Linguistics and Philosophy 35(6), 491–513 (2012)

[7] Magidor, O.: Categiry Mistakes. Oxford University Press (2013)

[8] Martin-Löf, P.: Intuitionistic type theory. Bibliopolis (1984)

[9] McCawley, J.D.: Concerning the base component of a transformational grammar. Foundations of Language 4(3), 243–269 (1968)

[10] Nunberg, G.: Transfers of meaning. Journal of Semantics 12(2), 109–132 (1995)

[11] Retoré, C.: The montagovian generative lexicon lambda tyn: a type theoretical framework for natural language semantics. In: 19th international conference on types for proofs and programs (TYPES 2013). pp. 202–229 (2014)

[12] Steedman, M.: Surface Structure and Interpretation. The MIT Press (1996)

Reference and Pattern Recognition: A Metasemantic Study

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In this talk I wish to address the question of how the reference of a proper name is determined. I will propose what might be regarded as a descendant of the "cluster theory" of descriptions: the reference of a proper name is determined on the basis of the properties each individual speaker associates to the name. Whereas the traditional cluster theory holds that the referent of a name must satisfy most of the properties a speaker ties to the name, I will argue that such properties just play the role of the basis on which the speaker decides whether a piece of information, whether it is given perceptually or verbally, is similar enough to them so that it comes from the same object to which they are associated. This is a task of "pattern recognition", widely studied in cognitive science and machine learning. The aim of this talk is to reconsider the question of reference from this perspective, and in doing this I will claim that a certain notion of stability of a concept is required in order for there to be a referent of community-wide uses of a name. Finally I will draw out some implications of this view for Kripke's causal account.

Interpretation of Wh-words in Aza-Irabu Miyakoan

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Abstract. The purpose of this paper is to discuss the semantic interpretation of wh-words of Aza-Irabu Miyakoan, a southern Ryukyuan language. In Aza-Irabu Miyakoan, focus particles show concord with a specific type of the sentence: *Ga* with wh-interrogatives, *ru* with yes/no-interrogatives and *du* with declaratives. In the complement of question embedding verbs, however, *du* can appear with bare wh-words particularly when the attitude holder knows the answer of the embedded question. I argue that such wh-words metalinguistically quantify variables over expressions.

Keywords: *kakarimusubi*, wh-words, focus particles, embedded questions, quotative marker, metalinguistic quantification

1 Introduction

Aza-Irabu Miyakoan[1] has *kakarimusubi* in the sense that focus particles appearing in the middle of the sentence show concords with sentence types:[2] *Du* with declaratives, *ru* with yes/no interrogatives, *ga* with wh-interrogatives as shown by each example in (1).

(1) a. Kincjaku=nu=**du** uti-taa.
wallet=NOM=FOC fall-PST
'A wallet fell.'

b. Kincjaku=nu=**ru** uti-taa?
wallet=NOM=FOC fall-PST
'Did the wallet fall?'

[1] Miyakoan is a language spoken in Miyako Islands, located about 180 miles southwest of Okinawa Island. Aza-Irabu is a variety of Miyakoan, spoken in the Irabu community of the Irabu island. Aza-Irabu Miyakoan, as well as other Ryukyuan languages, shares major typological features of morphosyntax with Japanese: Head final, SOV word order, dependent marking, accusative case alignment, agglutinative suffixal morphology (See [13] for grammatical sketches of Ryukyuan languages). Due to the combination of those typological similarities with cultural blending, the number of traditional speakers of Aza-Irabu is severely declining.

[2] See [9] for the definition of *kakarimusubi* adopted in this paper.

c. Noo=nu=**ga** uti-taa?
what=NOM=FOC fall-PST
'What fell?'

Replacing the focus particles in (1) changes the type of the sentence or leads to ungrammaticality: While using *du* instead of *ru* makes the sentence in (1-b) declaratives, the use of *du* in (1-c) makes it ungrammatical due to the incompatibility of *du* and wh-words as in (2).

(2) *Noo=nu=**du** uti-taa?
what=NOM=FOC fall-PST
'What fell?'

In embedded clauses, however, *du* as well as *ga* can be used despite the existence of wh-words as in (3). I mark embedded clauses by brackets hereafter.

(3) Obaa=ja [ici={**ga/du**} fuɨ kutu]=tii=ja sa-n.
Grandma=TOP when=FOC come SFP=QUOT=TOP know-NEG
'Grandma does not know when he will come.'

This paper discusses the semantic interpretation of wh-words and particles in such examples as in (3), clarifying the condition of the use of wh-words with a declarative marker *du*.

2 Data

To clarify the condition of the use of *du* with bare wh-words, I prepared the following set of embedding contexts, which is illustrated with the clause 'who came' embedded.

(4) A. **I don't know** who came.
B. **He doesn't know** who came.
C. **Do you know** who came?
D. **He knows** who came.
E. **I know** who came.

The contexts from A to E are arranged according to the attitude holder's certainty about the answer of the question embedded: While the attitude holders are uncertain about the answers of the questions in A and B, they are not in D and E; C is in-between. The difference between A and B, as well as D and E, is the referent of the attitude holder: A and E have the first person pronoun, i.e. the speaker, as the matrix subject whereas B and D have the third person. As the result, the embedded clause is similar to direct questions more in A than in B, provided that the speaker does not know the answer when he utters a question like 'Who came?' At the other end of the scale, E is the farthest from direct questions since it encodes that the speaker knows the answer. Thus, *ga*-markings are most expected to appear for wh-phrases of A and least expected for those of E, given the obligatory *ga*-marking in unembedded wh-interrogatives as in (1-c).

I conducted an experimental survey designed to confirm whether this expectation is on the right track. The informants are two native speakers of Aza-Irabu: TG born in 1935 and KK born in 1924[3]. Not informing the speakers of the intention of this research, I requested them to translate Japanese sentences into their dialect. The Japanese sentences provided in the survey are varied by embedding the following clauses into the contexts given in (4).

(5) a. **who** cleaned the room
b. **what** Grandma is reading
c. **where** this car is heading
d. **when** K comes
e. **which** one the children ordered
f. **why** Grandma is angry

The result of the experiment is given in Table 1. The capital alphabets in the top line correspond to those in (4) and the wh-words in the first column represent different sentences embedded in the complement of *know*, i.e. (5).[4]

Table 1. Distribution of *ga* and *du*

	A.	B.	C.	D.	E.
who	ϕ	ϕ	ϕ	*du*	*du*
what	*ga*	*ga*	*du/ga*	*du*	*du*
where	*ga*	*ga/du*	*du*	*du*	*du*
when	*ga/ϕ*	*du/ga/ϕ*	*ga*	*du*	*du*
which	*ga*	*ga*	*du/ga*	*du*	*du*
why	*ga*	*ga*	*du*	*du*	*du*

TG (male, 1935)

	A.	B.	C.	D.	E.
who	ϕ	ϕ	*du*	*du*	*du*
what	*ga*	*ga*	*ga*	*du*	*du*
where	*ga*	*ga*	*du*	*du*	*du*
when	*ga*	*ga/ du*	*du*	*du*	*du*
which	*ga*	*ga*	*ga*	*ga/ du*	*du*
why	*ga*	*ga*	*ga*	*du*	*du*

KK (male, 1924)

When the informant utters several sentences for one stimulus sentence and the use of focus particles is not consistent, I write them with slash indicating that the left one is more often uttered than the right. The shading indicates that the clause is marked by a quotative marker *tii* and the rest is marked by *gara*.

I give in (6) examples of the **what**'s line of Speaker TG.

(6) A. Abaa [mma=ʕa noo=ju=**ga** jummi bui]=**gara**
1SG.TOP Grandma=NOM what=ACC=FOC read CONT=Q
sa-n.
know-NEG
'I don't know what Grandma is reading.'

[3] I also conducted an experiment with a person born in 1941. But he always uses *ga* even in embedded contexts. The detailed research on the age and the social status which evoke this difference is left for another project.

[4] ϕ in Table 1 means that no focus particle attaches to wh-phrases. In the **who**'s line, the lack of focus particles is due to a morphophonological constraint of this dialect: /ga/ does not appear after the long vowel /aa/ or /aʕa/. The reason for the drop of particles in the **when**'s line of TG has not yet been made clear.

B. Mma=a [jarabi-taa noo=ju=**ga** jummi buɨ]=**gara**
Granma=TOP children-PL what=ACC=FOC read CONT=Q
sa-n noo-ham.
know-NEG EVI-ACOP
'Grandma doesn't know what the children are reading.'

C. Ja=a [mma=ʕa noo=nu hun=nu={**du**/**ga**}
2SG=TOP Grandma=NOM what=GEN book=ACC=FOC
jummi buɨ]=**tii**=ja sɨdzi=ru buɨ?
read CONT=QUOT=TOP know=FOC CONT
'Do you know what Grandma is reading?'

D. Uja ja=tigaa [mma=ʕa noo=ju=**du** jummi
father COP=COND Grandma=NOM what=ACC=FOC read
buɨ]=**tii**=ja sɨdzi=du buɨ=padzɨ.
CONT=QUOT=TOP know=FOC CONT=CONJ
'Our father knows what Grandma is reading.'

E. Abaa [mma=ʕa noo=ju=**du** jummi
1SG.TOP Grandma=NOM what=ACC=FOC read
buɨ]=**tii**=ja sɨdzi=du buɨ=suga ndzɨ
CONT=QUOT=TOP know=FOC CONT=CONC say
taf=fa nii-n.
OPT=TOP no-NEG
'I know what Grandma is reading but I don't want to say it.'

Table 1 exhibits the following: *Ga* and *du* are obligatorily used for A and E respectively; The use of *ga* is superior to that of *du* in B and vice versa in D; both *ga* and *du* are evenly used in C. This confirms our expectation that *ga* is most used in A and least in E. Moreover, I construe the above gradable distribution from A to E as indicating that *du* is used when the embedded clause denotes a proposition whereas *ga* is used when it denotes a set of proposition. This hypothesis is considered as a natural consequence of the fact that *du* and *ga* are used respectively for declaratives and interrogatives in matrix clauses. The distribution of *ga* and *du* in Table 1 can be elucidated from this viewpoint in the following way.

In context E, the attitude holder, i.e. the speaker, envisages a particular proposition since he/she knows the answer of the question. Thus, the use of *du* is obligatory. In context D, *du* is preferred because the attitude holder, not the speaker in this case, has a particular proposition in mind. But D differs from E in that the speaker might not know the answer of the question. Therefore, *ga* is exceptionally used as in the **which**'s line of KK. The attitude holder's stance toward the answer of the embedded question is neutral in context C. Depending on the speaker's presumption of the knowledge of the attitude holder, he/she uses *ga* or *du*. B and A can be explained by the mirror image of D and E respectively. Particularly, there is no possibility for *du* to appear in the context A. Under this context, the attitude holder=speaker does not associate any particular proposition to the embedded clause, since he/she is uncertain about the answer. Therefore, the semantic object of the embedded clause cannot be a proposition, but is a set of propositions.

Another generalization drawn from Table 1 is that *du* is used only when the complement is marked by a quotative marker *tii*. While there are examples of *ga* appearing within the quotative marker, there is only one exception of *du* appearing in a clause marked by *gara*, i.e. the **where**'s line of B in Speaker TG. It is not unreasonable to consider it as an exception, because this example is obtained after the speaker utters the sentences in the **where**'s line of C and, after the utterance, he corrected the relevant example replacing *du* with *ga*, which suggests that the questionable sentence is generated by the influence of the preceding utterance. Granting that the use of *du* in the **where**'s line of B is exceptional, the combination of focus particles and complementizers in Aza-Irabu is schematized as in (7).

(7) a. [....WH=ga....]=gara V.
b. [....WH=ga....]=tii V.
c. *[....WH=du....]=gara V.
d. [....WH=du....]=tii V.

Assuming that the status of *gara* as a question marker[5], the appropriateness of the pattern (7-a) naturally follows: *Ga* and *gara* cooperatively make the sentence questions. The acceptability of (7-b) is not surprising as well, because quotative markers can embed questions as well as statements. What prevents the pattern (7-c) from naturally occurring in the experiment? One reasonable explanation is to attribute it to the mismatch between the meaning of *du* and that of *gara*: *Gara* requires the clause to be a set of propositions as a question marker; the sentence with *du* nonetheless denotes a proposition as a declarative marker. This is reasonable because the hypothesis that the clause with *du* denotes a proposition also explained the distribution of focus particles in Table 1. If this line of reasoning is on the right track, the clause marked by *du* in (7-d) denotes a proposition though it includes a wh-word.

The above discussion allows us to rephrase the compatibility of *du* and wh-words as a problem of how to interpret a wh-word used in a proposition. I maintain in this article that wh-words in *du*-marked clauses are employed to hide the value of expressions for some reasons. For example in E of (6), the speaker knows the proposition but, as the context indicates, he is reluctant to convey the explicit information to the addressee, which leads him to use wh-words to hide the expression. In the case of D of (6), though the attitude holder knows the answer, i.e. a proposition, the speaker might not know it, which forces him to use wh-words to cover the value. In the next section, those uses of wh-words in declaratives are analyzed to be made possible by the function of a quotative marker *tii*, only by which the coexistence of *du* and wh-words is admitted.

[5] Although I have not found an example of *gara* used in direct questions in Aza-Irabu, the same form is adapted to mark self-addressed questions in many varieties of Miyakoan.

3 Account

Given the ungrammaticality of using *du* with wh-words in matrix questions, e.g. (2), there would be two simple ways to account for the compatibility of wh-words and *du* in some embedded clauses: One is to move *du* and the other to move wh-phrases, which result in interpreting them in different places. It is untenable to move *du*, however. Even if we move *du* to the matrix clause to circumvent the coexistence with wh-phrases, there are examples in which matrix clauses are also questions: Consider the use of *du* in C of (6).

I take an approach to move wh-phrases, using Karttunen's theory of questions ([8]). In [8], wh-phrases are treated in the same way as quantified expressions, which are introduced into a sentence through 'rules of quantification' in Montague semantics ([11]). Thus, wh-phrases need to move higher than the sentence they combine with.

The semantic object of the sentence which can be the complement of wh-phrases is called 'proto-questions' in [8]. In his theory, proto-questions are formed by an operator '?'; I assign the function of this operator to the question particle *ga* in Aza-Irabu. Thus, the basic idea of forming wh-questions is to let wh-phrases quantify into a set of propositions created by the particle *ga*.

To implement this idea, both wh-phrases and focus particles must move as in (8)[6], which is the LF representation of the example (1-c) and (2), repeated here as (9).

(8) $[_{\mathrm{IP}_1}$ noo $[_{\mathrm{IP}_2}$ $[_{\mathrm{IP}_3}$ t_8=nu uti-taa] {ga/du}]]

(9) Noo=nu={**ga**/***du**} uti-taa?
what=NOM=FOC fall-PST
'What did you drop?'

The trace of wh-phrases is interpreted as a variable with an index, which enables *ga* to take a proposition as the argument as in (10).

(10) a. $[\![\mathrm{t}_8 \text{ uti}]\!]^{M,w,g} = \lambda w'.fall(x_8)(w')$
b. $[\![\text{ga}]\!]^{M,w,g} = \lambda p.\lambda q[q(w) \ \&\ q = p]$
c. $[\![\text{ga}]\!]^{M,w,g}([\![\mathrm{t}_8 \text{ uti}]\!]^{M,w,g}) = \lambda q[q(w) \ \&\ q = \lambda w'.fall(x_8)(w')]$

[6] Aside from semantic interpretations, there is another evidence which advocates the movement of focus particles. As shown in the following example, when a wh-phrase resides within a syntactic island, *ga* cannot attach directly to that phrase and must appear outside the island. This is straightforwardly explained if we assume movement of focus particles to the sentence periphery. Similar movement is assumed for focus particles of Sinhala by [4] and [10].

(i) [Ndza=nkai(=***ga**) pii pɨtu]=nu=**ga** jamaʕasa-ha-taa?
where=ALL(=FOC) go human=NOM=FOC many-ACOP-PST
'Where$_i$ were there many [people that went t_i]?'

The result of applying *ga* to a proposition is a set of propositions as in (10-c), i.e. a proto-question, which will be combined with wh-phrases through Wh-quantification Rule given in (11).

(11) *Wh-quantification Rule*
If α is a branching node with $\beta \in P_{WH}$ and $\gamma \in P_Q$ containing an occurrence of t_i for some integer i,
then $[\![\alpha]\!]^{M,w,g} = \{p | [\![\beta]\!]^{M,w,g}(\lambda x_i.[[\![\gamma]\!]^{M,w,g}(p)])\}$

In Montague semantics, a set of phrases P for each category is defined and rules are applied to specific categories: The set of wh-phrases P_{WH} and the set of questions P_Q in this rule. Assuming that the semantic content of wh-phrases is an existential quantifier, we can correctly calculate the meaning of (9) with *ga* attached.

(12) $[\![\text{noo nu ga utitaa}]\!]^{M,w,g}$
$=\{p|\lambda P.\exists x[P(x)](\lambda y[\lambda q[q(w) \ \& \ q = \lambda w'.fall(y)(w')](p)])\}$
$=\{p|\exists x[p(w) \ \& \ p = \lambda w'.fall(x)(w')]\}$

The denotation of (12) is a set of true propositions of the form 'x fell.'

Since *du* appears in declarative clauses in matrix contexts, it is reasonable to assume the semantics of *du* simply as in (13-a), which is applied to (10-a) as in (13-b).

(13) a. $[\![\text{du}]\!]^{M,w,g} = \lambda p.p$
b. $[\![\text{du}]\!]^{M,w,g}([\![\text{t}_8 \text{ uti}]\!]^{M,w,g}) = \lambda w'.fall(x_8)(w')$

Since the result is just a proposition, it fails to satisfy any part of the rule in (11): Hence, the ungrammaticality of (9) with *du*.

Let us now proceed to the subordinate clause. As noted in the previous section, the question constituted by *ga* can be embedded both by *gara* and *tii* (see (7)). Let us consider *tii* here as a simple complementizer whose semantic contribution is vacuous. Recall that *gara*, derived from *ga*, was regarded to function as a question marker. Here I assume the semantics of *gara* as an identity function as in (15-a), i.e. a function from a set of propositions to a set of propositions. These assumptions allow us to treat the embedded clauses marked by *gara* and *tii* uniformly as a set of propositions. An illustration of combining those embedded clauses with its matrix is given in (15), where *gara* is reduced to 'ra' for a morphophonological constraint mentioned in footnote 4.

(14) Abaa [noo=nu=**ga** uti-taa]=**ra** sa-n.
1SG.TOP what=NOM=FOC fall-PST=Q know-NEG
'I don't know what fell.'

(15) a. $[\![\text{gara}]\!]^{M,w,g} = \lambda Q_{\langle st,t \rangle}.Q$
b. $[\![\text{noo nu ga utitaa ra}]\!]^{M,w,g} = \{p|\exists x[p(w) \ \& \ p = \lambda w'.fall(x)(w')]\}$
c. $[\![\text{sɨ}]\!]^{M,w,g} = \lambda Q_{\langle st,t \rangle}.\lambda x_e.\exists p \in Q[p(w) = 1 \ \& \ \text{DOX}^w_x \subseteq p]$
d. $[\![\text{sɨ}]\!]^{M,w,g}([\![\text{noo nu ga utitaa ra}]\!]^{M,w,g})([\![\text{an}]\!]^{M,w,g})$
$= \exists p \in \{p|\exists x[p(w)\& \ p = \lambda w'.fall(x)(w')]\}[p(w) = 1 \& \text{DOX}^w_{spkr} \subseteq p]$

e. $[\![\mathrm{n}]\!]^{M,w,g}([\![\text{abaa noo nu ga utitaa ra sa}]\!]^{M,w,g})$
$= \lambda q.\neg q(\exists p \in \{p|\exists x[p(w)\&p = \lambda w'.fall(x)(w')]\}$
$[p(w) = 1\&\mathrm{DOX}^{w}_{spkr} \subseteq p])$
$= \neg\exists p \in \{p|\exists x[p(w)\&p = \lambda w'.fall(x)(w')]\}$
$[p(w) = 1\&\mathrm{DOX}^{w}_{spkr} \subseteq p]$

In (15-c), I follow [15] in the treatment of the predicate *know* (*sɨ* in Aza-Irabu), which takes a set of propositions and a subject, and returns true iff the subject believes a true proposition in that set.[7] (15-d) is the result of *sɨ* taking the complement and the subject, which is eventually in the scope of negation as in (15-e). If *du* appears in the clause marked by *gara*, as in the pattern of (7-c), the ungrammaticality is predicted, since a proposition like (13-b) cannot be the argument of *gara* in (15-a).

In problematic cases such as (16), wh-words must move out of the subordinate clause, otherwise the semantic interpretation collapses due to the coexistence of *du* and wh-words in the same clause.

(16) Abaa [noo=nu=**du** uti-taa]=**tii**=ja sɨdzi=du
1SG.TOP what=NOM=FOC fall-PST=QUOT=TOP know=FOC
buɨ.
CONT
'I know what fell.'

It is still not possible to apply Wh-quantification Rule to (16) with the wh-phrase dislocalted, since the matrix clause does not have a question denotation but is a declarative sentence. This forces us to posit a new rule which can combine wh-words with some specific sorts of declarative sentences.

The basic idea behind the co-occurrence of wh-words and *du* is, as was argued in the previous section, that while *du* denotes a proposition, wh-words are used to cover the term, for example, not to reveal it to the addressee. I embody this idea in the notion of *metalinguistic quantification* developed by [14]: His proposal is that it is not only individuals but also expressions that can be quantified and thus quantification into quotational contexts is possible. The above idea is rephrased using this notion as follows: While *du* expresses that the attitude holder knows the proposition which constitutes the answer, wh-words are used to quantify the expression of the focus of the answer to hide it.

In a model theoretic fragment which utilizes metalinguistic quantification, a new atomic type u, the type of expressions, must be added to the basic inventry of types e, t and s, and the model is modified accordingly. In addition, expressions are also interpreted relative to $\mathcal{Q}$, which assign entities in D_u to variables over expressions. Although [14] conceives that specific types of predicates directly take expressions, assuming, for example, separate lexical entries for the propositional say_1 and the quotational say_2 (cf. [12]):

[7] To simplify the discussion, I do not include exhaustivity in the interpretation of embedded questions. See [6] and [2] for a method to induce exhaustive interpretations in the Karttunen's theory of questions.

(17) a. $[\![\mathrm{say}_1]\!]^{M,w,g,\mathcal{Q}} = \lambda p_{st}.\lambda y_e.\lambda w'.\ report(p)(y)(w')$
b. $[\![\mathrm{say}_2]\!]^{M,w,g,\mathcal{Q}} = \lambda X_u.\lambda y_e.\lambda w'.\ utter(X)(y)(w')$,

I contend that, in Aza-Irabu, expressions are integrated into semantic compositions by way of *tii*, which leads us to assume separate entries for it: Tii_1 for the complementizer use and tii_2 for the quotative use. Let $< \Phi\ ;\ \alpha >$ an arbitrary linguistic object, where Φ is a phonological representation and α is a syntactically structured object. Then, the function of each *tii* is described as in (18).

(18) a. $\mathrm{tii}_1(< \Phi\ ;\ \alpha >) = \alpha$
b. $\mathrm{tii}_2(< \Phi\ ;\ \alpha >) = \ulcorner \Phi \urcorner$

Note that $< \Phi;\ \alpha >$ itself does not belong to any type and thus is not an object of meaning language.[8] With the help of tii_2, the phonological content transforms to 'proto-expressions', which will be the argument of quotative $ndz\text{ɨ}_2$ (say_2) by the interpretation function $[\![\ ]\!]^{M,w,g,\mathcal{Q}}$. I use $\ulcorner\ \urcorner$ both for proto-expressions and expressions which is a member of D_u.

(19) $[\![\mathrm{ndz}\text{ɨ}_2]\!]^{M,w,g,\mathcal{Q}}([\![\mathrm{tii}_2(<\text{it's sunny};\ [it'[is'[sunny']]] >)]\!]^{M,w,g,\mathcal{Q}})$
$= [\![\mathrm{ndz}\text{ɨ}_2]\!]^{M,w,g,\mathcal{Q}}([\![\ulcorner \text{ it's sunny } \urcorner]\!]^{M,w,g,\mathcal{Q}})$
$= \lambda X_u.\lambda x_e.\lambda w'.[utter(X)(x)(w)](\ulcorner \text{ it's sunny } \urcorner)$
$= \lambda x_e.\lambda w'.[utter(\ulcorner \text{ it's sunny } \urcorner)(x)(w')]$

Identifying the *tii* in (16) with tii_2 gives us the following application.

(20) $\mathrm{tii}_2(< \text{noo nu du utitaa};\ [[noo']du']uti'] >)$
$= \ulcorner\text{noo nu du utitaa}\urcorner$

In order for expressions like (20) to be the argument of *sɨ* (know), the meaning of it must be ambiguated between the proposition-taking *know* and the expression-taking *know.*

(21) a. $[\![\mathrm{s}\text{ɨ}_1]\!]^{M,w,g,\mathcal{Q}} = \lambda p_{st}.\lambda y_e.[p(w)\ \&\ \mathrm{DOX}_y^w \subseteq p]$
b. $[\![\mathrm{s}\text{ɨ}_2]\!]^{M,w,g,\mathcal{Q}} = \lambda X_u.\lambda y_e.\exists p_{st}[p(w)\ \&\ \mathrm{DOX}_y^w \subseteq p\ \&\ p = Cont(X)]$

When $s\text{ɨ}_2$ takes expressions X as in (21-b), the attitude holder believes the true proposition p, which corresponds to the content of the expression X. *Cont* works to make this correspondence based on the syntactic information α of $< \Phi;\ \alpha >$.[9]

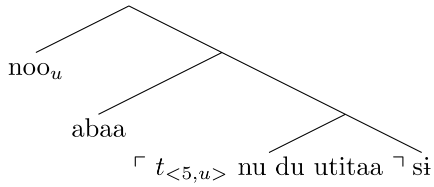
The result of applying tii_2 to a linguistic object constitutes a part of the syntactic structure of the whole sentence. Since the wh-word *noo* is employed here to quantify the expression, subscribed with u here, the phrase containing it

[8] [12] offers a grammar which generates triples $< \Pi\ ; \Sigma\ ;\ \alpha : \tau >$ where each member represents phonological, syntactic and semantic content. He considers $< \Pi\ ; \Sigma\ ;\ \alpha : \tau >$ itself also to be a member of D_u, which is the crucial difference from the assumption adopted here.

[9] If it is not possible to retrieve a propositional meaning from α, the function *Cont* is undefined.

moves to a place where it can take a set of expressions as its argument. Thus the LF representation of (16) looks like (22), in which the trace of the wh-phrase is indexed with an integer and a type ([14], [7]).

(22)



Based on this LF representation, the meaning of (16) is appropriately calculated as follows.

(23) $[\![\text{sɨ}_2]\!]^{M,w,g,\mathcal{Q}}([\![\ulcorner t_{<5,u>} \text{ nu du utitaa } \urcorner]\!]^{M,w,g,\mathcal{Q}})([\![\text{an}]\!]^{M,w,g,\mathcal{Q}})$
$= \lambda X_u.\lambda y_e.\exists p[p(w)\ \&\ \text{DOX}_y^w \subseteq p\ \&\ p = \mathit{Cont}(X)](\ulcorner Y_{<5,u>} \text{ nu du utitaa } \urcorner)(\mathit{spkr})$
$= \exists p[p(w)\ \&\ \text{DOX}_{spkr}^w \subseteq p\ \&\ p = \mathit{Cont}(\ulcorner Y_{<5,u>} \text{ nu du utitaa}\urcorner)]$

Finally, we need a rule for metalinguistic quantification to bind a variable left in the proposition.

(24) *Metalinguistic (Wh-)Quantification Rule*
If α is a branching node with $\beta \in P_{WH}$ and $\gamma \in P_t$ containing an occurrence of $t_{<i,u>}$ for some integer i,
then $[\![\alpha]\!]^{M,w,g,\mathcal{Q}} = [\![\beta]\!]^{M,w,g,\mathcal{Q}}(\lambda X_{<i,u>}.[[\![\gamma]\!]^{M,w,g,\mathcal{Q}}])$

The denotation of the wh-phrase is an existential quantifier as before, but differs from the previous version in that it quantifies over expressions, being extracted from inside expressions.

(25) a. $[\![\text{noo}_u]\!]^{M,w,g,\mathcal{Q}} = \lambda P_{ut}.\ \exists X_u.P(X)$
b. $[\![\text{noo}_u]\!]^{M,w,g,\mathcal{Q}}(\lambda Y_{<5,u>}.\ [\![\text{abaa } \ulcorner \text{ t}_{<5,u>} \text{ nu du utitaa } \urcorner \text{ sɨ}_2]\!]^{M,w,g,\mathcal{Q}})$
$= \lambda P_{ut}.\ \exists X_u[P(X)](\lambda Y_u.\ \exists p[p(w)\ \&\ \text{DOX}_{spkr}^w \subseteq p\ \&\ p = \mathit{Cont}(\ulcorner Y \text{ nu du utitaa}\urcorner)])$
$= \exists X_u \exists p[p(w)\ \&\ \text{DOX}_{spkr}^w \subseteq p\ \&\ p = \mathit{Cont}(\ulcorner X \text{ nu du utitaa}\urcorner)]$

The formula in (25-b) predicts the use of *du* in the embedded clause since it denotes a proposition *p*. Further, the account in this section predicts that *du* co-occurs with wh-words only within quotative clauses. This is because variables of type *u* are created only by the quotative marker *tii* and metalinguistic quantification by wh-phrases operates on this type of variables from the outside of the clause.[10]

[10] One might wonder whether the movement of wh-phrases is an indispensable part of our analysis. Since wh-movement is required by the [8]'s treatment of wh-phrases as quantifiers, it seems not impossible to get rid of the movement by adopting other theories on questions ([5], [3] etc): For example, it is a viable option that noo_u translates to a variable over expressions in-situ and the existential closure takes place at the top node. On the other hand, the change of the meaning of wh-phrases, i.e. the

4 Conclusion and further issue

In Aza-Irabu, focus particles agree with sentence types: *Du* with declaratives, *ru* with yes/no interrogatives and *ga* with wh-interrogatives. In embedded questions, however, *du* can be used particularly when the speaker or the attitude holder knows the answer of the question. In those examples, I considered the embedded clause to denote a proposition, a particular answer, and that wh-words are used to hide the value of expressions. I materialized this idea in the notion of metalinguistic quantification.

Metalinguistic quantification was first proposed by [14] for the analysis of wh-doublets in Japanese, which is similar to English *such-and-such* and *so-and-so*. Wh-doublets in Aza-Irabu seem to have functions similar to Japanese ones, appearing in quotational contexts as shown in (26).

(26) [**Taru taru**=ga niv-kam]=tii=du cɨmo=o idii
who who=NOM late-ACOP=QUOT=FOC heart=TOP out
bu-taa.
CONT-PST
'He complained that so-and-so is late.'

Different from Japanese, wh-doublets in Aza-Irabu can be used in questions, inducing plural interpretations (see [1] for extensive research of wh-doublets in Yaeyaman): (27-a) is an example of direct questions and (27-b) is a plural version of problematic examples.

(27) a. **Noo noo**=nu=ga uti-taa?
what what=NOM=FOC fall-PST
'What fell?' (plural interpretation)
b. Abaa [**noo noo**=nu=du uti-taa]=tii=ja
1SG.TOP what what=NOM=FOC fall-PST=QUOT=TOP
sɨdzi=du buɨ.
know=FOC CONT
'I know what fell.' (plural interpretation)

On the other hand, the use of wh-singlets in the context of (26) is not perfect according to the informants; they prefer proper nouns to wh-singlets in this context. But this is not a restriction imposed by the semantics of wh-singlets, but by pragmatics which requires explicit reference if there is only one individual. This means that contextual adjustments make it possible for wh-singlets to function like Japanese doublets. Though the precise condition of the use of wh-singlets has not yet been fully clarified, it is quite likely that the context in which the

usual *noo* and noo_u, is essential. This is because any accurate analysis must account for the compatibility of *du* and wh-phrases in the subordinate clause and the incompatibility of them in the matrix clause simultaneously. Therefore, the explanation of the incompatibility based on the meaning of *du* and wh-words automatically leads to transforming the meanings of those items. This is exactly what is achieved in this paper by the mechanism metalinguistic quantification.

speaker knows the proposition but is reluctant to inform it, c.f. (6)-E, is one of those adjusted contexts, which encourages the analysis of embedded questions using metalinguistic quantification.

Abbreviations

1sg = 1st person singular, 2sg = 2nd person singular, acc = accusative, acop = adjectival copula, all = allative, conc = concessive, cond = conditional, conj = conjecture, cont = continuous, cop = copula, evi = evidential, foc = focus, gen = genitive, neg = negation, nom = nominative, opt = optative, pl = plural, pst = past, q = question, quot = quotation, sfp = sentence final particle, top = topic

References

1. Davis, Christopher. 2015. Plurality and distribution in Yaeyaman wh-questions. In Sarah D'Antonio, Mary Moroney, & Carol Rose Little (eds.), *Proceedings of SALT 25*, 636–655. Washington and Ithaca: LSA & CLC Publications.
2. George, Benjamin. 2011. *Question embedding and the semantics of answers*. Los Angeles: UCLA dissertation.
3. Groenendijk, Jeroen & Martin Stokhof. 1982. Semantic analysis of wh-complements. *Linguistics and Philosophy* 5. 175–233.
4. Hagstrom, Paul. 1998. *Decomposing questions*. Cambridge: MIT dissertation.
5. Hamblin, Charles L. 1973. Questions in Montague English. *Foundations of Language* 10. 41–53.
6. Heim, Irene. 1994. Interrogative semantics and Karttunen's semantics for *know*. In Rhonna Buchalla & Anita Mitwoch (eds.), *The proceedings of the ninth conference and of the workshop on discourse*, 128–144. Jerusalem: IATL.
7. Heim, Irene & Angelika Krazter. 1998. *Semantics in generative grammar*. Oxford: Blackwell.
8. Karttunen, Lauri. 1977. Syntax and semantics of questions. *Linguistics and philosophy* 1. 3–44.
9. Kinuhata, Tomohide. 2016. Kakari-musubi of wh-questions in Miyako Ryukyuan: Focusing on the Aza-Irabu Dialect (in Japanese). *Gengo Kenkyuu* 149. 19–42.
10. Kishimoto, Hideki. 2005. *Wh*-in-situ and movement in sinhala questions. *Natural Language & Linguistic Theory* 23. 1–51.
11. Montague, Richard. 1974. The proper treatment of quantification in ordinary English. In Richmond H. Thomason (ed.), *Formal philosophy: Selected papers of Richard Montague*, 247–270. New Haven: Yale University Press.
12. Potts, Christopher. 2007. The dimensions of quotation. In Christopher Barker & Pauline Jacobson (eds.), *Direct Compositionality*, 405–431. Oxford: Oxford University Press.
13. Shimoji, Michinori & Thomas Pellard (eds.). 2010. *An introduction to Ryukyuan languages*. Tokyo: Research institute for languages and cultures of Asia and Africa.
14. Sudo, Yasutada. 2013. Metalinguistic quantification: Evidence from Japanese wh-doublets. An unpublished manuscript. Paris; Institute Jean Nicod.
15. Uegaki, Wataru. 2015. Content nouns and the semantics of question-embedding. *Journal of Semantics*. Advance Access published July 17, 2015.

Interpretations of Embedded Expressives: A View from the Japanese Comparative Expressive *Motto*

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Abstract. Recent studies of expressives have shown that when expressives like *damn* are embedded in the complement of an attitude predicate, they can be either speaker-oriented or non-speaker-oriented (Amaral et al. 2007; Harris & Potts 2009). Amaral et al. (2007) and Harris and Potts (2009) have suggested that this phenomenon is an instance of indexicality. In this paper, I will investigate the interpretations of embedded expressives on the basis of new data in terms of the Japanese comparative expressive *motto*, and argue that the interpretation of the embedded expressive is not merely a matter of indexicality. More specifically, I argue that (i) there can be a semantic shift from a conventional implicature to a secondary at-issue entailment at a clausal level in a non-speaker-oriented reading, and (ii) in some expressives, like the negative *motto*, a speaker-oriented reading can arise only when there is an appropriate speaker-oriented modal in the main clause.

Keywords: embedded expressives, *motto*, secondary at-issue entailment, projection via a modal support, consistency of a judge

1 Introduction

Potts (2005) has claimed that the meaning of expressives, such as *bastard* in (1), is a conventional implicature (CI) and that it is logically independent of "what is said":

(1) That bastard Kresge is famous. (Expressive/CI: Kresge is bad, in the speaker's opinion.)

However, recent studies have shown that when expressives are embedded in the complement of an attitude predicate, they can have either a non-speaker-orientation or a speaker-orientation (Amaral et al. 2007; Harris and Potts 2009; Tonhauser et al. 2013). For example, it has been observed that while *bastard* in (2) is speaker-oriented, *friggin'* in (3) is construed as subject-oriented:

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(2) Sue believes that that bastard Kresge should be fired. (#I think he's a good guy.) (Potts 2007)

(3) (Context: The speaker likes mowing the lawn.) Monty said to me this very morning that he hates to mow the friggin' lawn. (Amaral et al. 2007)

Amaral et al. (2007) and Harris and Potts (2009) have informally suggested that this phenomenon is an instance of indexicality. For example, Harris and Potts (2009) claimed that expressives (and appositives) are inherently underspecified for their orientation, and that there is a free variable for a judge (*j*) that is determined by context. Harris and Potts (2009) further claimed on the basis of corpus and experimental evidence that appositives and expressives are generally speaker-oriented, but certain discourse conditions can counteract this tendency (cf. Schlenker (2003, 2007) and Sauerland's (2007) semantic binding approach to a non-speaker-orientation).

In this paper, I will investigate the interpretation of embedded expressives on the basis of new data in terms of the Japanese comparative expressive *motto*, and argue that the interpretation of the embedded expressive is not merely a matter of indexicality. More specifically, as for the subject-orientation, I argue that (i) there can be a semantic shift from a CI to a secondary at-issue entailment at clausal level in a non-speaker-oriented reading. It will be shown that the semantic shift from a CI to a secondary at-issue entailment is a general phenomenon and that it can also be observed in typical expressives.

As for the speaker-oriented reading, I will argue that in some expressives, like the negative *motto*, a speaker-oriented reading can arise only when there is a modal in the main clause. I will argue that there is a specific type, a dependent projective content, which requires consistency between at-issue and CI meanings including a judge. The theoretical implication of this paper is that both semantic and pragmatic mechanisms are involved in the interpretation of embedded expressives.

2 The Expressive Property of the Japanese *Motto*

2.1 The Degree and Negative Uses of *Motto*

Before investigating the interpretation of the expressive *motto* in an embedded context, let us first discuss the meaning and use of the expressive *motto* in a non-embedded context. It has been observed in the literature that the Japanese comparative adverb *motto* has two different uses, namely a degree use and a negative/expressive use (Watanabe 1985; Sano 1998, 2004; Kinoshita 2001), as in (4):

(4) Kono mise-no keeki-wa motto oishi-katta.
This store-GEN cake-TOP MOTTO delicious-PAST

a. **Degree reading**: This store's cake was {even/still far} more delicious than a contextual store's cake.
b. **Negative reading**: This store's cake was delicious. (Implied: It is not delicious now.)

In the degree reading, the sentence is interpreted as an "elliptical" comparison. It conveys that although the given store's cake and a contextual store's cake were both delicious, the former was far more delicious. Thus, the degree *motto* has a positive meaning.

On the other hand, in the negative reading, *motto* conveys the speaker's complaint about the utterance context, i.e., the store's cake is not delicious now. The phenomenon we are going to focus on is this expressive (or negative) use. Let us consider the difference between the degree *motto* and the negative *motto* more closely.

2.2 The Meaning of the Degree *Motto*

The degree use of *motto* expresses an intensified comparison at the at-issue level and, in addition to this, there is a positive presupposition that the standard of comparison satisfies the standard of an adjective (i.e., y is A). Consider the example in (5) with the explicit standard *yori* PP: [1]

(5) Hanako-no keeki-wa Taro-no keeki-yori(-mo) motto oishi.
Hanako-GEN cake-TOP Taro-GEN cake-than-MO MOTTO delicious
'Hanako's cake is {still far/even} more delicious than Taro's cake'

We can analyze the meaning of sentence (5) as having two components, namely an at-issue component and a presupposition component, as in (6):

(6) The meaning of (5)
a. **At-issue**: Hanako's cake is much more delicious than Taro's cake.
b. **Presupposition**: Taro's cake is delicious.

We can then formalize the meaning of the degree *motto* as in (7), in which the underlined part represents the presupposition component:

(7) $[[motto_{DEGREE}]] = \lambda g_{\langle d,\langle e,\langle i\langle s,t\rangle\rangle\rangle\rangle}\lambda y\lambda x\lambda t\lambda w : \underline{\exists d[d \geq Stand \wedge g(d)(y)(t)(w)]}.$
$max\{d|g(d)(x)(t)(w)\} >!!max\{d|g(d)(y)(t)(w)\}$

In the case of an elliptical degree reading, like that in (4b), a standard of comparison (the second argument) is implicit, so we need to posit a slightly different lexical item for the degree *motto*. However, essentially the same semantic mechanism is involved in the case of the elliptical comparative (see Sawada (2014) for a detailed discussion).

2.3 The Negative Use of *Motto* is a CI/Expressive

Let us now consider the meaning of the negative *motto*, which is the main focus of this paper. Sawada (2014) claims that the expressive/negative use of *motto* is an expressive and that it conventionally implies that "the expected degree is much greater than a current degree," as in (8):

[1] Note that there is no negative reading in (5). If there is an explicit standard of comparison, we cannot get a negative reading (Sawada 2014).

(8) Taro-wa (mukashi-wa) motto majime-da-tta.
Taro-TOP old days-TOP MOTTO serious-PRED-PAST

At-issue: Taro was serious.
Expressive (CI): The degree of seriousness of Taro in the past is much greater than the current degree. (Expected degree = the past degree.) (=> Taro is not serious now (conversational implicature)

Sawada (2014) then claims that the speaker's negative attitude arises from the gap between the expected degree and the current degree (as a conversational implicature).

The comparative meaning triggered by the negative *motto* is a CI because it is independent of "what is said" (Grice 1975; Potts 2005). In (8), the expressive meaning is not within the semantic scope of the past tense. Furthermore, the expressive *motto* can also appear in an imperative, a conditional clause, or a modal sentence, but its expressive meaning cannot be within the semantic scope of these operators. For example, in (9), the negative *motto* is clearly outside the scope of the imperative:

(9) Motto hayaku hashi-re! (imperative)
MOTTO fast run-IMPERATIVE

a. Run even faster! (Degree reading)
b. Run fast! The expected speed of running is much higher than the current speed. (Implied: You are running slowly now.) (Negative reading)

Regarding the compositionality of the negative *motto*, Sawada (2014) claims that the negative *motto* is mixed content (McCready 2010; Gutzmann 2011) in that it has both an at-issue meaning and a CI meaning, as shown in (10)(The left side of ♦ is the at-issue component and the right side of ♦ is the CI component):[2]

(10) $[[motto_{EXPRESSIVE}]] : \langle G^a, \langle e^a, \langle i^a, \langle s^a, t^a\rangle\rangle\rangle\rangle \times \langle G^a, \langle e^a, \langle i^a, \langle s^a, t^s\rangle\rangle\rangle\rangle =$
$\lambda g \lambda x \lambda t \lambda w.\exists d[d \geq STAND \wedge g(d)(x)(t)(w)] \blacklozenge \lambda g \lambda x \lambda t \lambda w.max\{d|g(d)(x)(t)(w)\} \succ$
$!!max\{d|g(d)(x)(t_0)(w_0)\}$ (where t_0 = current time, w_0 = the actual world)

Roughly speaking, in the at-issue component, *motto* denotes that the degree associated with the gradable predicate is above a certain standard. In the CI component, it conventionally implies that the expected degree is far greater than the current degree.

3 Interpretations of Embedded *Motto*: Some Puzzling Facts

Let us now consider the interpretation of the embedded *motto*. Although previous studies have focused only on non-embedded cases of the negative *motto*, it has several puzzling properties in terms of its interpretation in an embedded environment.

[2] Superscript *c* is a CI type and superscript *a* is an at-issue type (Potts 2005). Superscript *s* is a type for a CI expression interpreted by a resource sensitive application (McCready 2010).

3.1 Puzzle 1

First, the expressive meaning triggered by *motto* is interpreted as at-issue if it is embedded under an attitude predicate and has a subject orientation as in (11): [3]

(11) (Negative/expressive reading)
Taro-wa motto isshoukenmei benkyoo-si-nakerebanaranai-to omo-tta.
Taro-TOP MOTTO seriously study-do-must-that think-PAST
At-issue: Taro thought that he must study hard.
Expressive (subject-oriented): Taro considered that the expected degree of seriousness of his study was much greater than the "current degree in the past."

The expressive meaning in (11) is at-issue because it is within the semantic scope of the past tense; it relates to Taro's past feeling. Notice, however, that the expressive meaning triggered by *motto* is not within the semantic scope of the embedded deontic modal *nakerebanaranai* 'must.' What does this mean?

3.2 Puzzle 2

A second puzzling characteristic of the embedded *motto* is that it can actually have speaker-orientation if a deontic modal occurs in the main clause:

(12) Taro-wa motto isshoukenmei benkyoo-si-nakerebanaranai-to
Taro-TOP MOTTO seriously study-do-must-that
omou-bekida.
think-should
At-issue: Taro should think that he must study hard.
Expressive 1 (subject-oriented): For all worlds *w"* compatible with the rule in w_0 and for all worlds *w'* compatible with Taro's beliefs in *w"*, the expected degree of seriousness of Taro's study is much greater than the current degree for Taro in *w'*.
Expressive 2 (speaker-oriented/CI): The expected degree of seriousness of Taro's study is much greater than the current degree for me.

The above asymmetry between (11) and (12) clearly shows that in the case of the expressive *motto*, the determination of a perspective is not merely a matter of context.

4 The Empirical Difference between Speaker-Oriented and Non-Speaker-Oriented Readings

How can we explain the above facts regarding the subject-oriented and speaker-oriented readings? One might think that the speaker-oriented reading in the embedded *motto*

[3] Note that there is also a degree reading in (11), i.e. 'Taro thought that he must study even harder (than now).' In the degree reading, there is a 'positive' presupposition that Taro has already studied hard. This clearly contrasts with the negative reading. Because the main focus is on the interpretation of the embedded expressive, we will not discuss the degree reading.

arises purely pragmatically because of the presence of the deontic modal *bekida*, i.e., speaker-orientedness pragmatically arises in addition to subject-orientedness. However, the two tests set out below clearly show that both speaker-oriented and subject-oriented readings exist in the logical structure.

First, if we add the discourse particle *koo* 'like' between the expressive *motto* and an adjective, the sentence only has a speaker-oriented reading, as in (13):

(13) (The example with the discourse particle *koo* 'like')
Taro-wa motto koo sikkarisita ronbun-o kaka-nakerebanaranai-to
Taro-TOP MOTTO like solid paper-ACC write-must-that
omou-bekida.
think-should
At-issue: Taro should think that he must write a solid paper.
Expressive (speaker-oriented, CI): The expected degree of solidness is much higher than the current degree for me.

In (13), the particle *koo* is used parenthetically to signal that the "speaker" is in the middle of thinking about what an appropriate adjective would be. The function is similar to that of the English *like*.

The second test regarding the distinction between a speaker-oriented and a subject-oriented reading is the insertion of the reflexive *zibun* 'self.' H. Sawada (1993) claims that if a reflexive *zibun* occurs in the embedded clause, the perspective of the embedded clause has to be the antecedent of *zibun* (i.e., the subject of the entire sentence). If we insert the reflexive *zibun* in the embedded clause, only a subject-oriented reading is possible, as in (14):

(14) (The example with *zibun* 'self')
Taro-wa motto jibun-wa sikkarisita ronbun-o kaka-nakerebanaranai-to
Taro-TOP MOTTO self-TOP solid paper-ACC write-must-that
omou-bekida.
think-should
At-issue: Taro should think that he must write a solid paper. **Expressive (subject-oriented)**: For all worlds *w"* compatible with the rule in w_0 and for all words *w'* compatible with Taro's beliefs in *w"*, the expected degree of seriousness of Taro's study is much greater than the current degree for Taro in *w'*.

5 Analyses

5.1 Subject-Oriented Reading of the Negative *Motto*: From a CI to a Secondary Entailment

Let us now try to explain the first puzzle above. In the previous section, we observed that when the negative *motto* is embedded under an attitude predicate, its meaning becomes at-issue, as in (15):

(15) (The negative *motto* = always subject-oriented)

Taro-wa	motto	isshoukenmei	benkyoo-si-nakerebanaranai-to	omo-ta.	
Taro-TOP	MOTTO	seriously	study-do-must-that	think-PAST	

At-issue: Taro thought that he must study hard.

Expressive (subject-oriented): Taro considered that the expected degree of seriousness of his study was much greater than the "current degree in the past."

The expressive meaning in (15) is at-issue because it is within the semantic scope of the past tense; it relates to Taro's past feeling. Notice, however, that the expressive meaning triggered by *motto* is not within the semantic scope of the embedded deontic modal *nakerebanaranai* 'must.' I propose that a semantic shift exists from a CI to a secondary entailment, as set out in (16):

(16) **Shifting from a CI to a secondary entailment**: A sentence S, which consists of an at-issue meaning of type t^a and a CI meaning of type t^c (or type t^s), can shift into an at-issue product type $\langle t^a \times t^a \rangle$ if and only if, S is embedded under an attitude predicate and the judge of S is the attitude holder of the predicate (where the first t^a is a primary entailment and the second t^a is a secondary entailment.)

The secondary entailment is at-issue but is not a primary at-issue meaning (Potts 2005). The embedded negative *motto* is an expressive and it conveys a subject's attitude, similar to the non-embedded negative *motto*.

The crucial point of this shift is that it applies at the root level of an embedded clause. Before the semantic shift applies at the root of the embedded clause, the expressive behaves as a CI triggering expression and it cannot be scoped over by any logical operators. This idea is supported by the fact that in (6), *motto* is not within the semantic scope of the embedded *nakerebanaranai* 'must.'

Let us now analyze the meaning of the subject-oriented reading of (17), which is ambiguous between the subject-oriented reading and the speaker-oriented reading.

(17)

Hanako-wa	kono	mise-no	keeki-wa	motto	oishi-katta-to
Hanako-TOP	this	store-GEN	cake-TOP	MOTTO	delicious-PAST-that

omo-bekida.
think-should

At-issue: Hanako should think that this store's cake was delicious.

Expressive 1 (subject-oriented, secondary at-issue): For all worlds w'' compatible with the rule in w_0 and for all worlds w' compatible with Hanako's beliefs in w'', the expected degree of deliciousness of this store's cake is much higher than the current degree for Hanako in w'.

Expressive 2 (speaker-oriented, CI): The expected degree of deliciousness of this store's cake is much higher than the current degree for me.

Inside the embedded clause, the negative *motto* behaves as a CI. The following figure shows the logical structure of the embedded clause: [4]

[4] Technically, the meaning of the negative *motto* and at-issue elements are combined via mixed application (McCready 2010; Gutzmann 2011):

(18) The logical structure of the embedded clause

Embedded S:
$\exists d[d \geq STAND \wedge delicious(this\ store's\ cake)(past)(w_0) = d]$
$\bullet$
$max\{d|delicious(this\ store's\ cake)(past)(w_0) = d\} >!! max\{d|delicious(thisstore'scake)(t_0)(w_0) = d\} for\ j$

$\lambda w.\exists d[d \geq STAND \wedge delicious(this\ store's\ cake)(past)(w) = d]$ $\blacklozenge \lambda w.max\{d|delicious(this\ store's\ cake)(past)(w) = d\} >!! max\{d|delicious(this\ store's\ cake)(t_0)(w_0) = d\} for\ j$ — **w**$_0$

$\lambda t\lambda w.\exists d[d \geq STAND \wedge delicious(this\ store's\ cake)(t)(w) = d]$ $\blacklozenge \lambda t\lambda w.w.max\{d|delicious(this\ store's\ cake)(t)(w) = d\} >!! max\{d|delicious(this\ store's\ cake)(t_0)(w_0) = d\} for\ j$ — **katta** ‘past’

kono mise-no keeki ‘this store’s cake’ — $\lambda x\lambda t\lambda w.\exists d[d \geq STAND \wedge delicious(x)(t)(w) = d]$ $\blacklozenge \lambda x\lambda t\lambda w.max\{d|delicious(x)(t)(w) = d\} >!! max\{d|delicious(x)(t_0)(w_0) = d\}$ for j

motto $\lambda g\lambda x\lambda t\lambda w.\exists d[d \geq STAND \wedge g(d)(x)(t)(w)]\blacklozenge \lambda g\lambda x\lambda t\lambda w.max\{d|g(d)(x)(t)(w)\} >!! max\{d|g(d)(x)(t_0)(w_0)\}$ — **oishi**‘delicious’ $\lambda d\lambda x\lambda t\lambda w.delicious(x)(t)(w) = d$

After the computation is complete, both the at-issue and CI meanings are gathered via parse tree interpretation, as in (19):

(19) Parsetree interpretation (McCready 2010)(cf. Potts 2005)
Let $\mathcal{T}$ be a semantic parsetree with the at-issue term $\alpha : \sigma^a$ on its root node, and distinct terms $\beta_1 : t^{\{c,s\}}, ..., \beta_n : t^{\{c,s\}}$ on nodes in it. Then, the interpretation of $\mathcal{T}$ is the $\langle [[\alpha : sigma^a]], [[\beta_1 : t^{\{c,s\}}]], ..., [[\beta_n : t^{\{c,s\}}]]\rangle$ (Based on McCready 2010: 32)

At this point, the speaker-oriented reading and the subject-oriented reading are the same in terms of meaning, as shown in (20):

(20) The final interpretation of the embedeed clause via parsetree interpretation
$\langle \exists d[d \geq STAND \wedge delicious(this\ store's\ cake)(past)(w_0) = d] : t^a,$
$max\{d|delicious(this\ store's\ cake)(past)(w_0) = d\} >!! max\{d|delicious$
$(this\ store's\ cake)(t_0)(w_0) = d\} for\ j_i : t^s\rangle$

(i) $\alpha(\gamma)\blacklozenge\beta(\gamma) : \tau^a \times \upsilon^s$

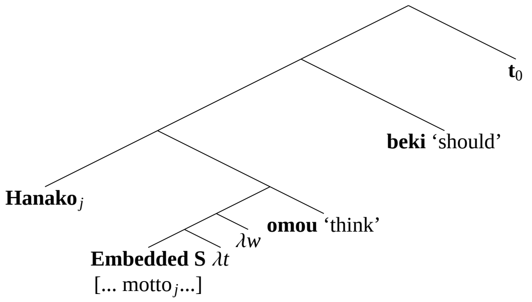
$\alpha\blacklozenge\beta : \langle\sigma^a, \tau^a\rangle \times \langle\sigma^a, \upsilon^s\rangle$ — $\gamma : \sigma^a$

However, after the parse tree interpretation, in the subject-oriented reading, the semantic shift from a CI to a secondary entailment applies, as shown in (21):

(21) After the semantic shift from CI to a secondary entailment
$\langle \exists d[d \geq STAND \wedge delicious(this\ store's\ cake)(past)(w_0) = d],$
$max\{d|delicious(this\ store's\ cake)(past)(w_0) = d\} >!!max\{d|delicious$
$(this\ store's\ cake)(t_0)(w_0) = d\}\ for\ j_i\rangle : \langle t^a \times t^a\rangle$

This meaning then interacts with the elements in the main clause. The figure in (22) shows the entire logical structure of sentence (17)(=subject-oriented reading):

(22) Interpretation of the entire sentence (subject-oriented reading)



The denotations of *omou* 'think' and *beki* 'should' are shown in (23) and (24):

(23) The denotation of *omou* 'think'
$\lambda p_{<s<i<t\times t>>>}\lambda x\lambda t\lambda w\forall w'\ compatible\ with\ x's\ beliefs\ in\ w : p(w')(t) = 1$

(24) The denotation of *beki* 'should'
$\lambda p_{<s<i<t\times t>>>}\lambda t.\forall w''\ compatible\ with\ the\ rules\ in\ w_0 : p(t)(w'') = 1\ for\ j$

If we put everything together, we get the following meaning in (25) as a final meaning:

(25) Final part of derivation (subject-oriented reading)
For all worlds *w"* compatible with the rule in w_0 and for all worlds *w'* compatible with Hanako's beliefs in *w"*:
$\langle \exists d[d \geq STAND \wedge delicious(this\ store's\ cake)(past)(w_0) = d],$
$max\{d|delicious(this\ store's\ cake)(past)(w_0) = d\} >!!max\{d|delicious$
$(this\ store's\ cake)(t_0)(w_0) = d\}\ for\ j_{Taro}\rangle\ at\ t_0\ in\ w' = 1\ for\ j\ (= speaker)$

One might propose that the shifting from a CI to a secondary at-issue entailment occurs at the lexical level. However, such an approach is problematic. As the above examples show, the embedded *motto* behaves as a CI inside the embedded clause. This seems to be natural, considering that it is the "expressive" feeling of a subject.

5.2 The Case of Subject-Oriented Reading in the English Expressives

The shift from a CI to a secondary entailment is pervasive in natural language and can also be observed in typical embedded expressives. (26) clearly shows that the embedded *friggin'* is within the semantic scope of the past tense:

(26) (Subject-oriented reading, friggin' = Monty's perspective)
Monty said to me two years ago that he hated to mow the friggin' lawn, but now, he doesn't mind. (Subject-oriented reading)[5]

On the subject-oriented reading, *friggin'* has to be within the scope of the matrix tense. On the sequence-of-tense reading, which is the most salient, the time of Monty's speech corresponds with the time of Monty's hating, i.e., the time at which Monty had a negative attitude toward the lawn, as in (27).

(27) Monty said to me two years ago that he hated to mow the friggin' lawn, but now, he doesn't mind. (embedded clause = past tense)

The important point, however, is that Monty's attitude is an expressive; it relates to Monty's attitude in the past. Thus, it is reasonable to consider that it is not a primary at-issue.

The question arises as to how we might analyze the meaning of the embedded *friggin'*, as in (28), which is similar to (3). It seems that the interpretation of embedded *friggin'* becomes complicated if the embedded clause has present tense.

(28) (Subject-oriented reading, *friggin'* = Monty's perspective)
Monty said to me two years ago that he hates to mow the friggin' lawn.

This is because this sentence has a "double access reading" (Ogihara 1996; Abush 1997, etc.), in which both a past situation and a present situation are relevant. Comrie (1985:115) has stated that (29b) is used "when the speaker is reporting a (real or imaginary) illness which he believes still has relevance."

(29) a. John said that he was ill.
b. John said that he is ill

This predicts that the expressive in (28) can be anchored to both the past and the present if the embedded clause has present tense. This prediction is borne out. The expressive *friggin'* in (28) is obligatorily anchored both to the present and the past (i.e., obligatory double access). This is supported by the fact the sentence in (30) sounds somewhat odd.

(30) ?? Monty said to me two years ago that he hates to mow the friggin' lawn, but now, he doesn't mind. (embedded clause = present tense)

This fact is consistent with the hypothesis that subject-oriented embedded expressives obligatorily give rise to the double access effect when the embedded tense is present.

5.3 Speaker-Orientation of the Negative *Motto*: The Existence of Dependent Projective Content

Let us now investigate the speaker-oriented reading of the embedded *motto*. The puzzle was that the embedded negative/expressive *motto* can only be speaker-oriented if there is a deontic modality in the main clause, as in (31):

[5] Note that there is also a speaker-oriented reading in which the speaker has a negative attitude toward the lawn.

(31) Hanako-wa kono mise-no keeki-wa mukashi-wa motto
Hanako-TOP this store-GEN cake-TOP old days-TOP MOTTO
oishi-katta-to omo-tta. (subject-oriented)
delicious-PAST-that think-PAST

At-issue: Hanako thought that this store's cake was delicious.
Secondary at-issue: Hanako thought that the expected degree of deliciousness (i.e. the deliciousness in the past) was much higher than the current degree.)

(32) Hanako-wa kono mise-no keeki-wa motto oishi-katta-to
Hanako-TOP this store-GEN cake-TOP MOTTO delicious-PAST-that
omo-bekida.
think-should (speaker-oriented/subject-oriented)

At-issue: Hanako should think that this store's cake was delicious.
Expressive 1 (subject-oriented, secondary at-issue): For all worlds *w"* compatible with the rule in w_0 and for all worlds *w'* compatible with Hanako's beliefs in *w"*, the expected degree of deliciousness of this store's cake is much higher than the current degree for Hanako in *w'*.
Expressive 2 (speaker-oriented, CI): The expected degree of deliciousness of this store's cake is much higher than the current degree for me.

This point is radically different from a typical expressive like *bastard*. As we observed in the Introduction, *bastard* can be speaker-oriented even if there is no external speaker-oriented element in the main clause, as in (33):

(33) Sue believes that that bastard Kresge should be fired. (#I think he's a good guy.) (Potts 2007)

How might we explain the "conditional" projective property of the embedded *motto* shown in the previous section? I argue that the embedded *motto* is a dependent projective content. Namely, it can be speaker-oriented only when a deontic modal exists in the main clause because it requires that the judge of the *motto* is consistent with the judge in the at-issue level. I posit such a constraint inside the lexical entry of *motto*, as in (34):

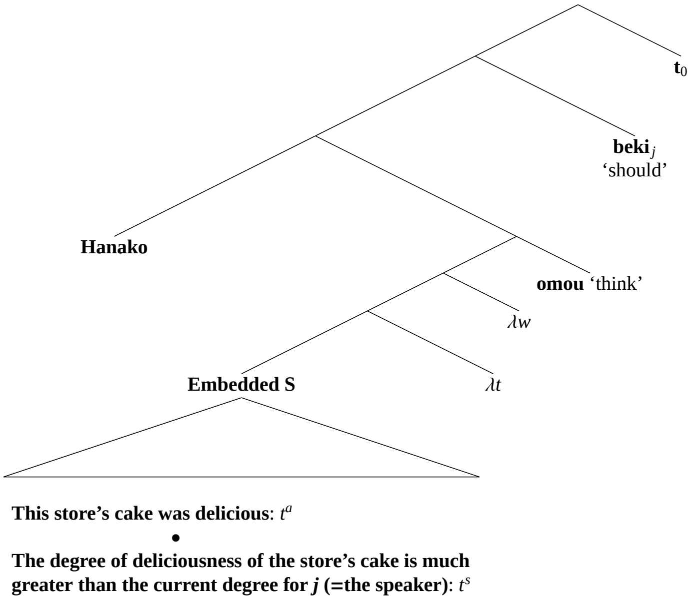
(34) $[[motto_{EXPRESSIVE}]] : \langle G^a, \langle e^a, \langle i^a, \langle s^a, t^a\rangle\rangle\rangle\rangle \times \langle G^a, \langle e^a, \langle i^a, \langle s^a, t^s\rangle\rangle\rangle\rangle =$
$\lambda g \lambda x \lambda t \lambda w.\exists d[d \geq STAND \wedge g(d)(x)(t)(w)] \blacklozenge \lambda g \lambda x \lambda t \lambda w.max\{d|g(d)(x)(t)(w)\} \succ$
$!!max\{d|g(d)(x)(t_0)(w_0)\}$ for j (where j is consistent with a judge in the at-issue level) (where t_0 = current time, w_0 = the actual world)

If there is no modal in the main clause, j of *motto* corresponds to the subject of the sentence (the attitude holder). This is because the sentence merely describes the subject's thoughts. However, if there is a deontic modal in the main clause, *motto* can be speaker-oriented because the modal *bekida* 'must' is a judge-sensitive expression (see also Stephenson (2007)), as shown in (37), and the judge variable of the embedded *motto* can correspond to the judge of *bekida*:

(35) $[[bekida]] = \lambda p_{\langle i^a, \langle s^a, t^a\rangle\rangle} \lambda t \forall w'$ compatible with the rules in w_0 : $p(w')(t) = 1$ for j

Thus, *motto* can be anchored to either a speaker or a subject in the sentence with *bekida*. The following figure shows the logical structure of the entire sentence:

(36) Interpretation of the entire sentence (speaker-oriented reading)



The following shows the final part of the derivation:

(37) **Final part of derivation (speaker-oriented reading)**
For all worlds w"compatible with the rule in w_0 *and for all worlds w' compatible with Hanako's beliefs in w"*: $\exists d[d \geq STAND \wedge delicious(this\ store's\ cake)(past)(w_0) = d]\ at\ t_0\ in\ w' = 1) = 1\ for\ j_{speaker} : t^a$

•

$max\{d|delicious(this\ store's\ cake)(past)(w_0) = d\} \succ !!max\{d|delicious(this\ store's\ cake)(t_0)(w_0) = d\}\ for\ j_{speaker} : t^s$

Note that the addition of the epistemic modality, such as *kamoshirenai* 'may' does not help the embedded *motto* become speaker-oriented, despite the fact that it is also a judge-sensitive expression (speaker-oriented), as is clear from (39):

(38) [[*kamoshirenai*]] = $\lambda p_{\langle s^a, t^a \rangle}.\exists$ *w'* compatible with *j*'s knowledge in w_0: *p*(*w'*) = 1 for *j*

(39) Taro-wa motto isshoukenmei benkyoo-si-nakerebanaranai-to
Taro-TOP MOTTO seriously study-do-must-that
omou-kamoshirenai.
think-should

At-issue: Taro may think that he must study hard.
Expressive (subject-oriented): For some worlds *w'* compatible with Taro's knowledge in w_0, the expected degree of seriousness of Taro's study is much

greater than the current degree for Taro in *w’*.

Why is it that the expressive *motto* cannot be speaker-oriented in (39). I would like to propose that this is because the meaning of the epistemic modality is not pragmatically consistent with the expressive meaning of the negative *motto*. In the case of (32) the deontic modality conveys a speaker’s complaint, and the negative *motto* also conveys a judge’s complaint. Thus, proposing that the judge of *motto* and the judge of the deontic modality are the same is natural. However, in the case of (33) no semantic consistency exists between *motto* and the epistemic modality.

6 Conclusion and Theoretical Implications

In this paper, I investigated the interpretations of embedded expressives on the basis of new data, namely the Japanese comparative expressive *motto*, and argued that the interpretation of the embedded expressive is not merely a matter of indexicality. More specifically, I argued that (i) there can be a semantic shift from a CI to a secondary at-issue entailment at a clausal level in a non-speaker-oriented reading, and (ii) in some expressives, like the negative *motto*, a speaker-oriented reading can arise only when there is an appropriate speaker-oriented modal in the main clause.

What do these claims imply theoretically? I think that these claims theoretically suggest the interpretation of embedded expressives involves both semantic and pragmatic mechanisms. Harris and Potts (2009) contrast a configurational approach and a contextual approach and support the contextual approach:

(40) a. **Configurational**: The source of non-speaker-oriented readings of appositives and expressives is semantic binding: their content can be bound by higher operators like attitude predicates, thereby shifting it away from the speaker (Schlenker 2003, 2007; Sauerland 2007).

b. **Contextual**: The source of non-speaker-oriented readings of appositives and expressives is the interaction of a variety of pragmatic factors. In general, these interactions favor speaker-orientation, but other orientations are always in principle available, regardless of syntactic configuration (Potts 2007).

However, the phenomenon of the embedded negative *motto* suggests that both semantic and pragmatic factors are involved. In this paper, I proposed that there is a semantic shift from a CI to a secondary entailment in the interpretation of a subject-oriented reading. This is clearly non-contextual, but at the same time, it is not purely semantic in that it maintains the meaning of subject-oriented expressives as secondary.

Furthermore, as for the speaker-oriented reading, I have proposed that there is a new class of projective content, i.e. a dependent projective content. The new class of projective content is semantic in the sense that whether or not it can project depends on the existence of a judge-sensitive element (i.e., a deontic modal in the case of *motto*). However, this dependency is also pragmatic in that the kind of external element it can support as a projection of an embedded expressive is pragmatically determined by the extent to which the external element semantically matches with the CI meaning of *motto*. In the case of the negative *motto*, an epistemic modal cannot support the projection of *motto* because its meaning does not match with the CI meaning of the negative *motto*.

In this paper, I focused only on the Japanese *motto* and certain English expressives. In future research, I would like to further investigate the interpretation of other embedded expressives and consider the variation from a broader perspective.

References

1. Abusch, Dorit. (1997). Sequence of tense and temporal de re. *Linguistics and Philosophy* 20: 1-50.
2. Amaral, Patricia, Craige Roberts, and Allyn Smith. (2007). Review of the logic of conventional implicatures by Chris Potts. *Linguistics and Philosophy* 30: 707-749.
3. Comrie, Bernard. (1985). *Tense*. Cambridge, UK: Cambridge University Press.
4. Grice, Paul. H. (1975). Logic and conversation. In Peter Cole and Jerry Morgan (eds.), *Syntax and Semantics, iii: Speech Acts*, 43-58. New York: Academic Press.
5. Gutzmann, Daniel. (2011). Expressive modifiers & mixed expressives. *Empirical Issues in Syntax and Semantics* 8, 123-141.
6. Harris, Jesee. A. and Potts, Christopher. (2009). Perspective-shifting with appositives and expressive. *Linguistics and Philosophy* 32: 523-552.
7. Kinoshita, Kyoko. (2001). Hikaku no fukushi motto ni okeru shukansei (The subjectivity in the comparative adverb motto). *Kokugogaku* 52: 16-29.
8. McCready, Eric. (2010). Varieties of conventional implicature: evidence from Japanese. *Semantics & Pragmatics*, 3, 1-57.
9. Ogihara, Toshiyuki. (1996). *Tense, attitudes and scope*. Dordrecht, Netherlands: Kluwer Academic Publishers
10. Potts, Christopher. (2007). The expressive dimension. *Theoretical Linguistics* 33: 165-197.
11. Potts, Christopher. (2005). *The Logic of Conventional Implicatures*. Oxford: Oxford University Press.
12. Sano, Yukiko. (1998) Hikaku ni kakawaru teido fukushi ni tuite (On the comparison-related degree adverbs). *Kokugogaku* 195: 1-14.
13. Sano, Yukiko. (2004). Motto no hiteiteki youhou ni tuite (On the 'negative usage' of motto). *Nihongo kagaku* 15: 5-21.
14. Sauerland, Uli. (2007). Beyond unpluggability. *Theoretical Linguistics* 33: 231—236.
15. Sawada, Harumi. (1993). *Siten to syukansei* (Viewpoint and Subjectivity). Tokyo: Hituzi Syoboo.
16. Sawada, Osamu. (2014). An utterance situation-based comparison. *Linguistics and Philosophy* 37: 205-248.
17. Sawada, Osamu. (2010). *Pragmatic Aspects of Scalar Modifiers*. Doctoral Dissertation, University of Chicago.
18. Schlenker, Philippe. (2007). Expressive presuppositions. *Theoretical Linguistics* 33: 237—245.
19. Schlenker, Philippe. (2003). A plea for monsters. *Linguistics and Philosophy* 26: 29—120.
20. Stephenson, Tamina. (2007). Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy* 30: 487—525.
21. Tonhauser, Judith, David Beaver, Craige Roberts, and Mandy Simons. (2013). Toward a taxonomy of projective content. *Language* 89: 66-109.
22. Watanabe, Minoru. (1985). Hikaku no fukushi: 'motto' o tyuushinni (The comparative adverbs in Japanese: The case of *motto*). *Bulletin of the Language Institute of Gakushuin University* 8: 65-74.

Non-canonical Coordination in the Transformational Approach

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Abstract. Recently introduced Transformational Semantics TS formalizes, restraints and makes rigorous the transformational approach epitomized by QR and Transformational Grammars: deriving a meaning (in the form of a logical formula or a logical form) by a series of transformations from a suitably abstract (tecto-) form of a sentence. TS generalizes various 'monad' or 'continuation-based' computational approaches, abstracting away irrelevant details (such as monads, etc) while overcoming their rigidity and brittleness. Unlike QR, each transformation in TS is rigorously and precisely defined, typed, and deterministic. The restraints of TS and the sparsity of the choice points (in the order of applying the deterministic transformation steps) make it easier to derive negative predictions and control over-generation.
We apply TS to right-node raising (RNR), gapping and other instances of non-constituent coordination. Our analyses straightforwardly represent the intuition that coordinated phrases must in some sense be 'parallel', with a matching structure. Coordinated material is not necessarily constituent – even 'below the surface' – and we do not pretend it is. We answer the Kubota and Levine challenge of analyzing RNR and gapping without using directional types, yet avoiding massive over-generation.

1 Introduction

Non-canonical coordination, and in particular gapping, (2) provides an unending stream of puzzles for the theory of semantics [8, 10]:

(1) John gave a book to Mary and a record to Sue.
(2) I gave Leslie a book and she a CD.
(3) John gave a present to Robin on Thursday and to Leslie on Friday.
(4) Mrs. J can't live in Boston and Mr. J in LA.

Interactions of coordination with scope-taking are particularly challenging: a competent theory needs to handle both narrow- and wide-scope reading of "a present" in (3) and the narrow- and wide-scope coordination in (4).

Recently in [8, 9], Kubota and Levin put forward new analyses of non-canonical coordination, applying hybrid categorial grammars they have been developing. In contrast, the analyses in [6] use plain old non-associative Lambek grammar. However, the main ideas of [6] are completely hidden behind thickets of complicated types and their interactions within a derivation. The intuition that coordinated structures must be parallel is thus lost in the details.

We present a new analysis of non-constituent coordination using the more intuitive and less round-about framework TS (formerly called AACG) [7], designed to take the 'hacking' out of tree-hacking. TS lets us talk about QR and other transformations towards some semantic form in a rigorous, formal, mostly deterministic way. We remind of TS in §2.

Our analyses re-expose ideas from the earlier approach of [6], but free them from the bondage of encoding. A notable feature of TS is the absence of directional types. We use it to answer the challenge posited by Kubota, Levin [10] and Moot (dubbed "the KLM problem" by Morrill): to analyze RNR within categorial-grammar–like formalisms without directional types, while avoiding massive overgeneration.

One may categorize the various approaches to non-canonical coordination based on what exactly is being coordinated. Are complete sentences being coordinated behind the scene, as in "John likes Bill" and "Mary hates Bill" with "Bill" being later elided? Or perhaps sentences with holes are being coordinated, as in "John likes hyp_{obj}"? (as done in [6, 8, 9].) Or perhaps we regard "John likes" and "Mary hates" as constituents and coordinate as such (as in CCG). In this paper we give another answer: we analyze "John likes and Mary hates Bill" as the coordination of the complete clause "Mary hates Bill" with the cluster "John" and "likes". The types of the cluster components and their order guide the transformation that picks the needed material from the clause "Mary hates Bill" to make the cluster the complete clause. The 'picking transformation' can be naturally supported within the existing setup of TS, using the same mechanism used in [7] to analyze quantification and inverse linking. §3 makes precise the intuition of 'picking' in the formal setting.

The structure of the paper is as follows. §2 reminds TS, in a different, clearer presentation. We then describe our approach to coordination: transforming non-canonical one to the ordinary coordination of clauses. §4 discusses the related work that forms the context of our approach. The rigorous nature of TS makes it easier to carry analyses mechanically, by a computer. In fact, the analyses in the paper have been so programmed and executed. The implementation, in the form of a domain-specific language embedded in Haskell – 'the semantic calculator' – is publicly available at `http://okmij.org/ftp/gengo/transformational-semantics/`.

2 TS Background

Traditional Categorial Grammar approaches draw parallels between proof systems and grammars: grammaticality is identified with the existence of a derivation. It is rather challenging however to prove the absence of a derivation, and to overview the space of possible derivations in general.

TS (formerly, AACG) [7] in contrast pursues the computational approach, harking back to Transformational Generative Grammars [2] of 1960s: Rather than trying to *deduce* a derivation, it tries to *induce* the meaning (the logical formula) by applying a sequence of precisely and formally defined transformations to a suitably abstract form of a sentence. The latter abstracts away the case and the number agreement, declination, etc. The transformations are deterministic; the order of their applications is generally not (there may still be dependencies between particular transformations imposing the order). The transformations are partial: the failure is taken as ungrammaticality of the original sentence.

Formally, TS deals with term languages that represent typed finite trees. Each T-language is a set of well-typed terms built from typed constants (function symbols) c. Types are

$$\begin{array}{lll} \text{Base types} & & \upsilon \\ \text{T-Types} & \sigma & ::= \upsilon \mid \sigma \to \sigma \end{array}$$

The set terms d is then inductively defined as: (i) each constant c of the type σ is a term; (ii) if c has the type $\sigma_1 \to \sigma$ and d is a term of type σ_1, then $c\ d$ is a term of type σ; (iii) nothing else is a term. The set of constants and their types is a (multi-sorted) algebraic signature; A T-language is hence a term language over the signature, which defines the language.

Table 1 shows three sample languages. T_S with the single base type string and numerous constants `"John"`, `"greet"`, `"every"`, etc. of that type describes the surface, "phonetic", form of a sentence. The constant $-\cdot- : \mathsf{string} \to \mathsf{string} \to \mathsf{string}$ (usually written as an infix operation) signifies string concatenation. The language T_A whose types are familiar categories represents the abstract form. T_L is the language of formulas of predicate logic, which describe the meaning of sentences. The (infinite) sets of constants $\mathsf{var_x}, \mathsf{var_y}, \ldots$ and the

	v	c
T_S	string	$\cdot$: $\mathsf{string} \to \mathsf{string} \to \mathsf{string}$ `"John"` , `"greet"` , `"every"` , … : string
T_A	S, NP, N, VP, PP, TV	John: NP $\mathsf{participant}$: N greet: TV cl: $NP \to VP \to S$ argp: $TV \to NP \to VP$ ppadv: $VP \to PP \to VP$ $\mathsf{every_x}, \mathsf{every_y}, \mathsf{a_z}$: $N \to NP$ $\mathsf{var_x}, \mathsf{var_y}, \ldots : NP$ $\mathsf{U_x}, \mathsf{U_y}, \ldots, \mathsf{E_x}, \mathsf{E_y}, \ldots : N \to S \to S$
T_L	e, t	conj , disj , … : $t \to t \to t$ john: e participant: $e \to t$ greet: $e \to t \to t$ app: $(\sigma_1 \to \sigma) \to \sigma_1 \to \sigma$ $\forall_x, \exists_y$: $t \to t$ $x, y, z, \ldots : e$

Table 1. Signatures of various T-languages

corresponding $\mathsf{U_x}, \ldots$ and $\mathsf{E_x}, \ldots$ represent (to be) bound variables and their binders. Unlike the conventional (lambda-bound) variables, they are not subject to substitution, α-conversion or capture-avoidance. T_L likewise has constants $x, y, z, \ldots$ of the type e and the corresponding sets of constants $\forall_x, \forall_y, \ldots, \exists_x, \exists_y, \ldots$ intended as binders.

As a way to introduce TS we show a sample analysis of quantification on "John greeted every participant". The sample sentence in the language T_A has the form

$$\mathsf{cl\ john\ (argp\ greet\ (every_x\ participant))}$$

The constant cl combines an NP and a VP into a clause. (Likewise, argp attaches an argument to a verb and ppadv attaches a prepositional phrase as a VP complement.) Quantifiers are uniquely labeled by x, y, z, etc. We assume it is the job of a parser to uniquely label the quantifiers in the abstract form.

The meaning is derived by applying a sequence of transformations to a T_A term. The transformation $\mathcal{L}_{Ux}$ gets rid of $\mathsf{every_x}$, introducing $\mathsf{var_x}$ and $\mathsf{U_x}$ instead. The transformation is context-sensitive. Therefore, we first define a context $C[]$, a term (tree) with a hole:

$$C[] = [] \mid \mathsf{cl}\ C[]\ d \mid \mathsf{cl}\ d\ C[] \mid \mathsf{argp}\ d\ C[] \mid \mathsf{ppadv}\ C[]\ d \mid \mathsf{ppadv}\ d\ C[]$$

where the meta-variable d stands for an arbitrary term. In words: a context is the bare hole [] or a clause (the cl term) that contains a hole in the subject or the predicate, or a complemented VP with the hole in the head or the complement, etc. We further distinguish two subsets of contexts $C_{cl}[]$ and $C_{ncl}[]$:

$$C_{cl}[] = \mathsf{cl}\ C_{ncl}[]\ d \mid cl\ d\ C_{ncl}[]$$
$$C_{ncl}[] = [] \mid \mathsf{argp}\ d\ C_{ncl}[] \mid \mathsf{ppadv}\ C_{ncl}[]\ d \mid \mathsf{ppadv}\ d\ C_{ncl}[]$$

Intuitively, $C_{cl}[]$ is the smallest context that has a hole within a clause.

The transformation $\mathcal{L}_{Ux}$ can now be stated as follows

$$\mathcal{L}_{Ux}[C_{cl}[\mathsf{every_x}\ \ d_r]] \quad \mapsto \quad \mathcal{L}_{Ux}[(\mathsf{U_x}\ \ d_r)]\ \mathcal{L}_{Ux}[C_{cl}[\mathsf{var_x}]]$$

The rule is written in the form reminiscent of extended top-down tree transducers: whenever a pattern, e.g., $C_{cl}[\mathsf{every_x}\ \ d_r]$, matches a branch of a tree, the branch is replaced with $(\mathsf{U_x}\ \ d_r)\ C_{cl}[\mathsf{var_x}]$ and the transformation is repeated on the branches. Here, d_r is a pattern variable that stands for an arbitrary subterm (tree branch). That is, $C_{cl}[\mathsf{every_x}\ d_r]$ on the left hand-side of the rule matches a tree that contains, somewhere inside, a sub-expression of the form $\mathsf{every_x}\ d_r$ (a branch headed by $\mathsf{every_x}$). On the right-hand side of the rule, $C_{cl}[\mathsf{var_x}]$ is the same tree in which $\mathsf{every_x}\ d_r$ subterm has been replaced with $\mathsf{var_x}$. If a tree does not match the left-hand side of any $\mathcal{L}_{Ux}$ clause, the transformation is applied to the child branches.

Our example matches the left-hand side of $\mathcal{L}_{Ux}$ immediately: d_r matches participant and $C_{cl}[]$ is john (argp greet []), The result of the transformation

$$(\mathsf{U_x}\ \ \mathsf{participant})\ \ (\mathsf{cl}\ \ \mathsf{john}\ \ (\mathsf{argp\ greet}\ \ \mathsf{var_x}))$$

is in effect the Quantifier Raising (QR) of "every participant", but in a rigorous, deterministic way. The intent of the new constants should become clear: $\mathsf{U_x}$ is to represent the raised quantifier, and $\mathsf{var_x}$ its trace. Unlike QR, the raised quantifier ($\mathsf{U_x}$ participant) lands not just on any suitable place. $\mathcal{L}_U$ puts it at the closest boundary marked by the clause-forming constant cl. $\mathcal{L}_U$, is type-preserving: it maps a well-typed term to also a well-typed term. Again unlike QR, we state the correctness properties such as type-preservation. The type preservation is the necessary condition for the correctness of the transformations. Finally, to derive the meaning we apply the transformation $\mathcal{L}_{sem}$ that produces the logical formula (a term in the language T_L)

$$\forall_x \text{ app participant } x \Rightarrow (\text{greet } x \text{ john})$$

by replacing john, etc. with the corresponding logical constants and $\mathsf{U_x}$ with the universal quantifier.

3 Coordination in TS

We now apply TS to the analysis of (non-canonical) coordination. As a warm-up, we take the non-problematic "John tripped and fell," which is an example of the conventional VP coordination. We analyze it differently, however, as 'left-node raising' so to speak, to introduce the technique to be later used in right-node raising (RNR), argument cluster coordination (ACC) and gapping [1].

The abstract form of our example is

$$\mathsf{and_{S,NP}}\ (\mathsf{cl\ john\ tripped})\ \mathsf{fell}$$

The new constant $\mathsf{and_{S,NP}}$ has the type $S \to NP \to S$. As common, we assume a whole family of constants $\mathsf{and_{X,Y}}$ of different types. The

[1] We may even analyze NP coordination as a sort of RNR: after all, "John and Mary left" can have the meaning of the conjunction of truth conditions of "John left" and "Mary left". Certainly, "John and Mary left" may also mean that "John and Mary", taken as a group, left. In the later case, the group can be referred as "they". Our analysis applies to the former (conjunction) case but not the latter. Hence we posit that 'and' is not only polytypic but also polysemic.

constant $\mathsf{and}_{\mathsf{S,NP}}$ is not in the domain of $\mathcal{L}_{sem}$. Therefore, to be able to derive the logical formula, we have to transform it away. The following transformation $\mathcal{L}_a$ does that:

$$\mathcal{L}_a[\mathsf{and}_{\mathsf{S,NP}}\ (\mathsf{cl}\ d_{NP}\ d_{VP})\ d] \quad \mapsto$$
$$\mathsf{and}\ \mathcal{L}_a[(\mathsf{cl}\ d_{NP}\ d_{VP})]\ \mathcal{L}_a[(\mathsf{cl}\ d\ d_{VP})]$$

The rule again is written in the form of extended top-down tree transducers: whenever the pattern on the left-hand side of a rule matches a branch of the tree, the branch is replaced with the right-hand-side of the rule. Again, d with various indices are meta-variables that stand for arbitrary subterms (tree branches). Applying the rule to our T_A term transforms it to

$$\mathsf{and}\ (\mathsf{cl\ john\ tripped})\ (\mathsf{cl\ john\ fell})$$

where and is the ordinary coordination, of the type $S \to S \to S$, which can be given the meaning of propositional disjunction and which hence is in the domain of $\mathcal{L}_{sem}$. The result is straightforward to transform to a logical formula T_L.

Our next example is a proper RNR: "John likes and Mary hates Bill", whose abstract form is

$$\mathsf{and}_{\mathsf{(NP,TV),S}}\ (\mathsf{john}, \mathsf{like})\ (\mathsf{cl\ mary}\ (\mathsf{argp\ hate\ bill}))$$

We have added to T_A tuples (d, d) and tuple types (σ, σ). The constant $\mathsf{and}_{\mathsf{(NP,TV),S}}$ has the type $(NP, TV) \to S \to S$. Whereas $(\mathsf{cl\ mary}$ $(\mathsf{argp\ hate\ bill}))$ is the complete sentence, $(\mathsf{john}, \mathsf{like})$ is certainly not. It is not even a constituent; it is just a sequence of words: a cluster. Since we added to T_A tuples and new constants, we may need to extend our earlier transformation rules, specifically, $\mathcal{L}_{syn}$ for transforming into the surface form of the sentence T_S:

$$\begin{array}{lcl} \mathcal{L}_{syn}[\mathsf{and}_{\mathsf{(NP,TV),S}}\ \ d_1\ d_2] & \mapsto & \mathcal{L}_{syn}[d_1] \cdot \texttt{"and"}\ \cdot \mathcal{L}_{syn}[d_2] \\ \mathcal{L}_{syn}[(d_1, d_2)] & \mapsto & \mathcal{L}_{syn}[d_1] \cdot \mathcal{L}_{syn}[d_2] \end{array}$$

Applying $\mathcal{L}_{syn}$ to our T_A clearly gives "John likes and Mary hates Bill". The 'phonetic' transformation is dull and uninteresting, in contrast to the higher-order phonetics of [8].

Let us derive the meaning, the T_L formula, from the same T_A term. Before we can apply $\mathcal{L}_{sem}$ we need to transform away $\mathsf{and}_{\mathsf{(NP,TV),S}}$, which is not in the domain of the latter transformation. We extend the $\mathcal{L}_a$ with a new clause:

$$\mathcal{L}_a[\mathsf{and}_{\mathsf{(NP,TV),S}}\ (d_1, d_2)\ (\mathsf{cl}\ d\ C[\mathsf{argp}\ d_4\ d_5])] \quad \mapsto$$
$$\mathsf{and}\ \mathcal{L}_a[(\mathsf{cl}\ d_1\ (\mathsf{argp}\ d_2\ d_5))]\ \mathcal{L}_a[(\mathsf{cl}\ d\ C[\mathsf{argp}\ d_4\ d_5])]$$

where d_1, d, d_5 have to be of the type NP and d_2 and d_4 of the type TV. The transformation is context-sensitive and type-directed. It may be regarded as matching of (d_1, d_2) against the complete sentence (the second argument of $\mathsf{and}_{(\mathsf{NP},\mathsf{TV}),\mathsf{S}}$). The matching is determined by the type of $\mathsf{and}_{(\mathsf{NP},\mathsf{TV}),\mathsf{S}}$. The parallel structure of the coordination is clearly visible.

Analyses of RNR without directional types (e.g, using ACG) run into trouble of over-generating "*John likes Bill and Mary hates". Although we can write the abstract form for that sentence as well:

$$\mathsf{and}_{\mathsf{S},(\mathsf{NP},\mathsf{TV})}\ (\mathsf{cl\ john\ (argp\ like\ bill)})\ (\mathsf{mary},\ \mathsf{hate})$$

we do not provide the $\mathcal{L}_a$ transformation with the constant $\mathsf{and}_{\mathsf{S},(\mathsf{NP},\mathsf{TV})}$. Since it remains uneliminated, $\mathcal{L}_{sem}$ cannot be applied and the meaning cannot be derived. In TS, transformations are partial and are not guaranteed to always succeed. The original sentence is considered ungrammatical then.

The same transformation idea also works for gapping and argument cluster coordination (ACC). Take for example, "Mary liked Chicago and Bill Detroit", or, in the abstract form:

$$\mathsf{and}_{\mathsf{S},(\mathsf{NP},\mathsf{NP})}\ (\mathsf{cl\ mary\ (argp\ liked\ chicago)})\ (\mathsf{bill}, \mathsf{detroit})$$

The transformational rule involving the constant $\mathsf{and}_{\mathsf{S},(\mathsf{NP},\mathsf{NP})}$ that picks a suitable subterm that can relate two NPs from the left conjunct

$$\begin{array}{l}\mathcal{L}_a[\mathsf{and}_{\mathsf{S},(\mathsf{NP},\mathsf{NP})}\ (\mathsf{cl}\ d\ C[\mathsf{argp}\ d_4\ d_5])\ (d_1, d_2)] \quad \mapsto \\ \qquad \mathsf{and}\ \mathcal{L}_a[(\mathsf{cl}\ d\ C[\mathsf{argp}\ d_4\ d_5])]\ \mathcal{L}_a[(\mathsf{cl}\ d_1\ (\mathsf{argp}\ d_4\ d_2))]\end{array}$$

turns our T_A term to

$$\mathsf{and\ (cl\ mary\ (argp\ liked\ chicago))\ \ (cl\ bill\ (argp\ liked\ detroit))}$$

with the clear meaning. The examples (1) and (2) of §1 are dealt with similarly. One may observe that the analysis of gapping is nearly the same as that of VP coordination, used in the warm-up example.

The interaction of non-canonical coordination with quantification is not much different from that of the ordinary coordination of two clauses. For example, take (3) of §1, whose abstract form is

$$\begin{array}{l}\mathsf{and}_{\mathsf{S},(\mathsf{PP},\mathsf{PP})} \\ \quad (\mathsf{cl\ speaker\ (ppadv\ (ppadv\ (argp\ gave\ (a_x\ present))\ (to\ robin))(on\ thu)))} \\ \quad (\mathsf{to\ leslie}, \mathsf{on\ fri})\end{array}$$

contains two components to be eliminated by transformations: $\mathsf{and}_{\mathsf{S},(\mathsf{PP},\mathsf{PP})}$ and the QNP ($\mathsf{a_x\ present}$). The latter is to be handled by $\mathcal{L}_E$, which

8

is analogous to $\mathcal{L}_U$ but for the existential quantifier. The transformations $\mathcal{L}_a$ and $\mathcal{L}_E$ can be applied in either order, which corresponds to the wide- and narrow-scope–readings of (3). The narrow scope happens when $\mathcal{L}_a$ goes first, producing

and
  (cl speaker (ppadv (ppadv (argp gave (a_x present)) (to robin))(on thu)))
  (cl speaker (ppadv (ppadv (argp gave (a_x present)) (to leslie))(on fri)))

In summary, we have presented the uniform analysis of both the canonical and non-canonical coordination, reducing the variety of coordination (VP, RNR, ACC, Gapping) to the choice of the coordinating constants $\mathsf{and}_{\mathsf{S,X}}$ or $\mathsf{and}_{\mathsf{X,S}}$ that adjoin material (often just a cluster of words) to a sentence. The transformation rules driven by the constants pick the pieces from the sentence to complete the material to a clause. There remains a question of a general principle/pattern that governs the choice of the constants. For example, the fact that in English the coordinated sentence appears on the right for RNR but on the left for ACC and Gapping boils down to the presence of $\mathsf{and}_{\mathsf{(NP,TV),S}}$ and $\mathsf{and}_{\mathsf{S,(NP,NP)}}$ and the absence of $\mathsf{and}_{\mathsf{S,(NP,TV)}}$ and $\mathsf{and}_{\mathsf{(NP,NP),S}}$. In contrast, one may say that this fact 'falls out' as a consequence of like-category coordination analyses in directional categorial grammars. One may also say that the like-category coordination is itself a postulate, which does not come from any general principle, but does have significant empirical justification. Like any empirical principle, it has exceptions: unlike-category coordination, e.g., "John saw the facts and that Mary had been right".

Since our TS approach is still new, we have not yet accumulated enough empirical data to discern patterns and formulate postulates that underlie the presence of coordination constants for some types and their absence for others. For now, we leave the question open.

4 Related Work

Our transformational approach is rooted in Transformational Generative Grammars [2, 3], later carried into Minimalism [4]. Our abstract form T_A is similar to spell-out of Minimalism. However, whereas spell-out is near culmination of a syntactic derivation for Minimalists, for us, it is just the beginning. We are not interested in how structure is created through a sequence of Merges from lexical selections. Rather, we consider our abstract form as given (by a parser) and investigate its transformations into a semantic form. Our transformations are hence all covert.

Closely related to TS is the work of Butler [1], who also obtains a semantic representation as a result of a transformation from a parsed tree. Unlike us, he has applied his approach to a wealth of empirical data in many languages and has truly achieved wide coverage. His transformations are rather complex and coarse, doing many things at once, and not typed. One may view TS as an attempt to re-engineer and understand Butler's approach and decompose his transformations into elementary steps.

We are grateful to the anonymous reviewer for pointing out the analysis of ACC and Gapping in [14].

> (1) The interpretation of an elliptical construction is obtained by uniformly substituting its immediate constituents into some immediately preceding structure, and computing the interpretation of the results. [14, p. 162, (119)]

We indeed share the underlying idea of picking and substituting of 'immediate constituents' into the coordinated material (understood at some level as an elliptical construction). The proposal of [14] remained rather informal; the present paper may be seen as an attempt to formalize the idea.

There have been other attempts to solve the KLM problem without directional types (within the ACG-like formalisms). Kanazawa [5] proposes 'regular constraints' to prevent over-generation (which recall structural constraints in Government and Binding). This amounts however to duplication of lexical entries. The approach [13] reins in the over-generation using subtyping. Either proposal can be classified as 'proof search' rather than computational like TS; in case of [13] with no guarantees that the proof search ever terminates (and, as the authors admitted, no good way to characterize the space of available derivations and detect over-generation).

5 Conclusions

We have demonstrated the transformational analyses of RNR and Gapping. The analyses make precise various eliding schemas, demanding type preservation. The asymmetry of the type of $\mathsf{and}_{(\mathsf{NP},\mathsf{TV}),\mathsf{S}}$ and similar constants is what lets us answer the Kubota and Levine challenge: how to prevent over-generation in analyses of RNR and gapping without directional types.

The idiosyncrasies of coordination are distilled to the ad hoc choice of constants $\mathsf{and}_{\mathsf{XY}}$. There are transformations for some types XY but not for the others. There may be a pattern there. Collecting the arbitrariness in one place might make the pattern easier to find.

It is interesting to consider interpreting the "sequence of words" as a discontinuous sentence in the sense of Morrill [12].

Acknowledgments We thank an anonymous reviewer for many very insightful and helpful comments. Numerous discussions with the participants of the workshop "New Landscapes in Theoretical Computational Linguistics" (Ohio State University, October 14-16, 2016) are gratefully acknowledged.

Bibliography

[1] Alastair Butler. *Linguistic Expressions and Semantic Processing - A Practical Approach.* Springer, 2015.

[2] Noam Chomsky. *Aspects of a Theory of Syntax.* MIT Press, Cambridge, Mass., 1965.

[3] Noam Chomsky. *Lectures on Government and Binding.* Foris, Dordrecht, 1981.

[4] Noam Chomsky. *The Minimalist Program.* The MIT Press, Cambridge, Massachusetts, 1995.

[5] Makoto Kanazawa. Syntactic features for regular constraints and an approximation of directional slashes in abstract categorial grammars. In Kubota and Levine [11], pages 34–70.

[6] Oleg Kiselyov. Canonical constituents and non-canonical coordination - simple categorial grammar account. volume 9067 of *Lecture Notes in Computer Science*, pages 99–113. Springer, 2014.

[7] Oleg Kiselyov. Applicative abstract categorial grammars in full swing. In *Proc. LENLS 12*. November 2015.

[8] Yusuke Kubota and Robert Levine. Gapping as like-category coordination. In Denis Béchet and Alexander Dikovsky, editors, *Logical Aspects of Computational Linguistics: 7th International Conference*, pages 135–150. Springer, 2012.

[9] Yusuke Kubota and Robert Levine. Gapping as hypothetical reasoning. To appear in *Natural Language and Linguistic Theory*, available at `http://ling.auf.net/lingbuzz/002123`, 2014.

[10] Yusuke Kubota and Robert Levine. Against ellipsis: Arguments for the direct licensing of 'non-canonical' coordinations. *Linguistics and Philosophy*, 38(6):521–576, 2015.

[11] Yusuke Kubota and Robert Levine, editors. *Proceedings for ESSLLI 2015 Workshop 'Empirical Advances in Categorial Grammar'*. University of Tsukuba and Ohio State University, 2015.

[12] Glyn Morrill, Oriol Valentín, and Mario Fadda. The displacement calculus. *Journal of Logic, Language, and Information*,

20(1):1–48, 2011.

[13] Carl Pollard and Chris Worth. Coordination in linear categorial grammar with phenogrammatical subtyping. In Kubota and Levine [11], pages 162–182.

[14] Ivan A. Sag, Gerald Gazdar, Thomas Wasow, and Steven Weisler. Coordination and how to distinguish categories. *Natural Language and Linguistic Theory*, 3(2):117–171, 1985.

From meaning representations to syntactic trees

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Abstract. This paper describes a method for generating natural language, going from meaning representations to syntactic trees. Starting from Davidsonian predicate logic formulas, parse trees are built following the Penn Parsed Corpus of Modern British English, from which the yield (i.e., the words) can be taken. The novel contribution is to highlight how arrangements of meaning content can inform decisions regarding the selection of language constructions.

1 Introduction

This paper pursues the idea that arrangement of information contained by a meaning representation can provide clues to drive a rule-based (pattern-action driven) generation of natural language. Generation will follow a series of transformations to construct parse trees. Trees constructed will conform to the annotation scheme of the Penn Parsed Corpus of Modern British English (PPCMBE; Kroch, Santorini and Diertani 2010), with the yield (i.e. terminal words) producing target sentences of English. The paper is structured as follows. Section 2 sketches the generation procedure with a simple example. Section 3 is the core of the paper, detailing different grammatical constructions and triggers for their creation. Section 4 outlines an implementation. Section 5 is a conclusion.

2 A sketch of the generation procedure

This section describes generation of a canonical sentence with a transitive verb:

(1) Girls see a boy.

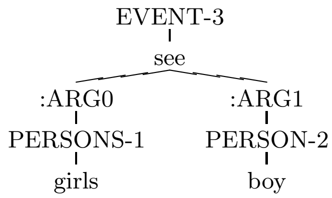
A typical Davidsonian (Davidson 1967) meaning representation for (1) is as follows:

```
∃ EVENT[3] PERSON[2] PERSONS[1] (
      girls(PERSONS[1])
    ∧ boy(PERSON[2])
    ∧ see(EVENT[3], PERSONS[1], PERSON[2]))
```

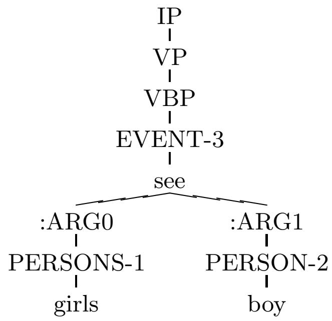
With information for argument roles (‘:ARG0’ and ‘:ARG1’) sourced from the arity of the ‘see’ predicate, the same content converted to a Penman representation (Matthiessen and Bateman 1991, Banarescu et. al. 2013) is as follows:

```
( EVENT-3 / see
      :ARG0 ( PERSONS-1 / girls)
      :ARG1 ( PERSON-2 / boy))
```

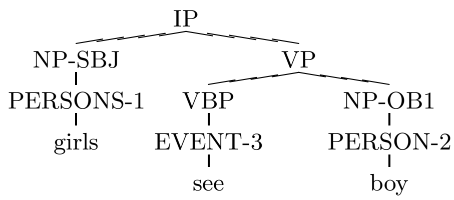
Already the Penman notation provides a base for growing tree structure:



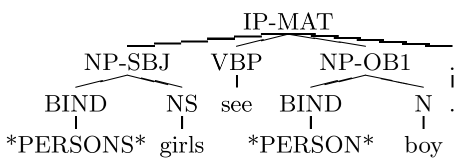
Clause structure is built by adjoining VBP, VP and IP layers to nodes beginning with ‘EVENT’.



Next, arguments of what has been made the main predicate are moved to populate the clause, with ‘`:ARG1`’ as the object inside VP, while ‘`:ARG0`’ creates the subject outside VP.



With arguments in place, it is safe to remove the VP layer and type the clause as IP-MAT (matrix clause), as well as add punctuation. Entity information is retained with BIND, which also contributes to the projection of noun part-of-speech tags (N; singular vs. NS; plural).



The resut leaves noun phrases bare if indefinite, requiring further post-processing to add an indefinite determiner (*a* or *an*) when the noun head is singular.

3 Different constructions

This section is concerned with how the make up of a meaning representation can determine generation of particular English language constructions as PPCMBE trees, with both meaning representation content and how the content is packaged influencing output.

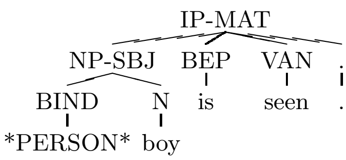
3.1 Passivisation

With '`:ARG0`' missing, but '`:ARG1`' present, the content of '`:ARG1`' can be taken to form the grammatical subject to create a passive clause with the verb tag altered to VAG (passive participle) and BEP (present tense copula) added.

(2) A boy is seen.

```
∃ EVENT[2] PERSON[1] (
      boy(PERSON[1])
    ∧ seen(EVENT[2], _, PERSON[1]))
```

```
( EVENT-2 / seen
      :ARG1 ( PERSON-1 / boy))
```



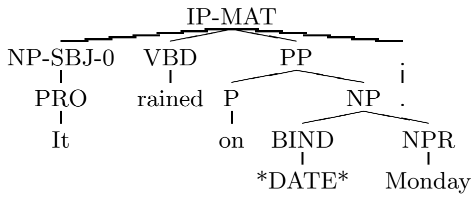
3.2 Expletive it

If there is no core argument ('`:ARG0`', '`:ARG1`', or '`:ARG2`') then expletive *it* should be created to fulfil the grammatical subject role.

(3) It rained on Monday.

```
∃ EVENT[1] (
      past(EVENT[1])
    ∧ rained(EVENT[1])
    ∧ on(EVENT[1]) = DATE[Monday])
```

```
( EVENT-1 / rained
      :MOD ( mod-1 / past)
      :ON ( DATE-Monday / DATE
             :name ( n-2 / name
                    :op1 "Monday")))
```



This example also demonstrates creation of a PP adjunct from an '`:ON`' argument, as well as the effect of `past` tense information altering the verb tag to VBD (past tense verb).

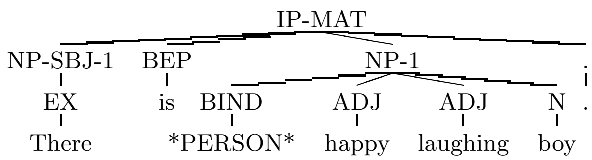
3.3 Existential construction

Another variation in clause construction occurs when the predicate is the copula and there is no '`:ARG1`', but '`:ARG0`' is present. This triggers creation of a *there* subject that is coindexed with the contribution of '`:ARG0`' captured as a noun phrase that immediately follows the copula.

(4) There is a happy laughing boy.

```
∃ ATTRIB[3] ATTRIB[2] EVENT[4] PERSON[1] (
      happy(ATTRIB[2])
    ∧ laughing(ATTRIB[3])
    ∧ is_boy_ATTRIBUTE(PERSON[1], ATTRIB[3])
    ∧ is_boy_ATTRIBUTE(PERSON[1], ATTRIB[2])
    ∧ copula_is(EVENT[4], PERSON[1]))
```

```
( EVENT-4 / copula_is
      :ARG0 ( PERSON-1 / boy
             :ATTRIBUTE ( ATTRIB-3 / laughing)
             :ATTRIBUTE ( ATTRIB-2 / happy)))
```



3.4 Discourse

A discourse is created when there is content for two or more clauses that are conjuncts of `multi-sentence` in the Penman representation.

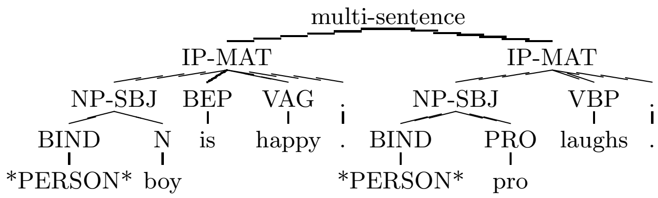
(5) A boy is happy. He laughs.

```
∃ PERSON[3] EVENT[2] EVENT[4] PERSON[1] (
      boy(PERSON[1])
    ∧ PERSON[3] = PERSON[1]
    ∧ is_happy(EVENT[2], PERSON[1])
    ∧ laughs(EVENT[4], PERSON[3]))
```

The presence of '`PERSON[3] = PERSON[1]`' is a sufficient clue when converting from the Davidsonian predicate language formula to create conjuncts conjoined with `multi-sentence`.

```
( CONJ-1 / multi-sentence
      :snt1 ( EVENT-2 / is_happy
             :ARG0 ( PERSON-1 / boy))
      :snt2 ( EVENT-4 / laughs
             :ARG0 PERSON-1))
```

With there being multiple argument roles for the same entity in distinct conjuncts, it should be the first instance in the Penman representation that is populated with information about the entity, e.g., `(PERSON-1 / boy)`, while subsequent instances are bare references, e.g., `PERSON-1`. It is such a bare reference that leads to the creation of a pronoun with the generated output.



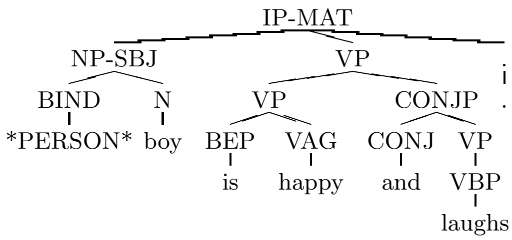
3.5 VP coordination

A relation projecting a label beginning `CONJ` is a relation of coordination. Such a relation name that is not `multi-sentence` is the foundation for forming intra sentential coordination. If the content for arguments that become subjects is shared between conjuncts then VP coordination is established to share the same subject.

(6) A boy is happy and laughs.

```
∃ EVENT[2] EVENT[3] PERSON[1] (
      boy(PERSON[1])
  ∧ (          is_happy(EVENT[2], PERSON[1])
      CONJ_and laughs(EVENT[3], PERSON[1])))
```

```
( CONJ-4 / and
      :op1 ( EVENT-2 / is_happy
             :ARG0 ( PERSON-1 / boy))
      :op2 ( EVENT-3 / laughs
             :ARG0 PERSON-1))
```



If there are other shared entities between conjuncts formed with the same argument role, then they are projected outside the VP layer, typically to the left, but to the right when the argument role is '`:ARG1`' and the verbs are active (also have '`:ARG0`' arguments), or when the argument is heavy, e.g., containing many terminal nodes.

3.6 Adverbial clause

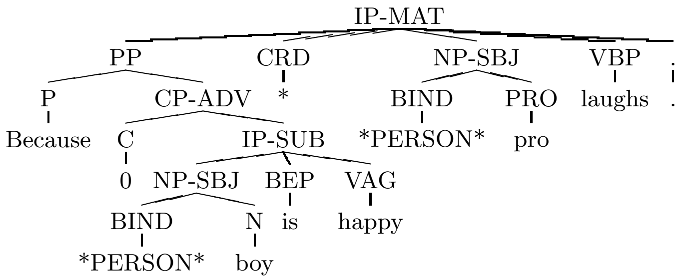
A relation connecting two conjuncts projecting a label beginning `CND` (conditional) or `CRD` (coordinating relation) can trigger creation of an adverbial clause from the first conjunct, with the relation name forming a subordinate conjunction (e.g., *if*, *when*, *unless*, *although*, *because*) that introduces the adverbial clause to a containing clause formed with the content of the second conjunct.

(7) Because a boy is happy he laughs.

```
∃ PERSON[3] EVENT[2] EVENT[4] PERSON[1] (
      boy(PERSON[1])
    ∧ PERSON[3] = PERSON[1]
    ∧ CRD_Because(
        is_happy(EVENT[2], PERSON[1]),
        laughs(EVENT[4], PERSON[3])))
```

With the same entity filling arguments in distinct conjuncts, it is the instance in what forms the adjunct clause that is populated with information about the entity, while subsequent instances are bare references, with bare references leading to the creation of pronouns.

```
( CRD-5 / Because
      :op1 ( EVENT-2 / is_happy
             :ARG0 ( PERSON-1 / boy))
      :op2 ( EVENT-4 / laughs
             :ARG0 PERSON-1))
```



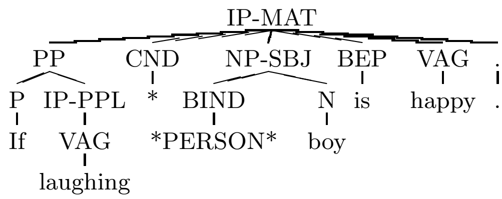
3.7 Participal clause

Instead of an adverbial clause being created from the first conjunct of a relation projecting a label beginning `CND` or `CRD`, a participal clause with a controlled subject is created when there is a bare reference for the '`:ARG0`' of the first conjunct that is coreferential with the content of a core argument ('`:ARG0`', '`:ARG1`' or '`:ARG2`') of the second conjunct.

(8) If laughing a boy is happy.

```
∃ EVENT[2] EVENT[3] PERSON[1] (
      boy(PERSON[1])
    ∧ CND_If(
        laughing(EVENT[2], PERSON[1]),
        is_happy(EVENT[3], PERSON[1])))
```

```
( CND-4 / If
      :op1 ( EVENT-2 / laughing
             :ARG0 PERSON-1)
      :op2 ( EVENT-3 / is_happy
             :ARG0 ( PERSON-1 / boy)))
```



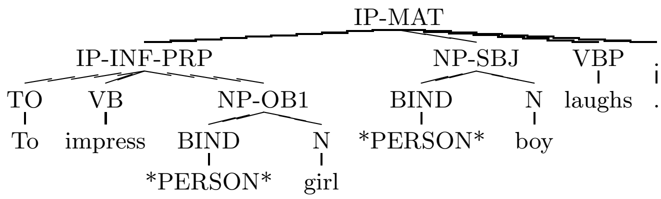
3.8 Purpose clause

A to-infinitive clause can function as an adverbial expressing ideas of purpose or outcome. The content for such a clause falls under a '`:PRP`' tag, while inside there is '`:ARG0`' containing a bare reference coreferential with (i.e., controlled by) the content of the '`:ARG0`' of the containing clause. This creates a subjectless IP-INF-PRP projection containing `(TO to)` and a non-finite verb (`VB`).

(9) To impress a girl a boy laughs.

```
∃ PRP[2] EVENT[4] EVENT[5] PERSON[3] PERSON[1] (
      boy(PERSON[1])
    ∧ girl(PERSON[3])
    ∧ is_FACT_THAT(PRP[2],
        impress(EVENT[4], PERSON[1], PERSON[3]))
    ∧ laughs(EVENT[5], PERSON[1])
    ∧ PRP(EVENT[5]) = PRP[2])
```

```
( EVENT-5 / laughs
      :ARG0 ( PERSON-1 / boy)
      :PRP ( EVENT-4 / impress
             :ARG0 PERSON-1
             :ARG1 ( PERSON-3 / girl)))
```



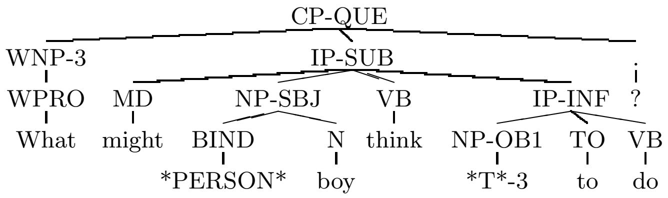
3.9 Infinitive clause with long distance dependency

Content for an embedded to-infinitive clause falls under a '`:TOCOMP`' tag, while inside there is '`:ARG0`' containing a bare reference coreferential with (i.e., controlled by) the content of the '`:ARG2`' if present, or alternatively '`:ARG1`' if present, or alternatively '`:ARG0`' of the containing clause. This creates a subjectless IP-INF projection containing `(TO to)` and a non-finite predicate.

(10) What might the boy think to do?

```
∃ EVENT[3] EVENT[2] PERSON[1] (
      boy(PERSON[1])
    ∧ QUEST(MD_might(think_TOCOMP(EVENT[2], PERSON[1],
          do(EVENT[3], PERSON[1], ENTITY[unknown])))))
```

```
( EVENT-2 / think
      :domain-of ( QUEST-5 / QUEST)
      :domain-of ( MD-4 / might)
      :ARG0 ( PERSON-1 / boy)
      :TOCOMP ( EVENT-3 / do
             :ARG0 PERSON-1
             :ARG1 ( ENTITY_UNK-6 / unknown)))
```



The example also illustrates how a long distance dependency is established with '`:ARG1 (ENTITY_UNK-6 / unknown)`' forming the foundation for an object noun phrase trace (`NP-OB1 *T*-3`) that is coindexed with a WH-phrase (`WNP-3 (WPRO What)`) that is placed as the highest constituent of the first commanding question scope marker: '`:domain-of (QUEST-5 / QUEST)`'.

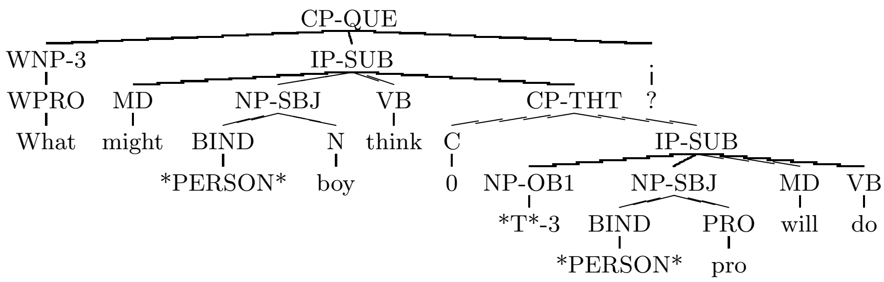
3.10 Embedded clause with long distance dependency

An embedded clause rather than a to-infinitive is established with clause content placed under '`:THAT`'. Inside the content for the embedded clause, bare references form foundations for pronouns.

(11) What might the boy think that he will do?

```
∃ PERSON[3] EVENT[4] EVENT[2] PERSON[1] (
      boy(PERSON[1])
    ∧ PERSON[3] = PERSON[1]
    ∧ QUEST(MD_might(think_THAT(EVENT[2], PERSON[1],
            MD_will(do(EVENT[4], PERSON[3], ENTITY[unknown]))))))
```

```
( EVENT-2 / think
      :domain-of ( QUEST-8 / QUEST)
      :domain-of ( MD-7 / might)
      :ARG0 ( PERSON-1 / boy)
      :THAT ( EVENT-4 / do
            :domain-of ( MD-6 / will)
            :ARG0 PERSON-1
            :ARG1 ( ENTITY_UNK-9 / unknown)))
```



The example again illustrates a long distance dependency established out of the embedding.

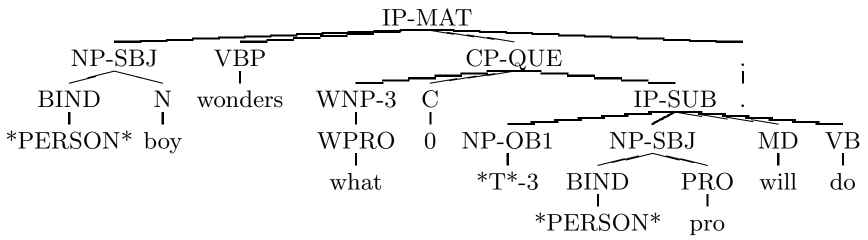
3.11 Embedded question

Having the scope marker for a question local to an embedded clause results in an embedded question.

(12) A boy wonders what he will do.

```
∃ PERSON[3] EVENT[4] EVENT[2] PERSON[1] (
      boy(PERSON[1])
    ∧ PERSON[3] = PERSON[1]
    ∧ wonders_THAT(EVENT[2], PERSON[1],
        QUEST(MD_will(do(EVENT[4], PERSON[3], ENTITY[unknown])))))
```

```
( EVENT-2 / wonders
      :ARG0 ( PERSON-1 / boy)
      :THAT ( EVENT-4 / do
             :domain-of ( QUEST-8 / QUEST)
             :domain-of ( MD-7 / will)
             :ARG0 PERSON-1
             :ARG1 ( ENTITY_UNK-9 / unknown)))
```



Together with the examples of section 3.9 and 3.10, this demonstrates how it is not enough for a meaning representation to mark a WH question with `unknown` alone, but that scope marking the level of structure to place a fronted WH phrase is also vital.

3.12 Small clause

Small clauses are embedded clauses that occur with the absence of a finite verb.

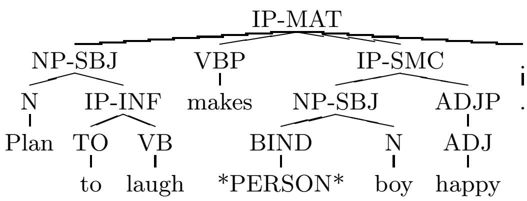
(13) A plan to laugh makes a boy happy.

Lack of a finite verb is reflected by creation of an unspecified '`xxx`' predicate that connects the subject *a boy* to the *happy* attribute in the following Davidsonian formula:

```
∃ ATTRIB[5] PERSON[4] ENTITY[1] EVENT[2] EVENT[6] EVENT[3] (
      is_plan_TOCOMP(ENTITY[1],
        laugh(EVENT[2]))
    ∧ boy(PERSON[4])
    ∧ happy(ATTRIB[5])
    ∧ makes_TOCOMP(EVENT[3], ENTITY[1],
        xxx(EVENT[6], PERSON[4], ATTRIB[5])))
```

```
( EVENT-3 / makes
      :ARG0 ( ENTITY-1 / plan
             :TOCOMP ( EVENT-2 / laugh))
      :TOCOMP ( EVENT-6 / xxx
             :ATTRIBUTE ( ATTRIB-5 / happy)
             :ARG0 ( PERSON-4 / boy)))
```

This leads to projection of an IP-SMC embedded clause with neither verb nor copula creation.



This example also illustrates creation of a nominal with an infinitive clause embedding, triggered by ‘TOCOMP’.

3.13 Relative clause

The example of this section demonstrates creation of a relative clause with a long distance dependency:

(14) A boy that a girl says is happy laughs.

There is nothing overt to signal a relative clause with the Davidsonian formula:

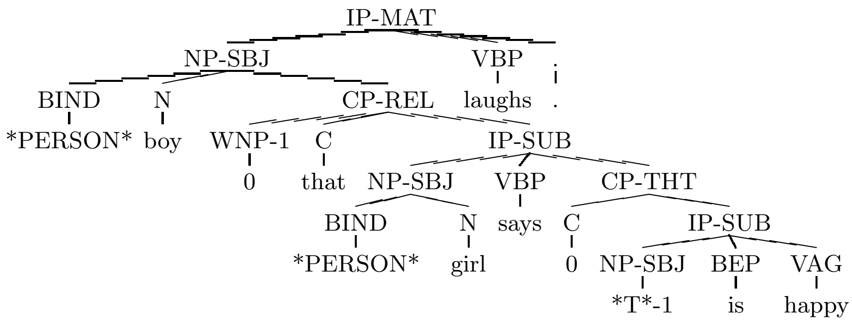
```
∃ EVENT[5] EVENT[4] EVENT[6] PERSON[3] PERSON[1] (
      girl(PERSON[3])
    ∧ says_THAT(EVENT[4], PERSON[3],
        is_happy(EVENT[5], PERSON[1]))
    ∧ boy(PERSON[1])
    ∧ laughs(EVENT[6], PERSON[1]))
```

But a base for realising a relative clause emerges with conversion to Penman notation:

```
( EVENT-6 / laughs
      :ARG0 ( PERSON-1 / boy
            :ARG0-of ( EVENT-5 / is_happy
                  :THAT-of ( EVENT-4 / says
                        :ARG0 ( PERSON-3 / girl)))))
```

A key requirement for Penman notation is to connect all content around a single rooted node. This privileged node typically forms the main predicate of a clause following generation. While not necessary for being a Davidsonian forumla, a convention can be followed to place such a privileged predicate as the most right-side predicate of the formula (so, '`laughs`' of the example). Connection to this single rooted predicate is possible by folding Penman material around 'inverse roles' (signalled by ending a role name with '`-of`'). This acts to compact to a local argument relation the long distance dependency of the relative clause.

Generation consists of unfolding the dependency, so taking '`-of`' content and reintegrating the content with clausal embedded structure:



3.14 It cleft

As a final example, consider generation of an it-cleft:

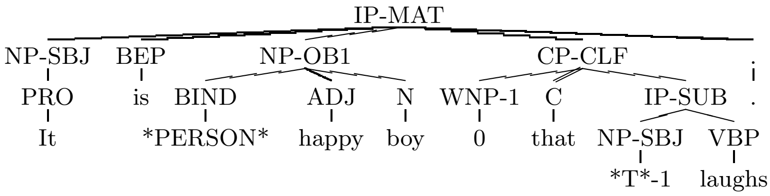
(15) It is the happy boy that laughs.

Such a cleft sentence leads to the presence of a copula predicate that connects '`ARG1`' content to an '`ARG0`' entity that, with conversion to Penman notation, is given a dummy '`ENTITY`' head and is inversed linked to material sufficient to create a clause:

```
∃ ATTRIB[2] ENTITY[3] EVENT[5] EVENT[6] PERSON[1] (
      happy(ATTRIB[2])
    ∧ is_boy_ATTRIBUTE(PERSON[1], ATTRIB[2])
    ∧ laughs(EVENT[5], ENTITY[3])
    ∧ copula_is(EVENT[6], ENTITY[3], PERSON[1]))
```

```
( EVENT-6 / copula_is
      :ARG0 ( ENTITY-3 / ENTITY
             :ARG0-of ( EVENT-5 / laughs))
      :ARG1 ( PERSON-1 / boy
             :ATTRIBUTE ( ATTRIB-2 / happy)))
```

This leads to generation of the following tree:



Internally, the it-cleft has the same structure as a relative clause, but externally, it is a daughter of IP.

4 Implementation

A program available from http://www.compling.jp/generation implements the generation of this paper. The assumed engine to transform trees is provided by tsurgeon (Levy and Andrew 2006). This works with tsurgeon scripts that contain patterns with associated actions. Patterns describe tree structure with the tree description language of tgrep (Pito, 1994) and actions transform the tree, e.g., moving, adjoining, copying or deleting auxiliary trees or relabelling nodes. Alternative programs with similar functionality to transform trees with scripts are CorpusSearch (Randall 2009) and TTT (Purtee and Schubert 2012).

5 Conclusion

This paper has focused on the generation of various English language constructions from meaning representations. Starting as Davidsonian predicate language formulas, a key step was conversion to Penman representations that formed the basis for generation to proceed with successive tree structure changes.

Language generation raises the issue of how to choose between the many ways a language offers to present content. The novel contribution of this paper has been to demonstrate how there can be a significant role for meaning representations to play in influencing the selection of grammatical constructions with the arrangement of information content, notably, handling distinctions of clause type as well as the choice between discourse, or (VP) coordination, or projection of an adverbial clause, or participial clause, or creation of a small clause, or infinitive embedding, or finite embedding, or relative clause, or it-cleft. This has simplified what it takes to create a generation component capable of rich, varied and natural output that preserves meaning.

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References

Banarescu, L., C. Bonial, S. Cai, M. Georgescu, K. Griffitt, U. Hermjakob, K. Knight, P. Koehn, M. Palmer, and N. Schneider. 2013. Abstract Meaning Representation for Sembanking. In *Proceedings of the 7th Linguistic Annotation Workshop and Interoperability with Discourse*, pages 178–186.

Davidson, Donald. 1967. The logical form of action sentences. In N. Rescher, ed., *The Logic of Decision and Action*. Pittsburgh: University of Pittsburgh Press. Reprinted in: D. Davidson, 1980. *Essays on Actions and Events*. Claredon Press, Oxford, pages 105–122.

Kroch, Anthony, Beatrice Santorini, and Ariel Diertani. 2010. *The Penn-Helsinki Parsed Corpus of Modern British English (PPCMBE)*. Department of Linguistics, University of Pennsylvania. CD-ROM, second edition, (http://www.ling.upenn.edu/hist-corpora).

Levy, Roger and Galen Andrew. 2006. Tregex and tsurgeon: tools for querying and manipulating tree data structure. In *5th International conference on Language Resources and Evaluation*.

Matthiessen, Christian and John A Bateman. 1991. *Text generation and systemic-functional linguistics: experiences from English and Japanese*. Pinter Publishers.

Purtee, A. and Lenhart K. Schubert. 2012. Ttt: A tree transduction language for syntactic and semantic processing. In *EACL 2012 Workshop on Applications of Tree Automata Techniques in Natural Language Processing (ATANLP 2012)*. Avignon, France.

Randall, Beth. 2009. CorpusSearch 2 Users Guide. (http://corpussearch.sourceforge.net/CS-manual/Contents.html).

Lambek Categorial Grammars as Abstract Categorial Grammars

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Abstract. This paper describes a construction that allows Lambek Categorial Grammars to be represented as Abstract Categorial Grammars. This construction is based upon a family of combinators that allows Lambek lexical entries to be interpreted as linear λ-terms.

1 Introduction

Abstract Categorial Grammars [2] (ACG, for short) differ from classical categorial grammars in an essential way: the ACG type system is based on a commutative logic (namely, the implicative fragment of multiplicative linear logic [4]). For this reason, it has been argued that the way of encoding wh-extraction in an ACG corresponds to an uncontroled form of extraction, which results in syntactic overgeneration. In particular, an ACG could not accomodate left and right peripheral extractions like a Lambek categorial grammar [9] (LG, for short) does.

This claim about ACG and LG is certainly not true at the level of the string languages. Indeed, Pentus' theorem [11, 12] states that every Lambek grammar can be transformed into a context-free grammar, and there is a canonical way of representing a context-free grammar as an ACG [3].

The claim is not quite true eiteher at the level of the derivations since Kanazawa and Salvati have shown that Pentus' construction preserve, in some sense, the derivations of the original Lambek grammar [6]. As a consequence, given a Lambek grammar G, it is possible to define an ACG that generates a set of λ-terms that correspond to the derivation of G. What is then needed in order to turn these derivations into strings is an appropriate way of interpreting the lexical entries of the original Lambek grammar as linear λ-terms. The main goal of this paper is to devise such an interpretation.

2 Mathematical preliminaries

Let A be a set of atomic types. The set $\mathscr{T}_A$ of the *simple types* (built upon A) is inductively defined according to the following rules:

$$\mathscr{T}_A \quad ::= \quad A \mid (\mathscr{T}_A \rightarrow \mathscr{T}_A)$$

The order of a simple type is inducively defined as follows:

1. $\mathrm{ord}(a) = 1$, for $a \in A$;
2. $\mathrm{ord}(\alpha \rightarrow \beta) = \max\{\mathrm{ord}(\alpha) + 1, \mathrm{ord}(\beta)\}$

A finite set of typed constants is called a *higher-order signature*. More formally, such a higher-order signature consists of a triple $\Sigma = \langle A, C, \tau \rangle$, where:

1. A is a finite set of atomic types;
2. C is a finite set of constants;
3. $\tau : C \rightarrow \mathscr{T}_A$ is a function that assigns to each constant in C a simple type in $\mathscr{T}_A$.

Let $\Sigma = \langle A, C, \tau \rangle$ be a signature. The order of Σ is defined to be $\max\{\mathrm{ord}(\tau(c)) \mid c \in C\}$.

Let X be an infinite countable set of λ-variables. Given a higher-order signature $\Sigma = \langle A, C, \tau \rangle$, the set $\Lambda(\Sigma)$ of the *linear λ-terms* built upon Σ is inductively defined as follows:

1. if $c \in C$, then $c \in \Lambda(\Sigma)$;
2. if $x \in X$, then $x \in \Lambda(\Sigma)$;
3. if $x \in X$, $t \in \Lambda(\Sigma)$, and x occurs free in t exactly once, then $(\lambda x.\, t) \in \Lambda(\Sigma)$;
4. if $t, u \in \Lambda(\Sigma)$, and the sets of free variables of t and u are disjoint, then $(t\, u) \in \Lambda(\Sigma)$.

$\Lambda(\Sigma)$ is provided with the usual notion of capture-avoiding substitution, α-conversion, β-reduction, and η-reduction [1]. we take the relation of $\beta\eta$-equivalence as the notion of equality between λ-terms.

Each λ-terms in $\Lambda(\Sigma)$ may be assigned a simple type according to the following type system:

$$\vdash_\Sigma \; c : \tau(c) \qquad\qquad x : \alpha \vdash_\Sigma \; x : \alpha$$

$$\frac{\Gamma, x : \alpha \vdash_\Sigma \; t : \beta}{\Gamma \vdash_\Sigma \; (\lambda x.\, t) : (\alpha \to \beta)} \qquad \frac{\Gamma \vdash_\Sigma \; t : (\alpha \to \beta) \quad \Delta \vdash_\Sigma \; u : \alpha}{\Gamma, \Delta \vdash_\Sigma \; (t\, u) : \beta}$$

Given two higher-order signatures $\Sigma_1 = \langle A_1, C_1, \tau_1 \rangle$ and $\Sigma_2 = \langle A_2, C_2, \tau_2 \rangle$, a morphism $\Phi : \Sigma_1 \to \Sigma_2$ consists of an interpretation of the atomic types of Σ_1 as types built upon A_2 together with an interpretation of the constants of Σ_1 as linear λ-terms built upon Σ_2. These two interpretations must be such that their homomorphic extensions commute with the typing relations. More formally, a *morphism between higher-order signatures*, $\Phi : \Sigma_1 \to \Sigma_2$, is defined to be a pair $\Phi = \langle \eta, \theta \rangle$ such that:

1. $\eta : A_1 \to \mathscr{T}(A_2)$ is a function that interprets the atomic types of Σ_1 as simple types built upon A_2;
2. $\theta : C_1 \to \Lambda(\Sigma_2)$ is a function that interprets the constants of Σ_1 as linear λ-terms built upon Σ_2;
3. the interpretation functions are compatible with the typing relation, *i.e.*, for any $c \in C_1$, the following typing judgement is derivable:

   $$\vdash_{\Sigma_2} \; \theta(c) : \hat{\eta}(\tau_1(c)),$$

   where $\hat{\eta}$ is the unique homomorphic extension of η.

Condition 3, in the above definition ensures that if $x_1 : \alpha_1, \ldots, x_n : \alpha_n \vdash_{\Sigma_1} \; t : \alpha$, then:

$$x_1 : \Phi(\alpha_1), \ldots, x_n : \Phi(\alpha_n) \vdash_{\Sigma_2} \; \Phi(t) : \Phi(\alpha)$$

where, according to the context, $\Phi(\cdot)$ denotes either the homomorphic extension of η or the homomorphic extension of η.

Let $\Phi : \Sigma_1 \to \Sigma_2$ be a morphism between signatures. The order of Φ is defined to be $\max\{\mathrm{ord}(\Phi(a)) \mid a \in A_1\}$.

We end this section by introducing a few additional notations that will be useful in the sequel of the paper. Let $I = \{i_1, \ldots, i_n\}$ be a totally ordered finite set of indices, and let $(x_i)_{i \in I}$ (respectively, $(\alpha_i)_{i \in I}$) be a sequence of λ-variables (respectively, simple types) indexed by I. We write $(x_i : \alpha_i)_{i \in I}$ for the typing environment $x_{i_1} : \alpha_{i_1}, \ldots, x_{i_n} : \alpha_{i_n}$. Similarly, we write $t[x_i := u_i]_{i \in I}$ for the simultaneous substitution in the λ-term t of the terms $u_{i_1}, \ldots, u_{i_n}$ for the free variables $x_{i_1}, \ldots, x_{i_n}$. In this latter case, we assume that the set of variables $\{x_i \mid i \in I\}$ corresponds exactly to the set of free variables of t.

Let Σ be an alphabet. As usual, we write Σ^* for the set of strings generated by Σ. We use ϵ to denote the empty string, and the infix operator '+' to denote string concatenation. Accordingly, given a sequence of strings $(w_i)_{i \in I}$, we write $\sum_{i \in I} w_i$ for the string $w_{i_1} \ldots w_{i_n}$.

Lambek Grammar
man : n
woman : n
some : $np \,/\, n$
every : $np \,/\, n$
loves : $(np \setminus s) \,/\, np$
who : $(n \setminus n) \,/ (np \setminus s)$
whom : $(n \setminus n) \,/ (s \,/\, np)$

Fig. 1.

3 Lambek categorial grammars

The classical notion of a Lambek categorial grammar is based on a deductive system known as the associative Lambek calculus [9]. This calculus may be seen as a non-commutative fragment of implicative linear logic [4].

Let A be a set of atomic formulas. The syntax of the Lambek formulas (built upon A) obeys the following formation rules:

$$\mathscr{F}_A \;::=\; A \mid (\mathscr{F}_A \setminus \mathscr{F}_A) \mid (\mathscr{F}_A \,/\, \mathscr{F}_A)$$

where formulas of the form $\alpha \setminus \beta$ correspond to left-to-right implications (i.e., α *implies* β), and formulas of the form $\alpha \,/\, \beta$ to right-to-left implications (i.e., α *is implied by* β). Lambek formulas are also called *syntactic types*.

The deduction relation is then specified by means of the following sequent calculus.

$$\alpha \vdash_L \alpha$$

$$\frac{\alpha, \Gamma \vdash_L \beta}{\Gamma \vdash_L \alpha \setminus \beta} \qquad \frac{\Gamma, \alpha \vdash_L \beta}{\Gamma \vdash_L \beta \,/\, \alpha}$$

$$\frac{\Gamma \vdash_L \alpha \quad \Delta, \beta, \Theta \vdash_L \gamma}{\Delta, \Gamma, \alpha \setminus \beta, \Theta \vdash_L \gamma} \qquad \frac{\Gamma \vdash_L \alpha \quad \Delta, \beta, \Theta \vdash_L \gamma}{\Delta, \beta \,/\, \alpha, \Gamma, \Theta \vdash_L \gamma}$$

It should be stressed that the above system does not include any structural rule. In particular, the non-commutativity of the systems is reflected by the absence of an exchange rule. This, in turn, explains the presence of two different implications.

A Lambek categorial grammar (L-grammar, for short) is defined to be a quadruple $G = \langle \Sigma, A, \mathcal{L}, s \rangle$ such that:

1. Σ is a finite set of terminal symbols;
2. A is a finite set of atomic types;
3. $\mathcal{L} : \Sigma \longrightarrow 2^{\mathscr{F}_A}$ is a *lexicon* that assigns to each terminal symbol a finite set of types built upon A;
4. $s \in A$ is a distinguished type, called the *initial type* of the grammar.

A word $a_0 a_1 \ldots a_n \in \Sigma^*$ belongs to the language generated by G if and only if there exist $\alpha_0 \in \mathcal{L}(a_0), \alpha_1 \in \mathcal{L}(a_1), \ldots \alpha_n \in \mathcal{L}(a_n)$ such that $\alpha_0, \alpha_1, \ldots \alpha_n \vdash_L s$ is derivable.

Figure 1 gives the lexicon of a Lambek categorial grammar that will serve as a running example throughout this paper. According to this grammar, the sentence "***every man who***

Context-free Grammar

$$
\begin{array}{rcl}
<np> & \rightarrow & <np/n> <n> \\
<s> & \rightarrow & <np> <(np\backslash s) / np> <np> \\
<n> & \rightarrow & <n> <(n\backslash n)/(np\backslash s)> <np\backslash s> \\
<n> & \rightarrow & <n> <(n\backslash n)/(s/np)> <s/np> \\
<np\backslash s> & \rightarrow & <(np\backslash s)/np> <np> \\
<s/np> & \rightarrow & <np> <(np\backslash s)/np> \\
<s/np> & \rightarrow & <np> <(np\backslash s) / np> <np/np> \\
<np/np> & \rightarrow & <np/n> <n/np> \\
<n/np> & \rightarrow & <n> <(n\backslash n)/(np\backslash s)> <(np\backslash s)/np> \\
<(np\backslash s)/np> & \rightarrow & <(np\backslash s)/np> <np/np> \\
<n> & \rightarrow & \textit{\textbf{man}} \\
<n> & \rightarrow & \textit{\textbf{woman}} \\
<np/n> & \rightarrow & \textit{\textbf{some}} \\
<np/n> & \rightarrow & \textit{\textbf{every}} \\
<(np\backslash s)/np> & \rightarrow & \textit{\textbf{loves}} \\
<(n\backslash n)/(np\backslash s)> & \rightarrow & \textit{\textbf{who}} \\
<(n\backslash n)/(s/np)> & \rightarrow & \textit{\textbf{whom}}
\end{array}
$$

Fig. 2.

loves some woman loves every woman" is grammatical because the following sequent is derivale:

$$np \,/\, n, n, (n \setminus n) \,/ (np \setminus s), (np \setminus s) \,/\, np, np \,/\, n, n, (np \setminus s) \,/\, np, np \,/\, n, n \vdash_L s$$

The above example illustrates that when dealing with a categorial grammar, parsing corresponds to proof-search. Consequently, a categorial parse structure amounts to a formal derivation. Then, using the Curry-Howard correspondance, it is possible to associate a simply typed (actually, linear) λ-term to any derivation of the Lambek calculus. This is realized by the following system:

$$x : \alpha \vdash_{\lambda L} x : \alpha$$

$$\frac{x : \alpha, \Gamma \vdash_{\lambda L} t : \beta}{\Gamma \vdash_{\lambda L} \lambda x. t : \alpha \setminus \beta} \qquad \frac{\Gamma, x : \alpha \vdash_{\lambda L} t : \beta}{\Gamma \vdash_{\lambda L} \lambda x. t : \beta \,/\, \alpha}$$

$$\frac{\Gamma \vdash_{\lambda L} u : \alpha \quad \Delta \vdash_{\lambda L} t : \alpha \setminus \beta}{\Gamma, \Delta \vdash_{\lambda L} t\, u : \beta} \qquad \frac{\Gamma \vdash_{\lambda L} t : \beta \,/\, \alpha \quad \Delta \vdash_{\lambda L} u : \alpha}{\Gamma, \Delta \vdash_{\lambda L} t\, u : \beta}$$

According to Pentus' theorem [11, 12], every Lambek grammar may be turned into an equivalent context-free grammar. The context-free grammar of Figure 2, for instance, is a grammar that generates the same language as the Lambek grammar of Figure 1.

In the grammar of Figure 2, in agreement with Pentus' construction, types of the Lambek calculus are used as non-terminal symbols. In addition, every production rule $<\alpha> \rightarrow <\alpha_1> \ldots <\alpha_n>$ corresponds to a derivable sequent $\alpha_1, \ldots, \alpha_n \vdash_L \alpha$ whose derivation, in turn, corresponds to a λ-term. These sequents, together with the λ-terms encoding their derivations, are given in Figure 3.

Derivable sequents

$$\begin{array}{rl}
x : np/n,\, y : n \vdash_{\lambda L} & x\, y : np \\
x : np,\, y : (np\backslash s)\,/\,np,\, z : np \vdash_{\lambda L} & y\, z\, x : s \\
w : n,\, x : (n\backslash n)/(np\backslash s),\, y : np\backslash s \vdash_{\lambda L} & x\,(\lambda z.\, y\, z)\, w : n \\
w : n,\, x : (n\backslash n)/(s/np),\, y : s/np \vdash_{\lambda L} & x\,(\lambda z.\, y\, z)\, w : n \\
x : (np\backslash s)/np,\, y : np \vdash_{\lambda L} & \lambda z.\, x\, y\, z : np\backslash s \\
x : np,\, y : (np\backslash s)/np \vdash_{\lambda L} & \lambda z.\, y\, z\, x : s/np \\
w : np,\, x : (np\backslash s)\,/\,np,\, y : np/np \vdash_{\lambda L} & \lambda z.\, x\,(y\, z)\, w : s/np \\
x : np/n,\, y : n/np \vdash_{\lambda L} & \lambda z.\, x\,(y\, z) : np/np \\
v : n,\, w : (n\backslash n)/(np\backslash s),\, x : (np\backslash s)/np \vdash_{\lambda L} & \lambda y.\, w\,(\lambda z.\, x\, y\, z)\, v : n/np \\
w : (np\backslash s)/np,\, x : np/np \vdash_{\lambda L} & \lambda yz.\, w\,(x\, y)\, z : (np\backslash s)/np
\end{array}$$

Fig. 3.

In some sense, a context-free grammar resulting from Pentus' construction preserves the parse structures of the original Lambek grammar. Consider, for instance, a sentence S belonging to the language generated by the Lambek grammar of Figure 1. Accordingly, there exists a derivation of S using the rules of the context-free grammar of Figure 2. Now, using this context-free derivation together with the λ-terms given in Figure 3 it is possible to compute the λ-term corresponding to the original derivation of S in the Lambek grammar of Figure 1. This has been shown by Kanazawa and Salvati [6].

4 Abstract categorial grammars

Abstract categorial grammars have been introduced in [2]. Contrarily to the case of most other notions of categorial grammar, they are based on a fully commutative logic.

Formally, an *abstract categorial grammar* is a quadruple $G = \langle \Sigma_1, \Sigma_2, \mathcal{L}, s \rangle$ where:

1. Σ_1 and Σ_2 are two higher-order signatures; they are called the *abstract vovabulary* and the *object vovabulary*, respectively;
2. $\mathcal{L} : \Sigma_1 \to \Sigma_2$ is a morphism between the abstract vovabulary and the object vovabulary; it is called the *lexicon*;
3. s is an atomic type of the abstract vocabulary; it is called the *distinguished type* of the grammar.

The *abstract language* generated by G, $\mathcal{A}(\mathcal{G})$, is defined as follows:

$$\mathcal{A}(G) = \{t \in \Lambda(\Sigma_1) \mid \;\vdash_{\Sigma_1} t\colon s \text{ is derivable}\}$$

In words, the abstract language generated by G is the set of closed linear λ-terms, built upon the abstract vocabulary Σ_1, whose type is the distinguished type s.

The *object language* generated by G, $\mathcal{O}(\mathcal{G})$, is then defined to be the image of the abstract language by the lexicon $\mathcal{L}$:

$$\mathcal{O}(G) = \{t \in \Lambda(\Sigma_2) \mid \exists u \in \mathcal{A}(G).\; t = \mathcal{L}(u)\}$$

The abstract language of an ACG may be thought of as its parse structures, and its object language as the language it generates. Using this intuition, it is not difficult to show that

Abstract Vocabulary

$\textsc{prod}_0 : <np/n> \to <n> \to <np>$
$\textsc{prod}_1 : <(np\backslash s) \,/\, np> \to <np> \to <np> \to <s>$
$\textsc{prod}_2 : <(n\backslash n)/(np\backslash s)> \to <np\backslash s> \to <n> \to <n>$
$\textsc{prod}_3 : <(n\backslash n)/(s/np)> \to <s/np> \to <n> \to <n>$
$\textsc{prod}_4 : <(np\backslash s)/np> \to <np> \to <np\backslash s>$
$\textsc{prod}_5 : <(np\backslash s)/np> \to <np> \to <s/np>$
$\textsc{prod}_6 : <(np\backslash s) \,/\, np> \to <np/np> \to <np> \to <s/np>$
$\textsc{prod}_7 : <np/n> \to <n/np> \to <np/np>$
$\textsc{prod}_8 : <(n\backslash n)/(np\backslash s)> \to <(np\backslash s)/np> \to <n> \to <n/np>$
$\textsc{prod}_9 : <(np\backslash s)/np> \to <np/np> \to <(np\backslash s)/np>$
$\textsc{man} : <n>$
$\textsc{woman} : <n>$
$\textsc{some} : <np/n>$
$\textsc{every} : <np/n>$
$\textsc{loves} : <(np\backslash s)/np>$
$\textsc{who} : <(n\backslash n)/(np\backslash s)>$
$\textsc{whom} : <(n\backslash n)/(s/np)>$

Fig. 4.

Object Vocabulary

$\mathsf{man} : n$
$\mathsf{woman} : n$
$\mathsf{some} : n \to np$
$\mathsf{every} : n \to np$
$\mathsf{loves} : np \to np \to s$
$\mathsf{who} : (np \to s) \to n \to n$
$\mathsf{whom} : (np \to s) \to n \to n$

Fig. 5.

every context-free grammar may be represented as an ACG [3]. Accordingly, the context-free grammar of Figure 2 could be turned into an ACG. Now, using a similar construction together with the result of Kanazawa and Salvati [6], one may take advantage of the λ-terms given in Figure 3 in order to devise an ACG that generates the λ-terms that encodes the derivations of the original Lambek grammar of Figure 1. Lets call this ACG $LDER$. Its abstract vocabulary and its object vocabulary are respectively given in Figure 4 and Figure 5. Its lexicon is defined by Figure 6 and Figure 7.

While every Lambek grammar is strongly lexicalized, $LDER$ is not (in the sense that some of its abstract constants are interpreted by pure combinators). In [7, 8], Kanazawa and Yoshinaka show that every second-order ACG (i.e., an ACG whose abstract vocabulary is

Lexicon: type interpretation
$<\!n\!> := n$
$<\!np\!> := np$
$<\!s\!> := s$
$<\!n/np\!> := np \to n$
$<\!np/n\!> := n \to np$
$<\!np/np\!> := np \to np$
$<\!np\backslash s\!> := np \to s$
$<\!s/np\!> := np \to s$
$<\!(np\backslash s)/np\!> := np \to np \to s$
$<\!(n\backslash n)/(np\backslash s)\!> := (np \to s) \to n \to n$
$<\!(n\backslash n)/(s/np)\!> := (np \to s) \to n \to n$

Fig. 6.

Lexicon: term interpretation
$\textsc{prod}_0 := \lambda xy.\, x\, y$
$\textsc{prod}_1 := \lambda xyz.\, x\, y\, z$
$\textsc{prod}_2 := \lambda wxy.\, w\, (\lambda z.\, x\, z)\, y$
$\textsc{prod}_3 := \lambda wxy.\, w\, (\lambda z.\, x\, z)\, y$
$\textsc{prod}_4 := \lambda xyz.\, x\, y\, z$
$\textsc{prod}_5 := \lambda xyz.\, x\, z\, y$
$\textsc{prod}_6 := \lambda wxyz.\, w\, (x\, z)\, y$
$\textsc{prod}_7 := \lambda xyz.\, x\, (y\, z)$
$\textsc{prod}_8 := \lambda vwxy.\, v\, (\lambda z.\, w\, y\, z)\, x$
$\textsc{prod}_9 := \lambda wxyz.\, w\, (x\, y)\, z$
$\textsc{man} := \mathsf{man}$
$\textsc{woman} := \mathsf{woman}$
$\textsc{some} := \mathsf{some}$
$\textsc{every} := \mathsf{every}$
$\textsc{loves} := \mathsf{loves}$
$\textsc{who} := \mathsf{who}$
$\textsc{whom} := \mathsf{whom}$

Fig. 7.

second-order) can be lexicalized. As a matter of fact, $LDER$ is a second-order ACG because it derives from a context-free grammar. We may therefore lexicalize it. The resulting ACG, which we call $LDER_{lex}$ shares with $LDER$ the same object vocabulary (Fig. 5), the same set of abstract atomic types, and the same type interpretation (Fig. 6). As for the new

Abstract Vocabulary (lexicalized grammar)

$$
\begin{aligned}
\text{MAN} &: <n> \\
\text{WOMAN} &: <n> \\
\text{SOME} &: <n> \to <np> \\
\text{SOME}_0 &: <n/np> \to <np/np> \\
\text{EVERY} &: <n> \to <np> \\
\text{EVERY}_0 &: <n/np> \to <np/np> \\
\text{LOVES} &: <np> \to <np> \to <s> \\
\text{LOVES}_0 &: <np> \to <np\backslash s> \\
\text{LOVES}_1 &: <np> \to <s/np> \\
\text{LOVES}_2 &: <np/np> \to <np> \to <s/np> \\
\text{LOVES}_3 &: <np/np> \to <(np\backslash s)/np> \\
\text{LOVES}_4 &: <(np\backslash s)/np> \\
\text{WHO} &: <np\backslash s> \to <n> \to <n> \\
\text{WHO}_0 &: <(np\backslash s)/np> \to <n> \to <n/np> \\
\text{WHOM} &: <s/np> \to <n> \to <n>
\end{aligned}
$$

Fig. 8.

Lexicon (lexicalized grammar)

$$
\begin{aligned}
\text{SOME} &:= \lambda x.\, \mathsf{some}\, x \\
\text{EVERY} &:= \lambda x.\, \mathsf{every}\, x \\
\text{LOVES} &:= \lambda xy.\, \mathsf{loves}\, x\, y \\
\text{WHO} &:= \lambda xy.\, \mathsf{who}\, (\lambda z.\, x\, z)\, y \\
\text{WHOM} &:= \lambda xy.\, \mathsf{whom}\, (\lambda z.\, x\, z)\, y \\
\text{LOVES}_0 &:= \lambda xy.\, \mathsf{loves}\, x\, y \\
\text{LOVES}_1 &:= \lambda xy.\, \mathsf{loves}\, y\, x \\
\text{LOVES}_2 &:= \lambda xyz.\, \mathsf{loves}\, (x\, z)\, y \\
\text{SOME}_0 &:= \lambda xy.\, \mathsf{some}\, (x\, y) \\
\text{EVERY}_0 &:= \lambda xy.\, \mathsf{every}\, (x\, y) \\
\text{WHO}_0 &:= \lambda wxy.\, \mathsf{who}\, (\lambda z.\, w\, y\, z)\, x \\
\text{LOVE}_3 &:= \lambda xyz.\, \mathsf{loves}\, (x\, y)\, z \\
\text{MAN} &:= \mathsf{man} \\
\text{WOMAN} &:= \mathsf{woman} \\
\text{LOVES}_4 &:= \lambda xy.\, \mathsf{loves}\, x\, y
\end{aligned}
$$

Fig. 9.

abstract constants togheter with their lexicalized interpretations, they are given in Figure 8 and Figure 9.

5 From string to terms and vice versa

In the previous section, we have defined a lexicalized ACG that generates the λ-terms corresponding to the derivations of a given Lambek grammar. It now remains to interpret these λ-terms as strings. To this end, we first associate to each lambek type, α, a simple type $\overline{\alpha}$, built over a type σ corresponding to a set of strings Σ^*:

$$\begin{aligned}\overline{a} &= \sigma, \text{ for } a \text{ atomic}\\ \overline{\alpha \setminus \beta} &= \overline{\alpha} \rightarrow \overline{\beta}\\ \overline{\beta \,/\, \alpha} &= \overline{\alpha} \rightarrow \overline{\beta}\end{aligned}$$

We then define two families of combinators, $\mathbb{E}_\alpha : \sigma \rightarrow \overline{\alpha}$ and $\mathbb{P}_\alpha : \overline{\alpha} \rightarrow \sigma$, which allow to transform a strings into a λ-terms and vice versa:

$$\begin{aligned}\mathbb{E}_a\, w &= w\\ \mathbb{E}_{\alpha \setminus \beta}\, w &= \lambda x.\, \mathbb{E}_\beta\,((\mathbb{P}_\alpha\, x) + w)\\ \mathbb{E}_{\beta \,/\, \alpha}\, w &= \lambda x.\, \mathbb{E}_\beta\,(w + (\mathbb{P}_\alpha\, x))\end{aligned}$$

$$\begin{aligned}\mathbb{P}_a\, t &= t\\ \mathbb{P}_{\alpha \setminus \beta}\, t &= \mathbb{P}_\beta\,(t\,(\mathbb{E}_\alpha\, \epsilon))\\ \mathbb{P}_{\beta \,/\, \alpha}\, t &= \mathbb{P}_\beta\,(t\,(\mathbb{E}_\alpha\, \epsilon))\end{aligned}$$

We first state and prove two technical lemmas.

Lemma 1. *For every string w and every type α, $\mathbb{P}_\alpha\,(\mathbb{E}_\alpha\, t) = t$.*

Proof. By induction on α. □

Lemma 2. *Let t be a λ-term in normal form such that $(x_i : \alpha_i)_{i \in I} \vdash_{\lambda L} t : \alpha$. Then, for every set of strings $\{w_i \mid i \in I\}$, the following properties hold:*

1. *if t is a variable or an application, then $t[x_i := (\mathbb{E}_{\alpha_i}\, w_i)]_{i \in I} = \mathbb{E}_\alpha\,(\sum_{i \in I} w_i)$;*
2. *if t is an abstraction, then $\mathbb{P}_\alpha\, t[x_i := (\mathbb{E}_{\alpha_i}\, w_i)]_{i \in I} = \sum_{i \in I} w_i$.*

Proof. By induction on t. □

The following lemma is the main property of the operators $\mathbb{E}_\alpha$ and $\mathbb{P}_\alpha$.

Lemma 3. *Let t be a λ-term such that $(x_i : \alpha_i)_{i \in I} \vdash_{\lambda L} t : \alpha$. Then, for every set of strings $\{w_i \mid i \in I\}$, $\mathbb{P}_\alpha\, t[x_i := (\mathbb{E}_{\alpha_i}\, w_i)]_{i \in I} = \sum_{i \in I} w_i$.*

Proof. Let t' be the normal form of t. The results follows from Lemma 2, Lemma 1, and the fact that $\mathbb{P}_\alpha\, t[x_i := (\mathbb{E}_{\alpha_i}\, w_i)]_{i \in I} = \mathbb{P}_\alpha\, t'[x_i := (\mathbb{E}_{\alpha_i}\, w_i)]_{i \in I}$. □

We then immediately obtain the following corollary.

Corollary 1. *Let t be a λ-term such that $(x_i : \alpha_i)_{i \in I} \vdash_{\lambda L} t : a$, where a is an atomic type. Then, for every set of strings $\{w_i \mid i \in I\}$, $t[x_i := (\mathbb{E}_{\alpha_i}\, w_i)]_{i \in I} = \sum_{i \in I} w_i$.* □

Interpretation of the object constants

$$\begin{aligned}
\mathsf{man} &:= \textbf{\textit{man}} \\
\mathsf{woman} &:= \textbf{\textit{woman}} \\
\mathsf{some} &:= \lambda x.\, \textbf{\textit{some}} + x \\
\mathsf{every} &:= \lambda x.\, \textbf{\textit{every}} + x \\
\mathsf{loves} &:= \lambda xy.\, y + \textbf{\textit{loves}} + x \\
\mathsf{who} &:= \lambda xy.\, y + \textbf{\textit{who}} + (x\, \epsilon) \\
\mathsf{whom} &:= \lambda xy.\, y + \textbf{\textit{whom}} + (x\, \epsilon)
\end{aligned}$$

Fig. 10.

6 From LG to ACG

Consider the signature given in Figure 5. It has been used as the object vocabulary of both the ACGs $LDER$ and $LDER_{lex}$. We now use it as the abstract vocabulary of another ACG whose lexicon, which is given in Figure 10, is obtained by applying the operator $\mathbb{E}_\alpha$. Let us call this new ACG GEN_{string}.

The fact that the object vocabulary of $LDER_{lex}$ is, at the same time, the abstract vocabulary of GEN_{string} allows the two ACGs to be composed. Let us call the ACG resulting from this composition CFG_λ. The picture is then the following:

$$\begin{tikzcd}
\bullet\ \text{Abstract Vocabulary (Fig. 4)} \arrow[d, "LDER_{lex}"] \arrow[dd, bend right, "CFG_\lambda"'] \\
\bullet\ \text{Object vocabulary (Fig. 5)} \arrow[d, "GEN_{string}"] \\
\bullet\ \text{Strings}
\end{tikzcd}$$

Now, in order to define an ACG that is strongly equivalent (up to a relabelling) to the original Lambek grammar, we need a last transformation. Consider the ACG CFG_λ, which has been obtained by omposition. It is a second-order ACG whose lexicon is third-order.[1] In fact, it is always possible to transform such an ACG in an object-language-equivalent third-order ACG whose lexicon is second order. Roughly speaking, the construction consists in replacing each atomic abstract type α, whose interpretation is $\alpha := a \to b$, by a type $\alpha_1 \to \alpha_2$ (where α_1 and α_2 are fresh symbols) together with the interpretations $\alpha_1 := a$ and $\alpha_2 := b$. Applying this transformation to CFG_λ, we end up with a grammar whose abstract vocabulary and lexicon are respectively given in Figure 11 and Figure 12. The object language of this ACG corresponds to the language generated by the original Lambek grammar. Its abstract language corresponds to the derivations of the original Lambek grammar (up to a relabelling, i.e., a homomorphism that is sending constants to constants).

[1] It is third-order because the type of the strings is defined to be the second-order type $o \to o$. Consequently, an apparently second-order interpretation such as $<\!n/np\!> := \sigma \to \sigma$ corresponds, in fact, to a third-order interpretation (namely, $<\!n/np\!> := (o \to o) \to o \to o$).

Abstract Vocabulary (final grammar)
$\text{MAN} : n$
$\text{WOMAN} : n$
$\text{SOME} : n \to np$
$\text{SOME}_0 : (np_0 \to n_0) \to np_1 \to np_2$
$\text{EVERY} : n \to np$
$\text{EVERY}_0 : (np_0 \to n_0) \to np_1 \to np_2$
$\text{LOVES} : np \to np \to s$
$\text{LOVES}_0 : np \to np_3 \to s_0$
$\text{LOVES}_1 : np \to np_4 \to s_1$
$\text{LOVES}_2 : (np_1 \to np_2) \to np \to np_4 \to s_1$
$\text{LOVES}_3 : (np_1 \to np_2) \to np_5 \to np_6 \to s_2$
$\text{LOVES}_4 : np_5 \to np_6 \to s_2$
$\text{WHO} : (np_3 \to s_0) \to n \to n$
$\text{WHO}_0 : (np_5 \to np_6 \to s_2) \to n \to np_0 \to n_0$
$\text{WHOM} : (np_4 \to s_1) \to n \to n$

Fig. 11.

Lexicon (final Grammar)
$\text{MAN} := \textbf{\textit{man}}$
$\text{WOMAN} := \textbf{\textit{woman}}$
$\text{SOME} := \lambda x.\, \textbf{\textit{some}} + x$
$\text{SOME}_0 := \lambda xy.\, \textbf{\textit{some}} + (x\, y)$
$\text{EVERY} := \lambda x.\, \textbf{\textit{every}} + x$
$\text{EVERY}_0 := \lambda xy.\, \textbf{\textit{every}} + (x\, y)$
$\text{LOVES} := \lambda xy.\, y + \textbf{\textit{loves}} + x$
$\text{LOVES}_0 := \lambda xy.\, y + \textbf{\textit{loves}} + x$
$\text{LOVES}_1 := \lambda xy.\, x + \textbf{\textit{loves}} + y$
$\text{LOVES}_2 := \lambda xyz.\, y + \textbf{\textit{loves}} + (x\, z)$
$\text{LOVES}_3 := \lambda xyz.\, z + \textbf{\textit{loves}} + (x\, y)$
$\text{LOVES}_4 := \lambda xy.\, y + \textbf{\textit{loves}} + x$
$\text{WHO} := \lambda xy.\, y + \textbf{\textit{who}} + (x\, \epsilon)$
$\text{WHO}_0 := \lambda wxy.\, x + \textbf{\textit{who}} + (w\, y\, \epsilon)$
$\text{WHOM} := \lambda xy.\, y + \textbf{\textit{whom}} + (x\, \epsilon)$

Fig. 12.

7 Conclusions and future work

We have demonstrated, on an example, how to represent a given Lambek grammar as an ACG. The generality of our method is somehow ensured by the several mathematical results on which it is based: weak equivalence between Lambek grammars and context-free grammars (Pentus [11, 12]), representing context-free grammars as ACGs (Pogodalla and de Groote [3]), lexicalization of second-order ACGs (Kanazawa and Yoshinaka [7, 8]), preservation of the derivations (Kanazawa and Salvati [6]). In practice, however, it will not always be the case that the resulting grammar will be strongly equivalent to the original Lambek grammar. This potential problem is due to Pentus' construction which produces highly redundant grammars exhibiting all the spurious ambiguities related to the associativity of the Lambek calculus.

In this paper, we circumvented the spurious ambiguity problem by implicitely using a particular instantiation of Pentus' construction. This more specific construction seems to be less general than Pentus'. Consequently, we do not know whether it can be put at work in every case. Investigating this question will be the subject of future work.

References

1. H.P. Barendregt. *The lambda calculus, its syntax and semantics*. North-Holland, revised edition, 1984.
2. Ph. de Groote. Towards abstract categorial grammars. In *Association for Computational Linguistics, 39th Annual Meeting and 10th Conference of the European Chapter, Proceedings of the Conference*, pages 148–155, 2001.
3. Ph. de Groote and S. Pogodalla. On the expressive power of abstract categorial grammars: Representing context-free formalisms. *Journal of Logic, Language and Information*, 13(4):421–438, 2004.
4. J.-Y. Girard. Linear logic. *Theoretical Computer Science*, 50:1–102, 1987.
5. M. Kanazawa. Syntactic features for regular constraints and an approximation of directional slashes in abstract categorial grammars. In Y. Kubota and R. Levine, editors, *Proceedings for ESSLLI 2015 Workshop 'Empirical Advances in Categorial Grammars' (CG 2015)*, pages 34–70, 2015.
6. M. Kanazawa and S. Salvati. The string-meaning relations definable by lambek grammars and context-free grammars. In G. Morrill and M.-J. Nederhof, editors, *Formal Grammar - 17th and 18th International Conferences, FG 2012, Opole, Poland, August 2012, Revised Selected Papers, FG 2013, Düsseldorf, Germany, August 2013, Proceedings*, volume 8036 of *Lecture Notes in Computer Science*, pages 191–208. Springer, 2013.
7. M. Kanazawa and R. Yoshinaka. The complexity and generative capacity of lexicalized abstract categorial grammars. In Ph. Blache, E. Stabler, J. Busquets, and R. Moot, editors, *Logical Aspects of Computational Linguistics, LACL 2005*, volume 3492 of *Lecture Notes in Artificial Intelligence*, pages 330–346. Springer Verlag, 2005.
8. M. Kanazawa and R. Yoshinaka. Lexicalization of second-order acgs. Technical Report NII-2005-012E, National Institute of Informatics, Tokyo, 2005.
9. J. Lambek. The mathematics of sentence structure. *Amer. Math. Monthly*, 65:154–170, 1958.
10. J. Lambek. On the calculus of syntactic types. In *Studies of Language and its Mathematical Aspects*, pages 166–178, Providence, 1961. Proc. of the 12th Symp. Appl. Math..
11. M. Pentus. Lambek grammars are context free. In *Proceedings of the eigth annual IEEE symposium on logic in computer science*, pages 429–433, 1993.
12. M. Pentus. Product-free Lambek calculus and context-free grammars. *Journal of Symbolic Logic*, 62(2):648–660, 1997.
13. S. Pogodalla and F. Pompigne. Controlling extraction in abstract categorial grammars. In Ph. de Groote and M.-J. Nederhof, editors, *Formal Grammar - 15th and 16th International Conferences, FG 2010, Copenhagen, Denmark, August 2010, FG 2011, Ljubljana, Slovenia, August 2011, Revised Selected Papers*, volume 7395 of *Lecture Notes in Computer Science*, pages 162–177. Springer, 2012.
14. C. Retoré and S. Salvati. A faithful representation of non-associative lambek grammars in abstract categorial grammars. *Journal of Logic, Language and Information*, 19(2):185–200, 2010.

Rich Situated Propositions: the 'right' objects for the content of propositional attitudes*

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Abstract We present a new account of linguistic meaning, *Rich Situated Semantics* [RSS], on which the meaning of sentential utterances is semantically *rich* and informationally *situated*. In virtue of its situatedness, the meaning of an utterance varies with the informational situation of the cognitive agent interpreting the utterance. In virtue of its richness, the meaning of an utterance contains information beyond the utterance's lexically encoded information. The agent-sensitivity of meaning solves a number of problems in semantics and the philosophy of language (cf. [15, 21, 25]). In particular, since RSS varies the granularity of utterance meanings with the interpreting agent's information state, it solves the problem of finding suitably fine- or coarse-grained objects for the content of propositional attitudes. In virtue of this variation, a layman will reason with more propositions than an expert.

Keywords: Information-sensitivity · interpreter-dependence · propositional attitude contents · richness of NL meaning · situated semantics

1 Introduction

The same utterance of a (non-indexical) sentence has a different meaning to different interpreting agents. This is due to the fact that different agents have different information about the sentence's subject matter, which is used in the utterance's agent-specific interpretation: Depending on the agent's background knowledge, the utterance of (1) in a particular context will be interpreted as an informationally rich proposition (e.g. as a proposition which contains the information that *the inhabitant of Gobbler's Knob* is a groundhog/that Punxsutawney Phil *is a member of the largest existing marmot species*) or as an informationally poorer proposition which does not contain this additional information.

(1) Punxsutawney Phil is a groundhog.

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Most formal theories of linguistic meaning (e.g. [8, 26, 27, 32]) restrict the meaning of sentential utterances to the utterances' lexical information (for (1): to the information that the referent of the name *Punxsutawney Phil* is a groundhog) and delegate all other available information about the utterance's subject matter to areas like pragmatics or psychology. However, Moltmann [25] has observed that they thus seriously underspecify the content of propositional attitudes. As a result, these theories cannot explain why an inference is valid for some agents, but invalid for others. The disregard of agents' information about the utterance's subject matter further conflicts with the (environ-)mental embeddedness of linguistic understanding that has been observed in Situated Cognition (cf. [6, 36]).

This paper solves the above problem by extending the meaning of sentential utterances to other relevant information that is available to the interpreter of the utterance at the time of the interpretation. Below, we first sketch our new account of linguistic meaning, called *Rich Situated Semantics* (in Sect. 2.2). We then present the rigid granularity problem for the content of propositional attitudes (in Sect. 3) and show how Rich Situated Semantics solves this problem (in Sect. 4). Section 5 identifies several other intensional phenomena that lend themselves to a rich situated semantic treatment. The paper closes with a discussion of the RSS-modelling of assertion and shared belief (in Sect. 6).

2 Rich Situated Semantics

Rich Situated Semantics [hereafter, RSS] (cf. [20, 21]) is a novel account of linguistic meaning on which the content of (utterances[1] of) declarative sentences is semantically *rich* and informationally *situated*. In virtue of its situatedness, the meaning of a sentence varies with the informational situation of the cognitive agent interpreting the sentence. In virtue of its richness, the meaning of a sentence contains information beyond the sentence's lexically encoded information.

Below, we first illustrate the richness and situatedness of sentence meanings and identify a number of theories from linguistics, philosophy, cognitive and computer science that suggest this richness and/or situatedness (in Sect. 2.1). We then specify the RSS-interpretation of sentences (in Sect. 2.2) and identify some notable consequences of this interpretation with respect to linguistic entailment and equivalence (in Sect. 2.3). The section closes with a definition of truth for Rich Situated Semantics (in Sect. 2.4).

2.1 Illustration and inspiration for RSS

To familiarize the reader with the core idea of RSS, we illustrate richness and situatedness of linguistic meaning through an example:

[1] We hereafter sometimes use the expression 'meaning of a sentence', instead of 'meaning of *an utterance of* a sentence'. This is merely a terminological shortcut. The reader is asked to keep in mind that sentences are uttered by a speaker (with certain background information) in a spatiotemporal and communicative situation and are directed at an addressee (with a certain, likely different, background information).

Consider the interpretation of (1) by Alf, Bea, and Chris. Assume that, *re* Punxsutawney Phil (here: 'Phil'), these agents have the following information:

Alf:	Phil lives in Gobbler's Knob.
Bea:	Phil is celebrated each February 2nd.
Chris:	Phil lives in Gobbler's Knob; Phil is celebrated each February 2nd.

Since Rich Situated Semantics assumes the inclusion-in-linguistic-meaning of the interpreter's information about the sentence's subject matter (here: Phil), the rich interpretation of (1) at Alf's informational situation contains the information that Phil is a groundhog who lives in Gobbler's Knob. The rich interpretation of (1) at Bea's and Chris' situation contain the information that Phil is a groundhog who is celebrated each February 2nd (Bea) and that Phil is a groundhog who lives in Gobbler's Knob and is celebrated each February 2nd (Chris). Thus, (1) is interpreted by Alf as (1.i), by Bea as (1.ii), and by Chris as (1.iii):

(1) i. Phil is a groundhog who/and lives in Gobbler's Knob.
[*alternatively:* Phil, who lives in Gobbler's Knob, is a groundhog.]
ii. Phil is a groundhog who is celebrated each February 2nd.
iii. Phil is a groundhog who lives in Gobbler's Knob and is celebrated each February 2nd.

The non-identity of the rich interpretations of (1) at Alf's, Bea's, and Chris' informational situation witnesses the *situatedness* of linguistic meaning in RSS. The greater informativeness of the interpretation of (1) at any of the above situations in comparison to the sentence's traditional, possible worlds-interpretation (which only contains the sentence's lexical information) witnesses the *richness* of linguistic meaning in RSS. In particular, the rich interpretation of (1) at Alf's informational situation contains the information that Phil lives in Gobbler's Knob, which is not contained in the sentence's the lexical information.

The situatedness of linguistic meaning is inspired by work in Situated Cognition and dynamic semantics ([10,44]). Dynamic semantics interprets sentences as *state transitions*, i.e. as functions from information states to the result of updating these states with the sentence's lexical information. Rich linguistic meanings are found in Fregean theories of belief content ([5,9]), in semantic descriptivism and generalized quantifier theory ([3,9,41]), and in frame semantics ([2,22]). The former assume that any adequate representation of belief contents involves the modes of presentation of the individuals the beliefs are about. The latter assume that proper names be interpreted analogously to definite NPs, i.e. as sets of properties of individuals. Frame semantics represents utterance meanings by rich recursive feature structures that account for the content of mental concepts.

2.2 The RSS-interpretation of sentential utterances

To capture the *situatedness* of linguistic meaning, RSS interprets sentences as *functions from interpreters' informational situations* to the sentences' meanings *at these situations* (i.e. to the sentences' <u>*situated*</u> *meanings*). These functions are objects of type $s \to \alpha$ (abbr. $s\alpha$), where α is the type for the sentence meanings.

The *richness* of situated sentence meanings is captured via (characteristic functions of) partial sets of situations (s.t. $\alpha := st$).[2] Such sets are familiar from the representation of sentence meanings in generalizations of possible world semantics, including some versions of situation semantics ([16, 27]). However, the set of situations that serves as the meaning of a sentence in RSS is generally smaller than the set of situations that serves as the meaning of this sentence in situational generalizations of possible world semantics. This is due to the fact that – in addition to being restricted to situations in which the sentence is true – the RSS-set is further restricted to situations which contain the interpreting agent's information about the sentence's subject matter. For example, while (1)-as-received-by-Alf[3] is interpreted as (2) in situational possible world semantics, it is interpreted as (3) in RSS. Below, i is a variable over situations, as reflected in the superscript s. The formulas $groundhog(phil)(i)$ and $livesinGK(phil)(i)$ assert that Phil is a groundhog in i and that Phil lives in Gobbler's Knob in i:

$$\lambda i^s[groundhog(phil)(i)] \tag{2}$$

$$\lambda i^s[groundhog(phil)(i) \wedge livesinGK(phil)(i)] \tag{3}$$

To capture the informational imperfection of cognitive agents, we identify situations with *partial* (i.e. informationally incomplete) spatio-temporal parts of worlds[4] in which the parts' individual inhabitants may fail to have some of the properties which they have at the relevant world-part. Situations in rich situated semantics are thus "partial specifications of some of the entities in the universe with [their] properties" [24, p. 614]. They are obtained from worlds by reducing the information about the world's inhabitants to the information available to the agent at the given point in time. As a result, situations are agent- and time-specific: the same agent may be in different situations at different points in time.

We assume that situated sentence meanings are *partially* (or *selectively*) *rich*, i.e. that they contain – next to the sentence's lexical information – all and *only* information *about the sentence's subject matter* that is available to the interpreter of the sentence at the time of the interpretation. As a result, RSS interprets any sentence p as a function from informational situations i to sets of situations whose members contain the lexical information of p together with all information from i which regards some individual about which p carries information. We hereafter call such sentences *aboutness-relevant*. Sentences that carry information about the same individuals are called *aboutness-identical*. The RSS-interpretation of p is given in (4).

In (4), x is a variable over individuals. The formula $\varphi^t \twoheadrightarrow \psi^t$ asserts that ψ contains the information of φ (i.e. that ψ is less partial/better defined than φ), s.t. ψ is true if φ is true and is false if φ is false (cf. [27, p. 50, 47]). $\varphi \twoheadrightarrow \psi$ is

[2] One can increase the granularity of situated sentence meanings by analyzing them instead as semantically primitive (i.e. non-analyzable) propositions (cf. [7, 28, 32, 43]). The development of hyperfine-grained RSS is left for another occasion.

[3] In fact, the interpreter of the utterance is irrelevant in possible world semantics.

[4] The inclusion of *impossible* worlds or situations (cf. [12, 34, 45]) captures the possibility of agents' misinformedness or false belief. We here neglect impossible worlds.

defined as $((\varphi \wedge \psi) \vee ((\varphi \vee \psi) \wedge *)) = \varphi$, where $*$ is the neither-true-nor-false formula. The introduction of $\twoheadrightarrow$ is made necessary by our association of t with the set of truth-*combinations*, the resulting existence of *two* orderings on the type-t domain (i.e. a truth- and an approximation-ordering), and the reference of the material conditional to the 'wrong' ordering for our purposes (i.e. to the truth-ordering; on this ordering, ψ is true if φ is true, *but φ is false if ψ is false*).

The formula $abt\,(x^e)(q^{st})$ asserts that q carries information about the referent of x. The behavior of abt is governed by a variant of the axioms from [29, p. 129] (cf. [19, p. 120–121]). These axioms include the aboutness-relevance (with respect to an individual) of atomic formulas that contain the designating constant for the individual as a constituent, the closure of aboutness-relevant propositions under non-contradictory conjunction,[5] the closure of aboutness-relevant propositions under disjunction (given that both disjuncts contain information about the subject matter), and the robustness of aboutness under semantic equivalence.

$$\lambda i^s \lambda j^s [p^{st}(j) \wedge (\forall q^{st}(q(i) \wedge \exists x^e[abt\,(x)(q) \wedge abt\,(x)(p)]) \twoheadrightarrow q(j))] \qquad (4)$$

To better understand the interpretation of sentences in Rich Situated Semantics, consider the rich meaning of (1) at Alf's informational situation, σ_{alf} (in (5)).

$$\lambda i\,[groundhog\,(phil)(i) \wedge (\forall q\,(q\,(\sigma_{alf}) \wedge \exists x^e[abt\,(x)(q) \wedge abt\,(x)(p)]) \twoheadrightarrow q(i))] \quad (5)$$
$$\equiv \lambda i\,[groundhog\,(phil)(i) \wedge (\forall q\,(q\,(\sigma_{alf}) \wedge abt\,(phil)(q)) \twoheadrightarrow q(i))]$$

We assume for simplicity that σ_{alf} only contains the information that Phil lives in Gobbler's Knob (cf. Sect. 2.1) and that Bea has red hair. Since only the first informational item regards Phil, (1) will be RSS-interpreted at σ_{alf} as (3).

We next identify a concrete candidate for the set of situations described by (3): Assume a universe consisting of four situations, σ_{alf}, σ_1, σ_2, and σ_3 and two individuals: Phil (abbreviated p) and Bea (abbreviated b). We assume that Phil lives in Gobbler's Knob (Kp) in σ_{alf}, σ_1, and σ_2, that Bea has red hair (Rb) in σ_{alf} and σ_2, and that Phil is a groundhog (Gp) in σ_1, σ_2, and σ_3 (cf. Fig. 1).

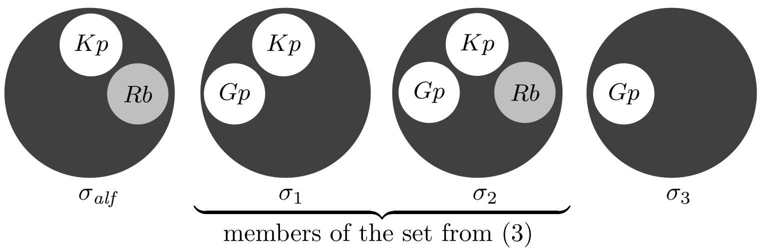

Figure 1. The meaning-at-σ_{alf} of (1).

[5] To avoid the inclusion of information that does not regard the subject matter, we demand (*contra* Perry) that *both* conjuncts be aboutness-relevant. This also avoids the problem of obtaining aboutness-'relevant' conjunctions by combining an aboutness-irrelevant proposition with a trivially aboutness-relevant *verum*, or with *falsum*.

Then, since the lexical information of (1) (i.e. Phil is a groundhog) and the Phil-relevant information from σ_{alf} (i.e. Phil lives is Gobbler's Knob) are included only in σ_1 and σ_2 (and in none of the other situations), the meaning of (1) at Alf's informational situation is represented by the set $\{\sigma_1, \sigma_2\}$ (underbraced in Fig. 1).

2.3 Consequences of RSS

The RSS-interpretation of sentences has a number of important consequences for the individuation of situated sentence meanings. In particular, since RSS updates the available information about a sentence's subject matter with the sentence's lexical information, it identifies the meanings of sentences at situations that differ only with respect to the inclusion of the sentence's lexical information. Consider the interpretation of (1) at Len's informational situation in which Phil is a groundhog and lives in Gobbler's Knob. (We assume that this situation does not contain any other information about Phil, s.t., as regards Phil, it is identical with σ_1): At this situation, (1) has the same meaning (i.e. $\{\sigma_1, \sigma_2\}$) as at σ_{alf}.

Note that, although (1) has the same rich meaning at Alf's and at Len's informational situation, its utterance has a different *effect* on Alf's than on Len's situation: While (1)-as-received-by-Alf updates Alf's information about Phil with the information that Phil is a groundhog (s.t. Alf's information is extended to the information from σ_2), it leaves Len's informational situation unchanged. The updating effect of (1) on Alf's Phil-specific information is witnessed by the fact that (the information associated with) the meaning of (1) at Alf's informational situation (i.e. $\{\sigma_1, \sigma_2\}$) is properly contained in (the information associated with) the meaning of (6) at Alf's situation (i.e. $\{\sigma_{alf}, \sigma_1, \sigma_2\}$).

(6) Punxsutawney Phil lives in Gobbler's Knob.

As a result of its interpretations' informational richness, RSS further identifies the meanings of different aboutness-identical sentences at situations which contain the sentences' lexical information. Consider the interpretation of (1) and (6) at σ_1: Since this situation already contains the lexical information of (1) and (6), these sentences have the same meaning (i.e. $\{\sigma_1, \sigma_2\}$) at this situation.

We will see in Section 4 that the identification-at-a-situation of the meanings of different aboutness-identical sentences solves the problem of finding suitably fine- or coarse-grained objects for the content of propositional attitudes. This problem is described in Section 3. However, to prepare our discussion of assertion in RSS (cf. Sect. 6), we first give a truth-evaluation procedure for RSS terms.

2.4 Truth-evaluation in RSS

We have described situated sentence meanings as the result of updating the available information about the sentence's subject matter with the sentence's lexical information. As a result of this description, situated sentence meanings in RSS contain much more information than sentence meanings in situational generalizations of possible world semantics. However, much of this information is irrelevant for the sentences' evaluation. For example, it does (or should) not matter for the truth of (1) whether Phil lives in Gobbler's Knob. Since *non*-situated sentence meanings (type ($s(st)$)) do not have the 'right' type for truth-evaluable objects (they do not yield a truth-value when applied to a world), we need to provide a custom truth-evaluation procedure for sentences in RSS:

To evaluate the truth of a sentence in Rich Situated Semantics, we check whether the world of evaluation w is a member of the union of the sentence's rich interpretations at all informational situations. The resulting truth-definition is given below. In this definition, we use denotation brackets, $[\![\cdot]\!]$, as a notational device for non-situated RSS-interpretations of sentences:

Definition 1 (Truth at a world). *In Rich Situated Semantics, a sentence p is* true *at a world w if $w \in \bigcup_{\sigma^s} [\![p]\!](\sigma)$, where $[\![p]\!](\sigma)$ is the rich interpretation of p at the situation σ.*

By taking the union of the rich interpretations, $[\![p]\!](\sigma)$, of p for each situation σ, we obtain the set of situations in which p is true. This set is a situational generalization of the classical Lewisian proposition denoted by p. The rationale behind the above strategy is as follows: Since we assume the existence of a situation for every consistent combination of information (including the 'empty' combination; cf. [31, 32]), the members of the above union will never share *more* than the lexical information of p (and of p's presuppositions). Since we identify the result of updating a situation's information via incompatible information with the empty set of situations[6], the members of this union will never share *less* than the lexical information of p. In particular, situations which contain the information that not-p will not contribute their information to the above union.

We next turn to the rigid granularity problem for the content of propositional attitudes. This problem lies in the fact that most theories of finely-grained meaning assume a single, uniform level of granularity for belief contents. As a result of this assumption, these theories cannot explain why an inference is valid for some epistemic agents (given their background knowledge), but invalid for others.

3 The Rigid Granularity Problem

To avoid predicting agents' logical omniscience, many theories of formal semantics (e.g. [8, 27, 32, 43]) assume hyperfine-grained sentence meanings that have stricter identity-conditions than sets of possible worlds. The level of granularity of these meanings is chosen in accordance with speakers' intuitions about synonymy (cf. [32, p. 553]). Since most speakers judge the meanings of many intensionally equivalent sentences (e.g. of (1) and (7)) to be non-identical, hyperfine-grained semantics distinguish the meanings of these sentences.[7]

[6] This is due to the fact that the available information about the sentence's subject matter at these situations will include the complement of the sentence's lexical information. Since we have excluded impossible situations from our considerations (cf. fn. 4), no situations contain both an item of information and its complement.

[7] The identification of the meanings of these sentences in possible world semantics is due to the fact that, in the actual world at the current time (cf. the adjective *existing* in (7)), groundhogs *are* the largest marmot species.

(7) Punxsutawney Phil is a member of the largest existing marmot species.

The success of these semantics is hampered by the fact that the above identity- (or *non*-identity-)judgements are not shared by all speakers for all sentence-pairs. This is due to the fact that speakers' judgements about sentential synonymy are influenced by their background information about the sentences' subject matter. Depending on their informational situation, speakers will thus identify or distinguish the meanings of the same sentences. Consider the case of (1) and (7): Since she knows that groundhogs are the largest existing marmot species, a groundhog expert (e.g. Eve in (8)) will identify the meanings of (1) and (7). Since he is unaware of this fact, a groundhog layman (e.g. Len in (9)) will treat the meanings of (1) and (7) as distinct. Any reasoner who is familiar with Eve and Len's level of groundhog expertise (s.t. (s)he knows that Eve assigns *the same* meaning to (1) and (3), while Len assigns different meanings to these sentences), will conclude (8b) from (8a), but not (9b) from (9a). Since hyperfine-grained semantics assume the same level of granularity of content for *all* agents interpreting a sentence, they cannot distinguish between the validity of these two inferences.

(8) a. Eve knows that Phil is a groundhog. **T**

b. Eve knows that Phil is a member of the largest existing marmot species. **T**

(9) a. Len knows that Phil is a groundhog. **T**

b. Len knows that Phil is a member of the largest existing marmot species. **F**

In particular, since hyperfine-grained semantics dinstinguish the meanings of (1) and (7), they will counterintuitively predict the invalidity of (8). Since traditional (coarse-grained) possible world semantics identifies the meanings of (1) and (7), it will counterintuitively predict the validity of (9).

4 Rich Situated Attitudes

Rich Situated Semantics solves the above problem by varying the granularity of meanings with the informational situation of the sentence's interpreter. This is possible since RSS identifies the meanings of different aboutness-identical sentences at situations which contain the sentences' lexical information (cf. Sect. 2.3).

4.1 Solving the rigid granularity problem

Since Eve's informational situation, σ_{eve}, contains the lexical information of (1), (6) and (7), RSS *identifies* the meanings of (1) and (7) at this situation (i.e. (11)). Since Len's situation does *not* contain the lexical information of (7) (s.t. this sentence is interpreted as an update on Len's information about Phil), RSS *distinguishes* the meanings of (1) (i.e. (3)) and (7) (i.e. (11)) at Len's informational situation. With respect to the relevant subject domain, a layman will thus reason with more sentence meanings than an expert.

$$\lambda i^s [groundhog(phil)(i) \wedge largestmarmot(phil)(i)] \tag{10}$$

$$\lambda i^s ([groundhog(phil)(i) \wedge largestmarmot(phil)(i)] \wedge livesinGK(phil)(i)) \tag{11}$$

The variation of sentences' semantic granularity with the epistemic agent's informational situation explains the intuitive validity of the inference from (8) and the intuitive invalidity of the inference from (9). However, this explanation presupposes the reasoner's familiarity with Eve and Len's level of expertise about Phil (cf. Sect. 3). Reasoners who are *not* familiar with the two agents' level of subject expertise (s.t. they are, in particular, unaware of Eve's coarse-grained interpretation of (1) and (7)) will not be able to make the inference from (8).[8]

To capture the dependence of (8) on the reasoner's awareness of the agent's subject expertise, we stipulate the following: when they occur in the complement of epistemic verbs like *know*, sentences are interpreted as sets of situations whose members only encode the agent's information about the sentence's subject matter *of whose availability to the agent the reasoner is aware*. For the occurrence of (1) from (8a), this set is specified in (12). There, r is a variable for the reasoner.

$$\lambda i\, [groundhog(phil)(i) \wedge (\forall q\, [\boldsymbol{aware(q(\sigma_{eve})(r))} \wedge abt(phil)(q)] \twoheadrightarrow q(i))] \tag{12}$$

We illustrate the reasoner-dependence of epistemic inferences by means of an example: Compare the interpretation of (8a) and (8b) by two reasoners, Dan and Fred, who have different degrees of familiarity with Eve's knowledge about Phil. In particular, Dan knows that, in Eve's situation, Phil is a groundhog, belongs to the largest existing marmot species, and lives in Gobbler's Knob. Fred only knows that Phil is a groundhog in this situation. The complements of the occurrences of *know* from (8a) and (8b) are then interpreted as (11) by Dan and as (2) (cf. (8a)) and (10) (cf. (8b)) by Fred. Since only Dan is, thus, aware of Eve's semantic identification of (1) and (7), only he will be able to make the inference from (8).

The above suggests the distinction between two types of validity: validity relative to an agent's informational situation (here called *situational validity*) and traditional validity (called *validity simpliciter*). The two types are defined below:

Definition 2 (Situational validity). *An inference is* valid relative to the informational situation σ *of some specific reasoner*[9] (*or is* valid-at-σ) *if the interpretation-at-σ of the inference's premise(s) entails the interpretation-at-σ of the inference's conclusion.*

Definition 3 (Validity simpliciter). *An inference is* valid simpliciter *if, at all informational situations σ, the interpretation-at-σ of the inference's premise(s) entails the interpretation-at-σ of the inference's conclusion.*

The condition from Definition 3 corresponds to requiring the entailment of the traditional, possible worlds-interpretation of the conclusion by the traditional, possible worlds-interpretation of the premise(s).

[8] The ability of (8b) to extend the reasoner's knowledge depends on this unfamiliarity.

[9] We assume as above that the reasoner's information contains information about the epistemic agent's level of subject expertise.

The different types of validity are illustrated by (8) and (13):

(13) a. Eve knows that Phil is a groundhog and lives in Gobbler's Knob. **T**
b. Eve knows that Phil is a groundhog. **T**

Since the situated interpretation of (8a) does not entail the situated interpretation of (8b) at some situations (e.g. σ_1), (8) is not valid *simpliciter*.

4.2 Consequences of situating attitudes

As a result of its rich situated interpretation of epistemic complements, RSS also predicts the validity of inferences between epistemic reports like (14), whose complements are not intensionally equivalent.

(14) a. Eve knows that Phil is a groundhog. **T**
b. Eve knows that Phil lives in Gobbler's Knob. **T**

The validity of these inferences may be justified by the reasoner's familiarity with the epistemic agent's level of subject expertise: A reasoner (e.g. Dan) who is aware of the agent's degree of informedness about the interpreted sentence's subject matter will follow the agent in identifying his/her situated interpretation of the complements of *know* from (14a) and (14b). However, intuitively, inferences like (14) have a different kind of validity from inferences like (8).

To block inferences of the form of (14), we modify the meaning of the epistemically embedded occurrence of (1) from (12) to the set of situations whose members only encode *the information contained in the complement's lexical information* of whose availability to Eve the reasoner is aware. This modification restricts the set of validly substitutable complements of epistemic verbs like *know* to CPs that are traditionally entailed[10] by the CP. For the complement of *know* from (8a), this is achieved by (15). There, w ranges over possible worlds.

$$\lambda i\,(\forall q\,[(\boldsymbol{\forall w[groundhog(phil)(w) \to q(w)]} \wedge aware(q(\sigma_{eve})(r))) \wedge abt(phil)(q)] \twoheadrightarrow q(i)) \tag{15}$$

Consider Dan's interpretation of the complements from (14a) and (14b). Following (15), these complements are interpreted as (10) (cf. (14a)) and (16) (cf. (14b)). Since the set of situations denoted by (10) is not contained in the set of situations denoted by (16), the inference from (14) is no longer valid on this interpretation.

$$\lambda i^s[livesinGK(phil)(i)] \tag{16}$$

The interpretation from (15) is in line with the understanding of propositional knowledge as focusing on *a particular item* of the agent's subject-relevant information (at a given point in time), rather than as surveying all of this information (at this point in time). It differs from most attitude treatments by extending propositional knowledge to the *union* of the sentence's lexical information and the available aboutness-relevant information of its traditional entailments.

Our previous considerations may have made it seem as if our interpretation of epistemic complements was only an *ad hoc* move to prevent counterintuitive

[10] Entailment is here defined in terms of (subset) inclusion of sets of possible worlds.

inferences of the form of (14). This is not the case. To the contrary: Since different verbs have differently strict requirements on the substitution of their complements (with some verbs like *remember* allowing the substitution of other than the traditionally entailed complements, see below), only RSS enables a modular account of complement restriction.

Consider the substitution properties of the complements of the verbs *say verbatim*, *know*, and *remember*: While *know* allows the substitution of its complement by sentences with the same subject matter to which the complement is traditionally equivalent (cf. the intuitive support for (8)), *say verbatim* does *not* allow such a substitution (cf. the intuitive support against (17), below). In contrast to the class of 'substitutable' complements of the verb *know*, the class of substitutable complements of the verb *remember* extends *beyond* the complement's traditional equivalents. The substitution-generality of the complement of *remember* is witnessed by the intuitive support for the inference from (18).[11] It is reflected in the neutrality of *remember* between taking a CP *or an NP* as its complement (cf. [13, 14, 37]) and by the semantic acceptability of the result of replacing the CP complement of *remember* by the CP's subject NP. This acceptability is illustrated in (19).[12]

(17) a. Eve said verbatim that Phil was a groundhog. **T**
———
b. Eve said verbatim that Phil was a woodchuck. **F**

(18) a. Dan remembered that Phil was nibbling at a dandelion. **T**
———
b. Dan remembered that Phil was endearing. **T**

(19) a. Dan remembered $[_{\text{CP}}$that Phil was endearing$]$.
b. Dan remembered $[_{\text{NP}}$Phil$]$.

5 Other Applications of RSS

We have shown above that Rich Situated Semantics solves the rigid granularity problem for the content of propositional attitudes. Our presentation of RSS suggests that this semantics can also be used to explain several other intensional phenomena. In particular, RSS helps solve some familiar problems of intensionality that have recently resurfaced in the philosophy of language. These include the cognitive accessibility problem for propositions (cf. [15, 25, 42]), the problem of rational illogical belief (cf. [4, 38, 40]), and the substitution problem for the objects and contents of propositional attitudes (cf. [1, 23, 25, 33]). Respectively, they regard the difficulty of most mainstream theories of semantic content to explain how communicative agents can grasp abstract propositions, how rational

[11] This inference assumes that the complements of the two occurrences of *remember* from (18) describe the same remembered situation. The intuitive validity of this type of inference is discussed in detail in [21, Sect. 4, 5].

[12] This substitution is not, in general, acceptable for other NP/CP-neutral verbs like *know*, which (assuming the same lexical entry) only accept *abstract* NP complements.

agents can jointly believe superficially contradictory propositions,[13] and how the *contents* of propositional attitudes (as denoted by the CP complements of epistemic verbs) differ from the *objects* of these attitudes (as denoted by the complements' NP nominalizations of the form *the proposition that* ___).

RSS solves these problems by incorporating the interpreting agents' information about the sentences' subject matter into the meaning of these sentences. This information corresponds to the agents' *mode of presentation* of the subject matter (cf. [5,9,39]). In RSS, an object's mode of presentation is thus represented by the set of situations (type *st*) in which the object has the properties that the agent associates with it.[14] Since rich situated sentence meanings depend on the information of the sentence's interpreting agent, they are cognitively accessible. The ability of agents to interpret different occurrences of the same NP w.r.t. their informational situations at different times further explains the possibility of rational illogical belief. The non-substitutability of CPs by their NP nominalizations in many contexts is explained by the situated (rich) interpretation of embedded CPs and the non-situated (poor) interpretation of their nominalizations.

The RSS-solution to the substitution problem is presented in [21, Sect. 7.3]. The detailed description of a rich situated solution to the remaining problems will be given in a sequel to this paper.

6 Assertion and Shared Belief

Our previous discussion has focused on applications of RSS that exploit the availability of *rich situated* sentence meanings. However, there are a number of linguistic phenomena whose explanation makes recourse to informationally poor(er) and/or non-situated meanings. These include the use of sentences to state facts (cf. Sect. 2.4), to update the receiver's current informational situation (or the conversational common ground), to give reasons (for actions and for facts), and to express shared belief. These uses are illustrated in (20) to (23):

(20) Eve asserted that Phil belonged to the largest existing marmot species.

(21) Eve taught Len that Phil belonged to the largest existing marmot species.

(22) Since Phil is a member of the largest existing marmot species, he weighs more than eight pounds.

(23) Len believes some of the things Eve believes, including the proposition that Phil is a groundhog.

In particular, Eve may utter (7) to communicate the fact that Phil belongs to the largest existing marmot species (rather than some other fact she knows about Phil) (cf. (20)) and to update Len's information about groundhogs by this fact (rather than by some other fact about Phil that is new to Len) (cf. (21)). Similarly, she may utter (7) to give a reason for Phil's high weight (cf. (22)). Many

[13] These include Pierre's simultaneous belief that London is pretty and that London is not pretty (cf. [18])

[14] This contrasts with the standard formal semantic representation of modes of presentation as sets of the object's properties (type $(et)t$) (cf. [3,17,26]).

other sentences which receive an identical interpretation at her informational situation (e.g. (6)) would not serve this purpose. An analogous observation holds for the belief report from (23), which does not warrant the substitution of (1) by (7).

The above examples illustrate the need to distinguish a sentence's lexical information from its rich situated interpretations. Remarkably, this distinction is readily available in Rich Situated Semantics: Section 2.4 has already identified a strategy for obtaining the lexical information of richly interpreted sentences. This strategy is supported by the fact that non-situated sentence meanings only contain the sentences' lexical information. It is complemented by the possibility of analyzing informational situations as situations that represent the discourse participants' common ground and by the resulting possibility of replacing interpreters' situating informational situations by common ground-representing situations. Such a replacement will be relevant in the explanation of justification and shared belief. The detailed treatment of the above phenomena is left as a project for future work.

References

[1] K. Bach, *Do belief reports report beliefs?*, Pac. Phil. Quarterly **78** (1997), 215–241.

[2] L.W. Barsalou, *Frames, concepts, and conceptual fields*, Frames, Fields, and Contrasts: new essays in semantic and lexical organization (A. Lehrer and E. F. Kittay, eds.), Erlbaum Associate, 1992.

[3] J. Barwise and R. Cooper, *Generalized quantifiers and natural language*, Linguistics and Philosophy **4/2** (1981), 159–219.

[4] D. Braun, *Illogical, but rational*, Noûs **40/2** (2006), 376–379.

[5] D. Chalmers, *The representational character of experience*, The Future for Philosophy (B. Leiter, ed.), Oxford UP, 2004.

[6] W.J. Clancey, *Situated Cognition: on human knowledge and computer representations*, Cambridge UP, 1997.

[7] C. Fox, S. Lappin, and C. Pollard, *A higher-order fine-grained logic for intensional semantics*, Proc. of the 7. International Symposium on Logic and Language, 2002.

[8] C. Fox and S. Lappin, *Foundations of Intensional Semantics*, Blackwell, 2005.

[9] G. Frege, *Über Sinn und Bedeutung*, Gottlob Frege: kleine Schriften, Edition Classic Verlag Dr. Müller, 2006, pp. 143–162.

[10] J. Groenendijk and M. Stokhof, *Dynamic predicate logic*, Linguistics and Philosophy **14** (1991), 39–100.

[11] ———, *Two theories of dynamic semantics*, Jelia '90. European Workshop on Logics in AI (J. van Eijck, ed.), LNCS, vol. 478, Springer, 1991.

[12] J. Hintikka, *Impossible possible worlds vindicated*, J. of Phil. Logic **4** (1975), 475–484.

[13] J.-B. Kim, *Grammatical interfaces in English object extraposition*, Linguistic Research **25/3** (2008), 117–131.

[14] J.-B. Kim and I. Sag, *It-extraposition in English: a constraint-based approach*, HPSG '2005 (A. Branco, F. Costa, and M. Sailer, eds.), University of Lisbon.

[15] J. King, S. Soames, and J. Speaks, *New Thinking about Propositions*, OUP, 2014.

[16] A. Kratzer, *An investigation into the lumps of thought*, Linguistics and Philosophy **12** (1989), 607–653.

[17] S. Kripke, *Naming and Necessity*, Harvard UP, 1980.

[18] ———, *A puzzle about belief*, Meaning and Use: papers presented at the Second Jerusalem Philosophy Encounter (A. Margalit, ed.), Reidel, 1979.
[19] K. Liefke, *A Single-Type Semantics for Natural Language*, Doctoral dissertation, Center for Logic and Philosophy of Science, Tilburg, 2014.
[20] ———, *Towards an account of rich situated semantic content*, Proceedings of *Cognitive Structures: linguistic, philosophical, and psychological perspectives* (Düsseldorf, 15–17 Sept., 2016) (Th. Gamerschlag *et al.*, ed.), 2016.
[21] K. Liefke and M. Werning, *Evidence for single-type semantics*: *an alternative to e/t-based dual-type semantics*, in revision.
[22] S. Löbner, *Evidence for frames from human language*, Frames and Concept Types (Th. Gamerschlag, ed.), Studies in Ling. and Philosophy, vol. 94, Springer, 2014.
[23] F. Moltmann, *Propositional attitudes without propositititions*, Synthese **135** (2003), 77–118.
[24] ———, *Part structures in situations*: *the semantics of* individual *and* whole, Linguistics and Philosophy **28** (2005), 599–641.
[25] ———, *Propositions, attitudinal objects, and the distinction between actions and products*, Canadian Journal of Philosophy **135** (2014), 1–23.
[26] R. Montague, *The proper treatment of quantification in ordinary English*, Formal Philosophy (R.H. Thomason, ed.), Yale UP, 1976.
[27] R. Muskens, *Meaning and Partiality*, Studies in Logic, Language and Information, CSLI Publications, 1995.
[28] ———, *Sense and the computation of reference*, Linguistics and Philosophy **28** (2005), 473-504.
[29] J. Perry, *From worlds to situations*, J. of Philosophical Logic **15** (1986), 83–107.
[30] ———, *Possible worlds and subject matter*, Possible Worlds in Humanities, Arts and Sciences (A. Sturé, ed.), Walter de Gruyter, 1989.
[31] C. Pollard, *Hyperintensions*, J. of Logic and Computation **18/2** (2008), 257–282.
[32] ———, *Agnostic hyperintensional semantics*, Synthese **192** (2015), 535–562.
[33] A. Prior, *Objects of Thought*, Clarendon Press, 1971.
[34] V. Rantala, *Quantified modal logic*: *non-normal worlds and propositional attitudes*, Studia Logica **41/1** (1982), 41–65.
[35] M. Richard, *Propositional Attitudes*: *an essay on thoughts and how we ascribe them*, Cambridge UP, 1990.
[36] P. Robbins and M. Aydede (eds.), *The Cambridge Handbook of Situated Cognition*, Cambridge UP, 2009.
[37] I. Sag, Th. Wasow, and E. Bender, *Syntactic Theory*: *a formal approach*, CSLI Publications, 2003.
[38] N. Salmon, *The resilience of illogical belief*, Noûs **40/2** (2006), 369–375.
[39] S. Schiffer, *The mode-of-presentation problem*, Propositional Attitudes: the role of content in logic, language, and mind (C.A. Anderson and J. Owens, eds.), CSLI Lecture Notes, vol. 20, CSLI Publications, 1990.
[40] ———, *Belief ascription*, The Journal of Philosophy **89/10** (1992), 499–521.
[41] J. Searle, *Proper names*, Mind **67** (1959), 166–173.
[42] S. Soames, *What is Meaning?*, Princeton UP, 2010.
[43] R.H. Thomason, *A model theory for the propositional attitudes*, Linguistics and Philosophy **4** (1980), 47–70.
[44] F. Veltman, *Defaults in update semantics*, J. of Phil. Logic **25/3** (1996), 221–261.
[45] E.N. Zalta, *A classically-based theory of impossible worlds*, Notre Dame Journal of Formal Logic **38/4** (1997), 640–660.

A Nonmonotonic Sequent Calculus with Downward Conditional

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Abstract. I present a nonmonotonic sequent calculus equipped with a new logical connective, the downward conditional. This system is motivated by the philosophical idea of logical expressivist inferentialism, and has the following two distinctive features. First, the system extends a nonmonotonic material consequence relation over an atomic language into the one over a logically complex language, demonstrating how the usage of logically complex formula can be determined by the usage of atoms. Since this extension is conservative, the resulting consequence relation is also nonmonotonic. Second, the system shows how logically complex language thus extended enables us to explicate the underlying material consequence relation. Especially, the new connective of downward conditional, together with the regular conditional, allows the consequence set of an arbitrary premise set to codify all the consequences of every other (finitely different) premise set. In this sense, the system is monadological.
Keywords: Sequent Calculus, Nonmonotonicity, Downward Conditional, Monadologicality.

1 Philosophical Motivations

Logical inferentialism claims that the meaning of a bit of logical vocabulary can be explained by specifying its inferential role, namely the rule according to which it should be used in the logical inference. My system[1] can be regarded as a product of such a logical inferentialist project. However, this system is also motivated by commitment to two distinctive philosophical ideas: Semantic inferentialism and logical expressivism, both of which are developed by Robert Brandom. Although these two ideas may not be as familiar to logicians as logical inferentialism, I believe that they shed new and interesting light on the characteristic role that the logical vocabulary plays in our inferential practice. Furthermore, they are the key to understand the two distinctive features of the system presented here, nonmonotonicity and the downward

[1] This paper owes a great deal to collaborative work of the research group of Prof. Robert Brandom.

conditional, neither of which has drawn too much attention in literature.

First, semantic inferentialism can be understood as a radical generalization of logical inferentialism. Semantic inferentialists claim that not only the meaning of a bit of logical vocabulary, but also the meaning of a bit of non-logical vocabulary are determined by its inferential role, namely the way it ought to be used, in connection with other relevant expressions, in our everyday inferential practice.[2]

It is crucial that "inferential practice" here is not understood narrowly as the logical one, but more broadly as "the game of giving and asking for reasons" in general. In this use-theoretic approach to the meaning, an inference that contributes to determine the meanings of given expressions is called a *material* inference (in contrast to a *formal* inference), since the correctness of that inference does not depend on its logical form, but rather on the way those expressions appear in it. For example, "A match is struck. Therefore, the match lights." is an instance of such material inferences, since this inference is correct not because it has a certain logical form, but because "match," "is struck," and "lights" are arranged in this particular manner in it. As exemplified in this instance, material inferences are often defeasible, and therefore nonmonotonic. Thus, for semantic inferentialists, who do not limit attention to the formal inference, monotonicity is no longer a feature of inference that can be assumed across the board.

In a word, for semantic inferentialists, the meaning of a given expression, whether it is logical or not, is determined by the role that it plays in the inferential practice broadly understood. At this stage, however, one may wonder on what basis semantic inferentialists demarcate some bits of vocabulary as logical from those that are not, and what is the relationship between those logical and non-logical bits of vocabulary. Logical expressivism is an answer to such a demarcation problem concerning logicality.

According to logical expressivism, there are two essential criteria to demarcate the logical vocabulary from the non-logical one. First, the ability to use logical vocabulary can be *algorithmically elaborated* from the ability to use non-logical vocabulary. In other words, if one already knows how to use a set of non-logical vocabulary in the underlying material inferential practice, one need not acquire any extra ability in order to come to know how to use a set of logical vocabulary. Second, the distinctive role that bits of logical vocabulary play is to *express* or *explicate* those material-inferential rules that we implicitly follow when we use bits of non-logical vocabulary.[3] To put differently, logical vocabulary enables us to explicitly talk about, within the object language, what we implicitly follow.

These background philosophical commitments put three substantial constraints on the logical system pursued in this paper. First, a *material* consequence relation that (according to

[2] Strictly speaking, this is what Brandom (1994) calls *strong* inferentialism.

[3] See Brandom (2008).

inferentialists) makes *non-logical* sentences mean what they mean should *not* be assumed to be *monotonic*. Second, the consequence relation between *logically complex* sentences must be determined by systematically extending this underlying nonmonotonic material consequence relation. The connective rules conducting this work demonstrates how the ability to use non-logical vocabulary can be algorithmically elaborated into the ability to use logical one. Third, for the inferential roles of logical connectives thus fixed to count as *explicating* the underlying material consequence relation, such extension of the underlying nonmonotonic material consequence relation must be (at least) conservative. Consequently, the extended consequence relation must also be nonmonotonic. Thus, logical inferentialist expressivists need a nonmonotonic logical system.

For logical expressivists, the paradigm of the logical (i.e., explicating) vocabulary is the conditional, since it enables us to *codify* (or "*talk about*"), within the object language, material implications if a new proposition is added to a premise set (i.e., $\Gamma \mid\sim A \rightarrow B$ *iff* $\Gamma, A \mid\sim B$, where $A \notin \Gamma$). In this paper, I enrich the current inventory of logical vocabulary with a new type of conditional——what I call *the downward conditional* ($^{-}\rightarrow$). The expressive role of this new conditional is the mirror image of that of the regular conditional: The downward conditional enables us to *codify* ("*talk about*") material implications if a proposition is *subtracted from* (as opposed to *added to*) a premise set (i.e., $\Gamma \mid\sim A {}^{-}\!\!\rightarrow B$ *iff* $\Gamma - \{A\} \mid\sim B$, where $A \in \Gamma$).[4]

2 Merits

It may seem that the downward conditional is a rather exotic connective from the standard truth-conditional semanticist viewpoint. However, from the logical expressivist inferentialist viewpoint, it is a natural counterpart of the regular conditional. Furthermore, there are at least two merits of having the downward conditional in our system. First, the downward conditional massively increases the expressive power of the system. Suppose Γ is given as a premise set. The regular conditional only allows us to talk about, within the object language, consequences of a premise set *(finitely) bigger* than Γ (i.e., $\Gamma \mid\sim U_1 \rightarrow \ldots \rightarrow (U_n \rightarrow A)$ *iff* $\Gamma, U_1, \ldots, U_n \mid\sim A$, where $U_1, \ldots, U_n \notin \Gamma$). With the downward conditional in hand, however, we become able to talk about consequences of an *arbitrary* premise set *(finitely) different* from Γ (i.e., $\Gamma \mid\sim D_1 {}^{-}\!\!\rightarrow \ldots {}^{-}\!\!\rightarrow (D_m {}^{-}\!\!\rightarrow (U_1 \rightarrow \ldots \rightarrow (U_n \rightarrow A)\ldots)\ldots)$ *iff* $\Gamma - \{D_1, \ldots, D_n\}, U_1, \ldots, U_m \mid\sim A$, where $D_1, \ldots, D_n \in \Gamma$ and $U_1, \ldots, U_m \notin \Gamma$). In this respect, each premise set is like Leibnizian monad in that "complete knowledge of it (i.e., knowledge of its consequences) gives us complete knowledge of every other (finite

[4] The idea of the downward conditional and the related idea of Monadologicality (to be explained below) are originally suggested by Bob Brandom. My contributions mainly consist in the technical results I am presenting here.

different) premise set (i.e., its consequences)."[5]

Apart from this logical expressivist inferentialist viewpoint, however, the downward conditional can also have wider appeal of its own. This unorthodox conditional enables us to formalize a special class of inferences that are made by *setting aside*, in contrast to *obtaining* or *assuming,* a certain set of knowledge. Although they have not drawn much attention from logicians, inferences of this class play an essential role in several philosophical arguments. Cartesian Skepticism (Descartes 1641/2008) and The Veil of Ignorance (Rawls 1971) are two of the most prominent examples. More casual instances can be found in everyday mind-reading practice. One example of them is the false belief task, in which one is required to infer from the viewpoint of another who lacks a piece of knowledge that one already has. Note that such downward inferences cannot be easily assimilated to those inferences already expressible by the regular logical vocabulary. After all, setting aside a premise A is different from either assuming $\neg$A or assuming A$\vee\neg$A.

3 The Base Consequence Relation

To begin with, let L_0^- be a set of atomic formula. From the perspective of logical expressivist inferentialism, the material implication and incoherence among atomic formula can be regarded as a given basis on which a semantic explanation can be built. Using "$\perp$," these material implication and incoherence can jointly be considered as a consequence relation from $P(L_0^-)$ to L_0, where $L_0 = L_0^- \cup \{\perp\}$. Such a consequence relation can be represented in either of the following manners:

$\Gamma_0 \mid\!\sim_0 p$; or $p \in Cn_0(\Gamma_0)$,

where if $p \neq \perp$, the premise set Γ_0 implies A, while if $p = \perp$, the premise set is incoherent. As discussed before, the material implication is supposed to be defeasible. Therefore, Weakening is not imposed as a structural rule in our system.[6] On the other hand, since the premise of the consequence relation is treated as a set, Contraction and Permutation are automatically built into the structure.

At this base level of material consequence relation, only the following two general properties

[5] I borrow this phrase from a remark given by Ulf Hlobil, a member of our study group.

[6] One might wonder if there is any connection between our system and relevance logic, since relevance logicians also give up Weakening. However, our reason to abandon Weakening (i.e., the deferability of the base material consequence relation) is independent from the consideration of relevancy. Furthermore, our base consequence relation is not supposed to be sensitive to the relevancy between the premise and conclusion, since as mentioned just below, we stipulate that Reflexivity holds with respect to an arbitrary premise.

are stipulated to hold:

Coherence of the empty set: It is not the case that $\emptyset \mid\!\sim_0 \bot$.
Reflexivity at the base: For any $\Gamma_0 \subseteq L_0^-$ and $p \in L_0^-$, $\Gamma_0, p \mid\!\sim_0 p$.

Let us call those material consequence relations that satisfy these conditions *proper*. Hereafter, we only deal with proper material consequence relations.

In the next section, it is shown how a given material consequence relation $\mid\!\sim_0$ over the atomic language L_0 can be extended into the consequence relation $\mid\!\sim$ over the logically complex language L according to our sequent calculus. In course of this construction, the following three aims are pursued. First, Reflexivity should be preserved at the logically extended consequence relation.

Preservation of Reflexivity: For any $\Gamma \subseteq L^-$ and $A \in L^-$, $\Gamma, A \mid\!\sim A$.

Second, for our extension of a given material consequence relation to count as logical at all, it has to conserve the original material consequence relation.

Conservativeness: For any $\Gamma_0 \subseteq L_0$ and $p \in L_0$, $\Gamma_0 \mid\!\sim p$, if and only if $\Gamma_0 \mid\!\sim_0 p$.

Finally, the main purpose of enriching our logical vocabulary with the downward conditional is to make the logically extended consequence relation *monadological* in that the consequence set of a given premise encodes all the information about arbitrary consequences of that premise's finite neighborhood.

Monadologicality: For any $\Gamma, \Gamma' \subseteq L^-$ such that $\Gamma - \Gamma'$ and $\Gamma' - \Gamma$ are finite, and for any A such that $\Gamma' \mid\!\sim A$, there is some B such that $\Gamma \mid\!\sim B$ *iff* $\Gamma' \mid\!\sim A$.

These three properties are proved in section 5.

4 The Construction

First, the syntax of our logically complex language is given as follows:

Syntax. $L =_{df.} L^- \cup \{\bot\}$, where L^- is inductively defined as follows:
(1) For any $p \in L_0^-$, $p \in L^-$, where L_0^- is the set of atomic formula;

(2) For any A, B $\in L^-$, ($\neg$A), (A$\to$B), ($A^-\to$B), (A&B), (A$\vee$B)$\in L^-$;

(3) These are the only formula that belong to L^-.

Next, let us call our sequent calculus the "nonmonotonic paired-conditional" system ("NMPC" for short). As semantic inferentialists, we regard the base material consequence relation as given. Such a given base forms the axioms of NMPC.

Axioms of NMPC.

Ax: If $\Gamma \mid\!\sim_0 A$, then $\Gamma \mid\!\sim A$ is an axiom.

These axioms are only concerned with the consequence relation over the atomic language. This consequence relation is extended to the one over the logically complex language by way of the following sequent rules:

Rules of NMPC.

$$\frac{\Gamma-\{A\} \mid\!\sim A\to B}{\Gamma, A \mid\!\sim B}\ \text{CCP} \qquad \frac{\Gamma, A \mid\!\sim B}{\Gamma-\{A\} \mid\!\sim A\to B}\ \text{CP}$$

$$\frac{\Gamma, A \mid\!\sim A^-\to B}{\Gamma-\{A\} \mid\!\sim B}\ \text{CCP}^- \qquad \frac{\Gamma-\{A\} \mid\!\sim B}{\Gamma, A \mid\!\sim A^-\to B}\ \text{CP}^-$$

$$\frac{\Gamma \mid\!\sim A}{\Gamma, \neg A \mid\!\sim \bot}\ \text{LN} \qquad \frac{\Gamma, A \mid\!\sim \bot}{\Gamma \mid\!\sim \neg A}\ \text{RN}$$

$$\frac{\Gamma-\{A\&B\}, A, B \mid\!\sim C}{\Gamma-\{A, B\}, A\&B \mid\!\sim C}\ \text{L\&} \qquad \frac{\Gamma \mid\!\sim A \qquad \Gamma \mid\!\sim B}{\Gamma \mid\!\sim A\&B}\ \text{R\&}$$

$$\frac{\Gamma-\{A\to B\}, A, B \mid\!\sim B}{\Gamma-\{B\}, A, A\to B \mid\!\sim B}\ \text{wwLC} \qquad \frac{\Gamma-\{A^-\to B\}, A, B \mid\!\sim C}{\Gamma-\{A, B\}, A^-\to B \mid\!\sim C}\ \text{wLC}^-$$

$$\frac{\Gamma-\{A\vee B\}, A \mid\!\sim C \qquad \Gamma-\{A\vee B\}, B \mid\!\sim C}{\Gamma-\{A, B\}, A\vee B \mid\!\sim C}\,\text{LV} \qquad \frac{\Gamma \mid\!\sim A}{\Gamma \mid\!\sim A\vee B}\,\text{RV}_1 \qquad \frac{\Gamma \mid\!\sim B}{\Gamma \mid\!\sim A\vee B}\,\text{RV}_2$$

$$\frac{\Gamma, A \mid\!\sim A}{\Gamma, A, B \mid\!\sim A}\,\text{COW}$$

The extended consequence relation is defined as the minimal set of those sequents derivable from the axioms via the rules as specified above.

Three comments are in order here. First, it would be noticed that CCP and CCP− are simplifying rules. As seen in the next section, the presence of these simplifying rules somehow complicates the proof of Conservativeness in NMPC. However, these rules cannot be dropped since, as also seen below, they play essential role in the proof of Monadologicality of NMPC.

Second, Cut is neither an explicit nor admissible rule of NMPC. This is because in the presence of CP and CCP, Cut collapses into Weakening (or Monotonicity).[7] Thus, the nonmonotonic base consequence relation cannot be conservatively extended by NMPC with Cut.

Finally, the so-called Deduction Theorem (hereafter DT) is not a metatheorem of NMPC. What holds instead is only the following slightly weaker version of DT: $\Gamma, A \mid\!\sim B$ *iff* $\Gamma-\{A\} \mid\!\sim A \to B$. The reason to give up full DT is the following. To ensure DT, CCP and CP must be strengthened into the following CCP* and CP*:

$$\frac{\Gamma \mid\!\sim A\to B}{\Gamma, A \mid\!\sim B}\,\text{CCP*} \qquad \frac{\Gamma, A \mid\!\sim B}{\Gamma \mid\!\sim A\to B}\,\text{CP*}$$

With CCP*, however, the following illicit derivation[8] can be made with the help of the downward conditional:

$$\frac{\dfrac{\Gamma, A, B \mid\!\sim C}{\Gamma, A, B, A\&B \mid\!\sim A\&B \,{}^{-}\!\!\to C}\,\text{CP}^{-}}{}\,\text{CP}$$

[7] See Hlobil (2016, sec. 4.3).
[8] This was originally pointed out by Daniel Kaplan, another member of our study group.

$$\dfrac{\dfrac{\dfrac{\Gamma, A, B \mid\sim A\&B \rightarrow (A\&B {}^{-}\!\!\rightarrow C)}{\Gamma, A\&B \mid\sim A\&B \rightarrow (A\&B {}^{-}\!\!\rightarrow C)}\ \text{L\&}}{\Gamma, A\&B \mid\sim A\&B {}^{-}\!\!\rightarrow C}\ \text{CCP*}}{\Gamma \mid\sim C}\ \text{CCP}^{-}$$

Thus, Conservativeness is violated. In fact, this derivation is a typical instance of how easily adding the downward conditional to an otherwise unproblematic set of connectives wrecks the resulting consequence relation. Adding the downward conditional to a system is a balancing act. Although it significantly enhances the expressive power of the system, it often interacts with other connectives of the system in such surprising manners that illegitimate derivations such as the one just presented are possible. For analogous reasons, the bottom premise of CP⁻ and the top premise of CCP− must be $\Gamma-\{A\}$ instead of Γ. The reason for which the top and bottom premises of the left rules need those set subtraction clauses as presented above is also similar.

5 Properties of the system

To begin with, NMPC preserves Reflexivity of the base consequence relation.

Proposition 1. Given that for any $\Gamma_0 \subseteq L_0^{-}$ and any $p \in L_0^{-}$, $\Gamma_0, p \mid\sim_0 p$, then for any (finite) $\Gamma \subset L^{-}$ and any $A \in L^{-}$, $\Gamma, A \mid\sim A$.

Proof. By induction on the complexity of the formulae on the right-hand side, A. Base case is immediate from Reflexivity at the base and COW. Induction step is also straightforward. ■

Next, let us show that NMPC is a monadological system. All that is needed to prove this property are the right and simplifying rules concerning the paired conditionals. First, notice that CP and CCP assures that Γ's implying B→A (i.e., $\Gamma \mid\sim B \rightarrow A$, where $B \notin \Gamma$) "says" that Γ together with B implies A (i.e., $\Gamma, B \mid\sim A$) in that the former implication holds *if, and only if* the latter holds. The situation is analogous with respect to the downward conditional. In the presence of CP⁻ and CCP⁻, Γ's implying C⁻→A (i.e., $\Gamma \mid\sim C {}^{-}\!\!\rightarrow A$, where $C \in \Gamma$) "says" that Γ minus C implies A (i.e., $\Gamma-\{C\} \mid\sim A$) in that the former implication holds *if, and only if* the lather holds.[9] Putting these two points together, Γ's implying C⁻→(B→A) (i.e., $\Gamma \mid\sim C {}^{-}\!\!\rightarrow (B \rightarrow A$, where

[9] Note that the right-to-left direction of this biconditional claim would not hold, if it were not for CCP⁻.

B ∉ Γ and C ∈ Γ) "says" that Γ minus C together with B implies A (i.e., $\Gamma - \{C\}, B \mid\sim A$), and so on and so forth. To generalize this point, any implication of any premise set that is reachable by way of adding/subtracting a finite number of members to/from Γ can be encoded as an implication of Γ by using these two conditionals jointly and iteratively.

Proposition 2. For any $\Gamma, \Gamma' \subset L$ such that $\Gamma - \Gamma'$ and $\Gamma' - \Gamma$ are finite, $\Gamma \mid\sim D_1{}^{-}\!\!\to \ldots {}^{-}\!\!\to (D_m{}^{-}\!\!\to (U_1 \to \ldots \to (U_n \to A)\ldots)\ldots)$ *iff* $\Gamma' \mid\sim A$, where $A \neq \bot$, $D_1, \ldots, D_n \in \Gamma - \Gamma'$, and $U_1, \ldots, U_m \in \Gamma' - \Gamma$.

Proof. Note that $((\Gamma - \{D_1, \ldots, D_n\}) \cup \{U_1, \ldots, U_m\} = \Gamma'$. The left-to-right direction is straightforward from repeated applications of CCP⁻and CCP. The right-to-left direction is also immediate from repeated applications of CP and CP⁻. ■

Finally, let us turn to the proof of Conservativeness of NMPC. Although the details of the following proof can seem complicated, the intuitive idea underlying it is relatively simple. Consider an arbitrary proof tree T whose root is $\Gamma_0 \mid\sim p$, where $\Gamma_0 \subseteq L_0{}^{-}$ and $p \in L_0$. We want to show that T does not violate Conservativeness——that is, (C) T's leaves already contain $\Gamma_0 \mid\sim p$. If $p = \bot$, the root can only come by Ax, and (C) immediately follows. However, if $p \neq \bot$, the root can come by COW, CCP or CCP⁻. If the root comes by CCP⁻ (one of our notorious simplifying rules!), the penultimate sequent is $\Gamma_0, A \mid\sim A^{-}\!\!\to p$ (where $A \notin \Gamma_0$). Since A can be of arbitrary complexity, apparently such a sequent can come by any of our left rules (except LN) along with the three rules as mentioned above. This makes it appear that we would have to consider intractably various types of cases in climbing up T to its leaves. The truth is, however, that no left rule can be applied on T, given that the left-hand side of T's root contains no connective (i.e., $\Gamma_0 \subseteq L_0{}^{-}$) (as shown by Lemma 2). This is because (roughly put) left rules always adds one more connective to the left-hand side of the snake turnstile, while in NMPC the total number of connectives on the left never decreases in descending a path of a proof tree as long as the formulae on the right-hand side remains the same (as shown by Lemma 1). This observation

For instance, consider the following derivation, where $p, q \in L_0{}^{-}$.

$$\dfrac{\dfrac{\dfrac{q, p \mid\sim p}{q^{-}\!\!\to p \mid\sim p}\ \text{wLC}^{-}}{q, q^{-}\!\!\to p \mid\sim q^{-}\!\!\to p}\ \text{CP}^{-}}{q, q \to (q^{-}\!\!\to p) \mid\sim q^{-}\!\!\to p}\ \text{wwLC}$$

If the right-to-left direction of the biconditional at issue holds, it must be the case that $q \to (q^{-}\!\!\to p) \mid\sim p$. However, this sequent cannot come by any rule other than CCP⁻.

simplifies the situation, and thereby allows us to specify a common form and properties shared by any possible sequents occurring in T (see Lemma 3). Based on those form and properties, we can show that every proof tree that concludes an atomic sequent satisfies (C) (Proposition 3).

To flesh out this proof sketch, we need to introduce a few definitions and prove several lemmas. To begin with, let us define the summation of complexity of a set Γ, Σc(Γ), as follows:

Definition 1. Let Γ be $\{B_1, \ldots, B_m\}$. $\Sigma c(\Gamma) =_{df} \sum_{i=1}^{m} c(Bi)$, where $c(B_i)$ is the complexity of B_i.

In other words, Σc(Γ) is the total number of the connectives contained in Γ. Note that it is often the case that $\Sigma c(\Gamma) \neq c(\Gamma)$, where c(Γ) is the complexity of Γ (i.e., the number of the connectives of the most complex formula(e) in Γ). Using this concept, our first lemma is articulated as follows:

Lemma 1. Suppose that $\Gamma \mid\sim A$ is proved at height h of a path of a proof tree, where $A \neq \bot$, and $\Gamma' \mid\sim A$ is proved at height h′ of the same path, where $h' \geqq h$. Then, $\Sigma c(\Gamma') \geqq \Sigma c(\Gamma)$.

Proof. By induction on the remainder of the height of $\Gamma' \mid\sim^{\uparrow x} A$ minus the height of $\Gamma \mid\sim^{\uparrow x} A$. Base case is immediate. In induction step, all the cases except for the ones in which the root comes by CP, CP^{-}, CCP, or CCP− are also straightforward. In the latter exceptional cases, however, the penultimate sequent does not share the same formulae on the right with the root. In such cases, we have to divide up the proof path into several sections so that the top and bottom sequents of each section share the same formulae on the right. Such divisions are made by appealing to the fact that $CP[/CP^{-}]$ is the only rule to introduce a regular[/downward] conditional to the right. Then, we apply the induction hypothesis to the top and bottom sequents of each section. ■

Next, let B be an arbitrary nested paired conditional, $A_n \overset{[-]}{\rightarrow} (A_{n-1} \overset{[-]}{\rightarrow} \ldots \overset{[-]}{\rightarrow} (A_1 \overset{[-]}{\rightarrow} p) \ldots)$. Two variants of the summation of complexity can be defined with respect to B: First, the summation of complexity of all those A_i that are *upward* antecedents of B, $\Sigma c^{uANT}(B)$; second, the summation of complexity of all those A_j that are *downward* antecedents of B, $\Sigma c^{uANT}(B)$.

Definition 2. let B be $A_n \overset{[-]}{\rightarrow} (A_{n-1} \overset{[-]}{\rightarrow} \ldots \overset{[-]}{\rightarrow} (A_1 \overset{[-]}{\rightarrow} p) \ldots)$.

$\Sigma c^{uANT}(B) =_{df.} \sum_{i=1}^{n} c^{uANT}(B, i)$,

$$\text{where } c^{uANT}(B, i) = \begin{cases} c(A_i) & : A_i \text{ is an upward antecedent of B} \\ 0 & : \text{otherwise.} \end{cases}$$

$\Sigma c^{uANT}(B) =_{df.} \sum_{i=1}^{n} c^{dANT}(B, i)$,

$$\text{where } c^{dANT}(B, i) = \begin{cases} c(A_i) & : A_i \text{ is a downward antecedent of B} \\ \\ 0 & : \text{otherwise.} \end{cases}$$ [10]

Based on Lemma 1 and Definition 1 and 2, the following lemma is provable, which plays an essential role in showing that no left rule can be applied in any proof trees that could lead to any violation of Conservativeness.

Lemma 2. Suppose $\Gamma \mid\!\sim^{\uparrow X} B$ is provable, where $B = A_n{}^{[-]}{\rightarrow}(A_{n-1}{}^{[-]}{\rightarrow}\ldots{}^{[-]}{\rightarrow}(A_1{}^{[-]}{\rightarrow}p)\ldots)$ and $p \in L_0$. Then, $\Sigma c(\Gamma) + \Sigma c^{uANT}(B) \geqq \Sigma c^{dANT}(B)$.

Proof. By induction on the complexity of the nested paired conditional on the right, B. In both base case and induction step, two cases are divided depending on whether the main connective of B is the regular or downward conditional. In those cases, Lemma 1 and (again) the fact that CP[/CP$^-$] is the only rule to introduce a regular[/downward] conditional to the right plays an essential role in showing the claim. I leave it to the reader to fill in the details of the proof. ■

We are in a position to prove the lemma specifying the form and properties shared by any sequents consisting of a proof tree suspectable of the Conservativeness violation. Before that, however, let us define one last thing. Let B be $A_m{}^{[-]}{\rightarrow}(A_{m-1}{}^{[-]}{\rightarrow}\ldots(A_1{}^{[-]}{\rightarrow}p)\ldots)$ again. $\cup/-(\Gamma, B)$ is defined as the set obtained by *adding* to Γ those *upward* antecedents of B and *subtracting* from Γ those *downward* antecedents of B in the order from m to 1. That is:

Definition 3. Let B be $A_m{}^{[-]}{\rightarrow}(A_{m-1}{}^{[-]}{\rightarrow}\ldots(A_1{}^{[-]}{\rightarrow}p)\ldots)$.

$$\cup/-(\Gamma, B)^{11} =_{df} \begin{cases} (\ldots((\Gamma \cup [/-]\{A_m\}) \cup [/-]\{A_{m-1}\}) \cup [/-]\ldots) \cup [/-]\{A_1\} & : m \neq 0 \\ \\ \Gamma & : m = 0 \end{cases}$$

10 More rigorously, $c^{uANT}(B, n)$ and $c^{dANT}(B, n)$ are inductively defined as follows. (Base case) If A_n is the main upward antecedent of B, then $c^{uANT}(B, n) = c(A_n)$; otherwise, $c^{uANT}(B, n) = 0$. If A_n is the main downward antecedent of B, then $c^{dANT}(B, n) = c(A_n)$; otherwise, $c^{dANT}(B, n) = 0$. Let B_{n-1} be $A_{n-1}{}^{[-]}{\rightarrow}(A_{n-2}{}^{[-]}{\rightarrow}\ldots{}^{[-]}{\rightarrow}(A_1{}^{[-]}{\rightarrow}p)\ldots)$. (Induction step) If A_{n-m} is the main upward antecedent of B_{n-m}, then $c^{uANT}(B, n-m) = c(A_{n-m})$; otherwise, $c^{uANT}(B, n-m) = 0$. If A_{n-m} is the main downward antecedent of B_{n-m}, then $c^{dANT}(B, n-m) = c(A_{n-m})$; otherwise, $c^{dANT}(B, n-m) = 0$. Let $B_{n-(m+1)}$ be $A_{n-(m+1)}{}^{[-]}{\rightarrow}(A_{n-(m+2)}{}^{[-]}{\rightarrow}\ldots{}^{[-]}{\rightarrow}(A_1{}^{[-]}{\rightarrow}p)\ldots)$.

11 Here is the more rigorous definition of $\cup/-(\Gamma, B)$: If $B = p$, then $\cup/-(\Gamma, B) = \Gamma$; otherwise, $\cup/-(\Gamma, B) = S_0$, where S_0 is inductively defined as follows. (Base case) If A_m is the main upward antecedent of B, then $S_{m-1} = \Gamma \cup A_m$. If A_m is the main downward antecedent of B, then $S_{m-1} = \Gamma - A_m$. Let B_{m-1} be $A_{m-1}{}^{[-]}{\rightarrow}(A_{m-2}{}^{[-]}{\rightarrow}\ldots{}^{[-]}{\rightarrow}(A_1{}^{[-]}{\rightarrow}p)\ldots)$. (Induction step) If A_{m-n} is the main upward antecedent of B_{m-n}, then $S_{m-(n+1)} = S_{m-n} \cup A_{m-n}$. If A_{m-n} is the main downward antecedent of B_{m-n}, then $S_{m-(n+1)} = S_{m-n} - A_{m-n}$. Let $B_{m-(n+1)}$ be $A_{m-(n+1)}{}^{[-]}{\rightarrow}(A_{m-(n+2)}{}^{[-]}{\rightarrow}\ldots{}^{[-]}{\rightarrow}(A_1{}^{[-]}{\rightarrow}p)\ldots)$.

Now, let us prove the crucial lemma.

Lemma 3. Suppose that T is a proof tree whose root is $\Gamma_0 \mid\sim p$, where $\Gamma_0 \subseteq L_0^-$ and $p \in L_0^-$. Also suppose that T's height is n. For any height $n-m$ $(1 \leqq m \leqq n-1)$, any sequent of T has the following form (F) with the following two properties (P1 and P2):

F: $\Gamma_m \mid\sim B_m$, where $B_m = A_l^{[-]} \rightarrow (A_{l-1}{}^{[-]} \rightarrow \ldots (A_1^{[-]} \rightarrow p) \ldots)$ and $0 \leqq l \leqq m$.

P1: $\Sigma c(\Gamma_m) + \Sigma c^{uANT}(B_m) = \Sigma c^{dANT}(B_m)$.

P2: Either $\cup/-(\Gamma_m, B_m) = \Gamma_0$; or $\cup/-(\Gamma_k, B_k) \subset \Gamma_0$ and $p \in \Gamma_m$.

Proof. By induction on the distance from the root, m (where $n-m$ is the height of the sequent whose form and properties are at issue). Base case is immediate. Induction step is also straightforward. Note that in induction step Lemma 2 plays a crucial role in assuring that the sequent whose form and properties are at issue can never come by any of our left rules. ■

It immediately follows from Lemma 3 that NMPC is conservative.

Proposition 3. $\Gamma_0 \mid\sim p$, if and only if $\Gamma_0 \mid\sim_0 p$, where $\Gamma_0 \subseteq L_0^-$ and $p \in L_0$.

Proof. (The right-to-left direction) Straightforward from Ax and the fact that NMPC only extends (i.e., never shrinks) the consequence relation. (The left-to-right direction) Take an arbitrary proof tree T whose root is $\Gamma_0 \mid\sim p$, where $\Gamma_0 \subseteq L_0^-$ and $p \in L_0$. We want to show that $\Gamma_0 \mid\sim_0 p$. Now, either $p = \bot$ or not.

(Case 1) $p = \bot$. Since $\Gamma_0 \subseteq L_0^-$, $\Gamma_0 \mid\sim \bot$ cannot come by any rules. Thus, $\Gamma_0 \mid\sim \bot$ is an axiom. By Ax, $\Gamma_0 \mid\sim_0 \bot$.

(Case 2) $p \neq \bot$. Take an arbitrary leaf of T (i.e., an arbitrary sequent at height 0 of T). By Lemma 3, any sequent of T has the following form with the following two properties:

F: $\Gamma_m \mid\sim B_m$, where $B_m = A_l^{[-]} \rightarrow (A_{l-1}{}^{[-]} \rightarrow \ldots (A_1^{[-]} \rightarrow p) \ldots)$ and $0 \leqq l \leqq m$.

P1: $\Sigma c(\Gamma_m) + \Sigma c^{uANT}(B_m) = \Sigma c^{dANT}(B_m)$.

P2: Either $\cup/-(\Gamma_m, B_m) = \Gamma_0$; or $\cup/-(\Gamma_m, B_m) \subset \Gamma_0$ and $p \in \Gamma_m$.

However, no sequent with a complex formulae on the right-hand side can appear at height 0. Thus, by (F), that given leaf must have the form: $\Gamma_m \mid\sim p$. By (P2) and Definition 3, either it is the case that $\Gamma_m = \Gamma_0$, or that $\Gamma_m \subset \Gamma_0$ and $p \in \Gamma_m$.

(Case 2.1) $\Gamma_m = \Gamma_0$. $\Gamma_0 \mid\sim p$ at height 0. Since this can only come by Ax, $\Gamma_0 \mid\sim_0 p$.

(Case 2.2) $\Gamma_m \subset \Gamma_0$ and $p \in \Gamma_m$. $\Gamma_m \mid\sim p$ at height 0. Since this can only come by Ax, $\Gamma_m \mid\sim_0 p$. Since $\Gamma_m \subset \Gamma_0$, $p \in \Gamma_m$, and it is stipulated that Reflexivity holds at the base consequence relation, $\Gamma_0 \mid\sim_0$ p. ■

6 Conclusion

As a semantic inferentialist, I started with a nonmonotonic material consequence relation over an atomic language. As a logical expressivist, I then presented a sequent calculus that maps this base consequence relation onto the one over a logically complex language. My sequent calculus preserves Reflexivity of the base consequence relation, and extends the base consequence relation conservatively. Thus, the resulting logically complex consequence relation is also nonmonotonic. Furthermore, the newly introduced logical connective of the downward conditional, along with the regular conditional, makes it possible to explicate, within the object language, an arbitrary consequence of an arbitrary premise set that is finitely reachable from a given premise set. In this sense, the extended consequence relation is monadological.

References

Brandom, R. B. (1994), *Making it Explicit*, Cambridge, MA: Harvard University Press.

Brandom, R. B. (2008), *Between Saying and Doing: Towards an Analytic Pragmatism*, Oxford: Oxford University Press.

Descartes, R. (1641/2008), *Meditations on First Philosophy: With Selections from the Objections and Replies*, trans. Michael Moriarty, Oxford: Oxford University Press.

Hlobil, U. (2016), "A Nonmonotonic Sequent Calculus for Inferentialist Expressivists." In Pavel Arazim and Michal Dančák (eds.) *The Logica Yearbook 2015*, pp. 87-105, College Publications: London.

Rawls, J. (1971), *A Theory of Justice*, Cambridge, MA: Harvard University Press.

Game semantics and vagueness in natural language

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Abstract We take up the challenge to define a procedure to systematically evaluate natural language statements involving vagueness, as is the case for "About half days are nice.", which is quantified with a vague proportional quantifier and applied to a vague predicate in the scope. Our approach is embedded into an analytic game semantic framework, which extends Giles's game for Łukasiewicz logic by means of a randomization operator and the Baaz-Delta operator.

Keywords game semantics; fuzzy logic; rough sets; natural language processing; vagueness; proportional quantifiers; vague predicates; multi agents

1 Introduction

The modeling of vague proportional quantifier expressions, like "about half", "almost all", and "at least about a third", and of vague predicates, like "tall", and "nice" is a great challenge taken up by many different researchers and communities, amongst which we have linguists [3,20,29], philosophers [7,32], computer scientists [1,28], and logicians [3,12,26]. There is a huge amount of literature on fuzzy quantification, summarized in the recent survey article [6], while there is also a whole monograph of Glöckner [15] about this topic. Also, recent developments in the field of mathematical fuzzy logic [4,5] contribute to the matter by addressing it using a game semantic framework, extending Giles's game ($\mathcal{G}$ game) for Łukasiewicz logic by a randomization operator [11]. We here intend to further extend the existing analytic game semantic framework, while following the systematic approach of Liu and Kerre [24], who proposed to split the problem of generalized quantification into four steps in the following way[1] :

Type I: the quantifier as well as the scope predicate is crisp;
Type II: the quantifier is crisp, but the scope predicate is vague;
Type III: the quantifier is vague, but the scope predicate is crisp;
Type IV: the quantifier as well as the scope predicate are vague.

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[1] Crisp here means binary, hence zero or one valued.

A Type I statement may have the form "All mothers are women.", or "There is a man being more than 180 cm tall.", where the quantifiers, $\forall$ and $\exists$, are crisp in the sense that they only take one of the two classical truth values, zero or one. The same goes for the properties, "being a mother", "being a man/woman", or "being more than 180 cm tall", which are also taken as crisp, as each object either completely fulfills them or not. A general Type IV statement may then be of the more complicated form "Almost all beaches in Thailand are beautiful.", where the quantifier "almost all" and the scope predicate "beautiful" are vague.

To be able to model vague proportional quantifiers and vague predicates adequately, we show how we can define a projection operator, also known as Baaz-Delta, in our game semantic setting. Then Type I quantification already becomes a rather easy task, and we continue to show how vague predicates can be conceived (Type II), following particular ideas from rough set theory or analytic philosophy [27] respectively, where vague concepts depend on several crisp ones, as well as on finitely many agents. We then define vague proportional quantifiers, again following ideas from rough set theory, namely the one of granular levels [37]. This goes back to Zadeh [38], and has attracted some intensified attention during the last years [21,35]. We apply this idea in the context of proportional quantification (Type III) and eventually combine it with our vague predicates, to arrive at the general level of Type IV proportional quantification.

The rest of the paper is organized as follows: Section 2 illustrates the core aspects of the used game semantic framework and defines an important new operator, the so called Baaz-Delta. Section 3 to Section 6 follow exactly the hierarchy of Liu and Kerre, which means each section corresponds to a respective type of quantification, as introduced just above. Section 7 summarized the contribution and describes what has to be done to further augment the presented material.

2 Giles's game and extensions thereof

Giles's game for Łukasiewicz logic is a two player zero sum game of perfect information, where the players are called **P** (proponent) and **O** (opponent). In contrast to the more classic Hintikka game [17], in Giles's game it is possible for both players to have asserted multisets of formulas at each state of the game. This feature results from the following implication rule [13]:

Game Rule 1 ($R_\rightarrow$) *If **P** asserts $F \rightarrow G$ then **O** may attack by asserting F, obliging **P** to assert G.*

In this way, any game state of the form $[F_1, ..., F_n \mid G_1, ..., G_m]$, where the $F's$ are **O**'s and the $G's$ are **P**'s asserted formulas, gets decomposed into a state of the form $[A_1, ..., A_n \mid B_1, ..., B_m]$, where the $A's$ are atoms which **O** eventually has to take responsibility for and the $B's$ are those for which **P** has to account for [4]. Taking up responsibility for an assertion of an atom means to accept having to pay 1€ to the opponent player in case the atom is evaluated to false with respect to a given interpretation I (over a finite domain $U = \{a_1, \ldots, a_n\}$, with $n \in \mathbb{N}$) and risk value assignment $\langle\rangle_I$, i.e. for every atomic formula A we let $\langle A \rangle_I$ be its risk and have $v_I(A) = 1 - \langle A \rangle_I$. Hence, the final risk, from **P**'s perspective, of a game is computed as:

$$\langle A_1, ..., A_n \mid B_1, ..., B_m \rangle = \sum_{1 \leq i \leq m} \langle B_i \rangle_I - \sum_{1 \leq j \leq n} \langle A_j \rangle_I \tag{1}$$

Note that the truth function corresponding to the previous game rule matches the well known truth function of Łukasiewicz implication:

$$v_I(F \to G) = \min(1, 1 - v_I(F) + v_I(G))$$

The negation of a formula F, defined as $(F \to \bot)$, introduces role switch of the players, and the following rule for strong conjunction the principle of limited liability [4]:

Game Rule 2 ($R_{\&}$) *If **P** asserts $F\&G$, then, if **O** attacks, **P** has to either assert F as well as G, or else $\bot$.*

Again, the truth function turns out to correspond to the known one,

$$v_I(F\&G) = \max(0, v_I(F) + v_I(G) - 1),$$

since, as well as **O** need not attack a by **P** asserted formula, also **P** can hedge her/his loss of asserting more than one formula, due to the definition of the game rule. Here, with the strong conjunction rule it is stated explicitly that **P** can assert $\bot$ instead of both F and G, in case they are both wrong. It be understood that this so called principle of limited liability always be in place throughout this paper, although it sometimes remains implicit [4]. We can give a characterization of strong Łukasiewicz logic via $\mathcal{G}$-games as follows:

Theorem 1 ([10]). *For every atomic formula A let $\langle A \rangle$ be its risk and let I be the Ł-interpretation given by $v_I(A) = 1 - \langle A \rangle$. Then, if both, **P** and **O**, play rationally, any game starting in state $[|F]$ will end in a state where **P**'s final risk is $1 - v_I(F)$.*

To generalize the game for Łukasiewicz logic Ł, the authors of [12] introduce the following randomizing quantifier rule in contrast to the ones for the existential and universal quantifiers, where either the proponent or the opponent can choose a witnessing constant[2] c:

Game Rule 3 (R_Π) *If **P** asserts $\Pi x F(x)$ then **P** has to assert $F(c)$ for a randomly picked c.*

[2] For simplicity we identify objects from the domain with unique constants from a set called U again.

It's truth function turns out to be the following:

$v_I(\mathit{\Pi} x F(x)) = \frac{\sum_{c\in U} v_I(F(c))}{|U|} = Prop_x F(x)$

We now also define the so called Baaz-Delta [5], which leaves the truth value of F unchanged in case it is 1, and projects all others to 0, in the following way[3]:

Game Rule 4 **(R_Δ)** *If* $\boldsymbol{P}$ *asserts* ΔF *then,* $\boldsymbol{O}$ *attacks by choosing* $j \in \mathbb{N}$*, obliging* $\boldsymbol{P}$ *to assert* F^j*.*

Theorem 2. *A Ł(R_Δ)-sentence ΔF, for a Ł formula F is evaluated to $v_M(\Delta F) = x$ in an interpretation I iff every $\mathcal{G}$-game for ΔF augmented by rule (R_Δ) is $(1-x)$-valued for $\boldsymbol{P}$ under risk value assignment $\langle\cdot\rangle_I$, i.e. $\boldsymbol{P}$ has a strategy to make his/her final risk at most $1-x$, and $\boldsymbol{O}$ has a strategy to make $\boldsymbol{P}$'s final risk at least $1-x$.*

Now we define the following Nabla operator, which, complementary to the Baaz-Delta, leaves the truth vahlue of F unchanged in case it is 0, and projects all others to 1:

Definition 1. $\nabla F := \neg\Delta\neg F$

The Delta and Nabla Operator can be seen as tools to create discontinuous truth functions, as they represent the limiting case of what can be expressed in ordinary Łukasiewicz logic, where all truth functions are continuous.

3 Type I - the most simple case

As mentioned in the introduction we follow the systematics of Liu and Kerre, and start with defining crisp quantifiers which may only be applied to crisp predicates. We define them in terms of our game semantic connectives, which we have introduced in section 2. The resulting quantifiers behave as expected, in the sense that they are merely a reformulation of respective quantifiers as they are well known from the literature [15]. The main difference here, from the more computational approaches there, is that we embed quantifiers into an analytic framework, instead of just giving the computation rule without relating them to a logical theory. The proportional quantifiers which we define in the present section evaluate for a given number $k \in [0,1]$ to true if and only if the proportion of elements of the domain that fulfill the scope predicate is k, and to false otherwise, and we denote them by $Q^{[=k]}$:

Definition 2. $\forall k \in [0,1]$ *we define* $Q^{[=k]} x\hat{F}(x) = \Delta(\mathit{\Pi} x\hat{F}(x) \leftrightarrow \overline{k})$

$\hat{F}$ denotes a crisp formula, i.e. one composed of crisp (zero or one valued) predicates only. For formulas F and G, $F \leftrightarrow G$ is defined as $(F \to G)\&(G \to F)$, and for $k \in [0,1]$, $\overline{k}$ denotes the truth constant with the value k.

[3] For a formula F, F^j means $F\&\ldots\&F$, j times.

Note that we can express the universal and existential quantifier as follows:

$\forall x F(x) = \Delta(\Pi x F(x))$

$\exists x F(x) = \nabla(\Pi x F(x))$

So far, we can deal with statements like "Exactly half (of the elements of the domain) are students.", or "Exactly a third (of the elements of the domain) are mothers.", as being a student, or mother respectively, is supposed to be a crisp property fully possessed or not by each element of the domain. The next section is devoted to augmenting the framework to predicates which need not be of this kind.

4 Type II - adding vague predicates

Vague predicates, such as "tall", "friendly", or "nice" are notoriously difficult to model, since it is not objectively determinable what makes competent speakers judge objects to possess such a property. There is a lot discussion to be found in the literature, coming from the linguists side [19] as well as from the side of fuzzy logicians, who also give models for vague predicates [23]. We here relate our approach to the so called rough set theory [31], and show how we can model vague predicates within our game semantic framework, following the convincing ideas of rough sets theorists [27,36], as well as those of linguists and philosophers [19,32], or computer scientists [14], still staying in a neat and uniform logical framework. We understand vague predicates as dependent on a finite number of crisp predicates in the following way:

Definition 3. *A vague predicate P_v comes with a set $\{P_v^1, \ldots, P_v^{m_v}\}$ of crisp predicates, where $m_v \in \mathbb{N}$, such that for all $c \in U$ it holds:*

$v_I(P_v(c)) = \frac{r}{m_v}$ *iff* $v_I(P_v^i(c)) = 1$ *for r indices $i \in \{1, \ldots, m_v\}$*

As game rule this may be formalized as follows:

Game Rule 5 ($R^1_{P_v}$) *If **P** asserts $P_v(c)$, then, if **O** attacks, $i \in \{1, \ldots, m_v\}$ gets picked randomly, and then **P** has to assert $P_v^i(c)$.*

The corresponding truth function can be determined as:

$v_I(P_v(c)) = \frac{\sum_{i=1}^{m_v} v_I(P_v^i(c))}{m_v}$

For simplicity we here assume the influence of each crisp predicate to be the same, hence describe an unweighted scenario. The weighted case can be achieved through allowing for multiple occurrences of the same indices in the sample space of the above rule.

Hence, a vague atomic formula evaluates to (completely) true if and only if all crisp atomic formulas relevant to the vague one evaluate to true. In this sense we can talk about lower approximations in the sense of rough set theory. Also, a vague atomic formula evaluates to a truth value greater than zero, if and only if there is a crisp atomic formula relevant to it, which is true. This corresponds to the idea of an upper approximation with regard to rough set theory. Only in case all relevant crisp atomic formulas evaluate to false, the respective vague atomic formula also evaluates to false. We use the following example, to illustrate how this translates into the language of the theory:

4.1 Example

We define the vague predicate "tasty" through the following three crisp predicates: "salami", "mushrooms", and "garlic". Let two universes U_1, U_2 consist of 100 meals each. For the first universe we have 50 portions of pasta, and 50 pizzas, all of which have salami, mushrooms, and garlic on top (hence are tasty!). For the second universe we have 50 portions of pasta, 17 pizzas with only salami and garlic on top, and 33 with only mushrooms on top. Now consider the following statement:

"Exactly 50% of all meals are (fully) tasty pizzas."

With respect to U_1 this statement is true, since the defining properties of "tasty" are (completely) fulfilled for exactly half of the elements of the domain. Formally, we express this statement as:

$$\Delta(\Pi x(\Delta P_{tasty}(x) \wedge pizza(x)) \leftrightarrow \overline{.5}))$$

With respect to U_2 we still have exactly 50% meals with a tasty-value greater than zero, hence we can still evaluate the following statement as true:

"Exactly 50% of all meals are kind of tasty pizzas."

This "kind of" - hedge gets formally represented in the following way:

$$\Delta(\Pi x(\nabla P_{tasty}(x) \wedge pizza(x)) \leftrightarrow \overline{.5})$$

We are now able to express a much greater range of statements, namely also those that involve vague predicates. Also we have described the vague linguistic hedge "kind of", which in natural language expresses a certain kind of uncertainty of the speaker regarding the definition of vague predicates present in some utterance. This becomes possible through the nabla operator ∇, defined in section 2, using the delta operator Δ.

4.2 Definitions and a multi agent extension

Definition 4. *crisp quantifier, vague predicates/formulas*

$$Q^{[=k]}x(\mathit{fully\_F}(x)) := \Delta(\Pi x(\Delta F(x)) \leftrightarrow \overline{k})$$

$$Q^{[=k]}x(\mathit{kind\ of\_F}(x)) := \Delta(\Pi x(\nabla F(x)) \leftrightarrow \overline{k})$$

We can also think of the following generalization of the just described approach to vague predicates. Instead of taking one particular set of crisp predicates, relevant to describe the meaning of a vague predicate, we may think of many different such, reflecting the fact that different agents (competent speakers) may have different reasons to judge an object as "nice", or the like. This more general setting can be achieved through simply changing the respective game rule in the following way, after we fixed some notation, to be able to refer to different agents:

Definition 5. *An agent a_j, $j \in \{1, \ldots, m_a\}$ comes with a set of crisp properties $\{P^1_{v,a_j}, \ldots, P^{m_{v,a_j}}_{v,a_j}\}$ for any vague predicate P_v, $m_{v,a_j} \in \mathbb{N}$, such that for all $c \in U$ it holds:*

$$v_I(P_{v,a_j}(c)) = \tfrac{r}{m_{v,a_j}} \text{ iff } v_I(P^i_{v,a_j}(c)) = 1 \text{ for } r \text{ indices } i \in \{1, \ldots, m_{v,a_j}\}$$

The general game rule for vague predicates now is:

Game Rule 6 ($R^2_{P_v}$) *If **P** asserts $P_v(c)$, then, if **O** attacks, $j \in \{1, \ldots, m_a\}$ gets picked randomly, followed by a random pick of $i \in \{1, \ldots, m_{v,a_j}\}$, and then **P** has to assert $P^i_{v,a_j}(c)$.*

Similarly to the first rule for vague predicates, we can determine the truth function as:

$$v_I(P_v(c)) = \frac{\sum_{j=1}^{m_a} \sum_{i=1}^{m_v} v_I(P^i_{v,a_j}(c))}{m_a m_{v,a_j}}$$

Again, the weighted case, where the influence of crisp predicates to a vague one, or of the different agents to the evaluation respectively, is not uniform, can easily be achieved, as described above.

5 Type III - vague proportional quantifiers, crisp scope

Modeling vague proportional quantifiers is, as argued by Liu and Kerre [24], as well as by Glöckner and others [12,15], best performed in a step by step manner, first focusing on the quantifiers and only later showing how they can then be applied to formulas which may involve vague atomic subformulas. Following this approach, we now develop a way of evaluating vague proportional quantifiers, again being inspired by rough set theory, while staying in our game semantic framework. Hence, in this section, we assume the scope formulas of quantifiers to be crisp again, and only combine it all together in the next section.

An idea going back to Zadeh [38], being carried out much in recent years, is granular computing [2]. The idea is to attach a level of granularity to certain scenarios, hence making objects indistinguishable with respect to some (equivalence) relation. This idea, applied to vague concepts [37], is here extended to vague quantification in an seemingly obvious way. We apply the simple idea of tolerance intervals around some crisp value. Take the quantifier expression "about half", which can be associated to several such, e.g. $[37,5\%, 62,5\%]$, [45%,55%], [49,5%,50,5%], or others. We can partition the unit interval in many different ways, where each partitioning then corresponds to some level of granularity. Having several such levels, we can talk about a granular hierarchy [21,35,37]. However, following everyday experience, we propose the following systematic refinement procedure:

- 3-partitioning: This can be associated to the common classification into three categories, e.g. "'small","medium", and "large"
  - partitioning intervals: $[0, \frac{1}{3}), [\frac{1}{3}, \frac{2}{3}), [\frac{2}{3}, 1]$
- 5-partitioning: Five categories, say "tiny", "small", "medium", 'large", "huge"
  - partitioning intervals: $[0, \frac{1}{5}), \ldots, [\frac{4}{5}, 1]$
- 7-partitioning: E.g. "almost none","few","several","about half","most", "many","almost all"
  - partitioning intervals: $[0, \frac{1}{7}), \ldots, [\frac{6}{7}, 1]$
- tenner-partitioning: (About) $0\%, 10\%, 20\%, \ldots, 90\%, 100\%$
  - partitioning intervals: $[0, \frac{1}{20}), [\frac{1}{20}, \frac{3}{20}), \ldots, [\frac{17}{20}, \frac{19}{20}), [\frac{19}{20}, 1]$
- fiver-partitioning: (About) $0\%, 5\%, 10\%, 15\%, \ldots, 90\%, 95\%, 100\%$
  - partitioning intervals: $[0, \frac{1}{40}), [\frac{1}{40}, \frac{3}{40}), \ldots, [\frac{37}{40}, \frac{39}{40}), [\frac{39}{40}, 1]$
- oner-partitioning: (About) $0\%, 1\%, 2\%, 3\%, \ldots, 98\%, 99\%, 100\%$
  - partitioning intervals: $[0, \frac{1}{200}), [\frac{1}{200}, \frac{3}{200}), \ldots, [\frac{197}{200}, \frac{199}{200}), [\frac{199}{40}, 1]$
- decimal place-partitioning: (About) $0\%, 0.1\%, 0.2\%, 0.3\%, \ldots, 99.8\%, 99.9\%, 100\%$
  - partitioning intervals: $[0, \frac{1}{2000}), [\frac{1}{2000}, \frac{3}{2000}), \ldots, [\frac{1997}{2000}, \frac{1999}{2000}), [\frac{1999}{2000}, 1]$

All these classifications are, of course, somehow freely defined, and may hence be changed accordingly. To describe the semantics of some vague proportional quantifier Q, we need to fix a finite number of such levels of granularity, say $GL_1, \ldots, GL_{m_Q}$, $m_Q \in \mathbb{N}$, with respect to which we can evaluate respective statements. In the present case, for statements "about half (of the domain elements) fulfill property $\hat{F}$" we then have acceptance intervals[4] as follows:

- $[\frac{1}{3}, \frac{2}{3})$ (3-partitioning)
- $[\frac{2}{5}, \frac{3}{5})$ (5-partitioning)
- $[\frac{3}{7}, \frac{4}{7})$ (7-partitioning)
- $[45, 55)$ (tenner-partitioning)
- $[47.5, 52.5)$ (fiver-partitioning)
- $[49.5, 50.5)$ (oner-partitioning)
- $[49.95, 50.05)$ (decimal place-partitioning)

Definition 6. *A granularity level GL corresponds to a partitioning of the real unit interval $[0, 1]$ into finitely many disjoint intervals $z_1, \ldots, z_{m_{GL}}$, such that $\bigcup_{i=1}^{m_{GL}} z_i = [0, 1]$.*

Definition 7. *A vague proportional quantifier Q comes with a set $\{GL_1, \ldots, GL_{m_Q}\}$ of granularity levels, where $m_Q \in \mathbb{N}$, such that each such level has an unique acceptance interval for Q, i.e. for all granularity levels GL_i, $i \in \{1, \ldots, m_Q\}$ there is exactly one interval z_{Q,GL_i} of the corresponding partitioning such that it holds:*

$$v_I(Q_{GL_i} x \hat{F}) = (\Delta(\Pi x \hat{F}(x) \to \overline{z^+_{Q,GL_i}}))\&(\Delta(\overline{z^-_{Q,GL_i}} \to \Pi x \hat{F}(x)))$$

If for some fixed vague proportional quantifier Q and granularity level GL, $z_{Q,GL}$ is the acceptance interval for Q, we set $z^+_{Q,GL}, z^-_{Q,GL}$ the upper, and lower, boundary of the interval. Q_{GL_i} denotes the quantifier Q restricted to one particular granularity level GL_i.

As a game rule, we can express this definition in the following way:

Game Rule 7 ($R^{[\approx k]}_{GL,III}$)
*If **P** asserts $\mathrm{Q}x\hat{F}(x)$, then, if **O** attacks, $i \in \{1, \ldots, m_Q\}$ gets chosen randomly, and then **P** has to assert $(\Delta(\Pi x \hat{F}(x) \to \overline{z^+_{Q,GL_i}}))\&(\Delta(\overline{z^-_{Q,GL_i}} \to \Pi x \hat{F}(x)))$.*

The corresponding truth function is the following:

$$v_I(Qx\hat{F}(x)) = \frac{\sum_{i=1}^{m_Q} v_I(\Delta(\Pi x \hat{F}(x) \to \overline{z^+_{Q,GL_i}})\&\Delta(\overline{z^-_{Q,GL_i}} \to \Pi x \hat{F}(x)))}{m_Q}$$

The range of statements we can express now includes all those that start with quantifier expression like "about halt", "about a third", or "almost all". By means of combining them, using the logical $\vee$ connective, we can also express

[4] I.e. the statement is true if $Prop_x \hat{F}(x)$ is an element of this acceptance interval.

statements like "at least about half", or "at most about a third", by simply linking respective statements together. This allows for an even wider range of quantification than Type II, and hence augments the applicability enormously, as many natural language statements in real life are of this form. In a last remaining step we combine Type II and Type III quantifiers, and end up with the final Type IV quantifiers, which are able to systematically evaluate statements that are vaguely quantified and have vague scope formulas at the same time.

6 Type IV - combining it all together

As our game semantic framework analytically decomposes formulas down to atomic subformulas, quantification, with respect to formulas potentially build by vague atomic subformulas, is rather straightforward, as we only need to combine game rule 5 (or 6) with game rule 7 of the present paper. Hence we may restate game rule 7 with the only adjustment of dropping the hat of the formerly crisp scope formula $\hat{F}$.

Game Rule 8 ($R^{[\approx k]}_{GL,IV}$)
If ***P*** *asserts* $\mathrm{Q}xF(x)$*, then, if* ***O*** *attacks,* $i \in \{1, \ldots, m_Q\}$ *gets chosen randomly, and then* ***P*** *has to assert* $(\Delta(\Pi xF(x) \rightarrow \overline{z^{+}_{Q,GL_i}}))\&(\Delta(\overline{z^{-}_{Q,GL_i}} \rightarrow \Pi xF(x)))$.

If we want to determine the truth function, we find the following:

$$v_I(QxF(x)) = \frac{\sum_{i=1}^{m_Q} v_I(\Delta(\Pi xF(x) \rightarrow \overline{z^{+}_{Q,GL_i}})\&\Delta(\overline{z^{-}_{Q,GL_i}} \rightarrow \Pi xF(x)))}{m_Q}$$

We are now able to evaluate all sort of vague statements, like "At most about a third (of all domain elements) are nice", or "Almost all (of the domain elements) are friendly or tall", as long as the involved quantifiers and predicates are well defined. This allows for a great deal of flexibility, and particularly embeds evaluations into a neat logical machinery, which is fully linked to recent developments in the field of mathematical fuzzy logic [4].

7 Conclusion and Outlook

We defined a way to systematically evaluate natural language statement within an analytic game semantic framework. Our approach follows the hierarchy prescribed by Liu and Kerre and focuses first on crisp quantification with respect to crisp scope formulas. In a next step we described how vague predicates can be defined and show how we can quantify over vague formulas. We then introduce granular levels and define vague proportional quantification based on this notion, followed by a final step, where we combine all together.

The presented procedure may be extended into at least two important directions. One of them is the multi arity of quantifiers, as usually natural language statements are at least binary, as is the case with "Almost all children are friendly.". It has been pointed out [15] that even higher arities may be of importance, and hence we focus on this aspect in ongoing work. Another important augmentation is introducing non-proportional quantifiers, which may depend on intensional matters, like "many" and "few" [22,25]. These are closely linked to modal logics, but may also be integrated into the present game semantic setting as we will show in future work.

References

1. S. Aguzzoli, P. Codara, D. Valota, T. Flaminio, and B. Gerla. Querying with Łukasiewicz logic. In *Fuzzy Systems (FUZZ-IEEE), 2015 IEEE International Conference on*, pages 1–8. IEEE, 2015.
2. Andrzej Bargiela and Witold Pedrycz. *Granular computing: an introduction*, volume 717. Springer Science & Business Media, 2012.
3. J. Barwise and R. Cooper. Generalized quantifiers and natural language. *Linguistics and philosophy*, 4(2):159–219, 1981.
4. P. Cintula, C.G. Fermüller, and C. Noguera, editors. *Handbook of Mathematical Fuzzy Logic - Volume 3*. College Publications, 2015.
5. P. Cintula, P. Hájek, and C. Noguera, editors. *Handbook of Mathematical Fuzzy Logic*. College Publications, 2011.
6. Miguel Delgado, M Dolores Ruiz, Daniel Sánchez, and M Amparo Vila. Fuzzy quantification: a state of the art. *Fuzzy Sets and Systems*, 242:1–30, 2014.
7. C.G. Fermüller. Revisiting Giles's game. In *Games: Unifying logic, language, and philosophy*, pages 209–227. Springer, 2009.
8. C.G. Fermüller. Combining fuzziness and context sensitivity in game based models of vague quantification. In *Integrated Uncertainty in Knowledge Modelling and Decision Making - 4th International Symposium, IUKM 2015, Nha Trang, Vietnam, October 15-17, 2015, Proceedings*, pages 19–31, 2015.
9. C.G. Fermüller and R. Kosik. Combining supervaluation and degree based reasoning under vagueness. In *Logic for Programming, Artificial Intelligence, and Reasoning*, pages 212–226. Springer, 2006.
10. C.G. Fermüller and G. Metcalfe. Giles's game and the proof theory of Łukasiewicz logic. *Studia Logica*, 92(1):27–61, 2009.
11. C.G. Fermüller and C. Roschger. Randomized game semantics for semi-fuzzy quantifiers. In S. et.al. Greco, editor, *Advances in Computational Intelligence*, volume 300, pages 632–641. Springer, 2012.
12. C.G. Fermüller and C. Roschger. Randomized game semantics for semi-fuzzy quantifiers. *Logic Journal of the IGPL*, 223(3):413–439, 2014.
13. R. Giles. A non-classical logic for physics. *Studia Logica*, 33(4):397–415, 1974.
14. Kinga Glinka and Danuta Zakrzewska. Effective multi-label classification method for multidimensional datasets. In *Flexible Query Answering Systems 2015*, pages 127–138. Springer, 2016.
15. I. Glöckner. *Fuzzy quantifiers: A computational theory*, volume 193 of *Studies in Fuzziness and Soft Computing*. Springer Verlag, 2006.
16. Madan M Gupta, Rammohan K Ragade, and Ronald R Yager. *Advances in fuzzy set theory and applications*. North-Holland Publishing Company, 1979.
17. J. Hintikka. *Logic, language-games and information: Kantian themes in the philosophy of logic*. Clarendon Press Oxford, 1973.

18. J. Hintikka and G. Sandu. Game-theoretical semantics. In A. ter Meulen and J. van Benthem, editors, *Handbook of Logic and Language*, pages 361–410. Elsevier, 2010.
19. Rosanna Keefe and Peter Smith. Vagueness: A reader, m assachusetts, 1997.
20. E. L. Keenan and J. Stavi. A semantic characterization of natural language determiners. *Linguistics and Philosophy*, 9(3):253–326, 1986.
21. C Maria Keet. From granulation hierarchy to granular perspective. In *Granular Computing, 2009, GRC'09. IEEE International Conference on*, pages 306–311. IEEE, 2009.
22. S. Lappin. An intensional parametric semantics for vague quantifiers. *Linguistics & Philosophy*, 23(6):599–620, 2000.
23. Jonathan Lawry. A voting mechanism for fuzzy logic. *International Journal of Approximate Reasoning*, 19(3):315–333, 1998.
24. Y. Liu and E.E. Kerre. An overview of fuzzy quantifiers.(i). interpretations. *Fuzzy Sets and Systems*, 95(1):1–21, 1998.
25. M.F.J.Hofer. On the context dependence of many. In *Modeling and Using Context; 9th International and Interdisciplinary Conference, CONTEXT 2015 Larnaca, Cyprus, November 2-6, 2015 Proceedings*, pages 346–358, 2015.
26. V. Novák. Reasoning about mathematical fuzzy logic and its future. *Fuzzy Sets and Systems*, 192:25–44, 2012.
27. Zdzisław Pawlak. Rough sets and intelligent data analysis. *Information sciences*, 147(1):1–12, 2002.
28. Martín Pereira-Fariña, Juan C Vidal, F DÍaz-Hermida, and A BugarÍn. A fuzzy syllogistic reasoning schema for generalized quantifiers. *Fuzzy Sets and Systems*, 234:79–96, 2014.
29. S. Peters and D. Westerståhl. *Quantifiers in Language and Logic*. Oxford University Press, USA, 2006.
30. P.L. Peterson. *Intermediate Quantifiers; Logic, linguistics, and Aristotelian semantics*. Ashgate, 2000.
31. Andrzej Skowron and Zbigniew Suraj. *Rough Sets and Intelligent Systems-Professor Zdzisław Pawlak in Memoriam*, volume 2. Springer Science & Business Media, 2012.
32. Nicholas JJ Smith. *Vagueness and degrees of truth*. Oxford University Press, 2008.
33. J. van Benthem. *Logic in games*. MIT Press, 2014.
34. D. Westerståhl. Quantifiers in formal and natural languages. In *Handbook of philosophical logic*, pages 1–131. Springer, 1989.
35. Xibei Yang, Yuhua Qian, and Jingyu Yang. On characterizing hierarchies of granulation structures via distances. *Fundamenta Informaticae*, 123(3):365–380, 2013.
36. YY Yao. A comparative study of fuzzy sets and rough sets. *Information sciences*, 109(1):227–242, 1998.
37. YY Yao. Information granulation and rough set approximation. *International Journal of Intelligent Systems*, 16(1):87–104, 2001.
38. L. A. Zadeh. Fuzzy sets and information granularity. In *Advances in fuzzy set theory and applications*, pages 3–18. North-Holland Publishing Company, 1979.
39. L. A. Zadeh. A computational approach to fuzzy quantifiers in natural languages. *Computers & Mathematics with applications*, 9(1):149–184, 1983.

Proper names in interaction

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We present an argument that proper names need to be treated from a dialogical perspective and that they present a simple argument that language is a dynamic system in a state of flux with changes taking place in the language during the course of a dialogue. The analysis illuminates the nature of dialogue information states as well as their relation to other cognitive components such as long term memory and representation of the visual scene.

The analysis also gives us an interesting perspective on some old philosophical puzzles such as Kripke's discussion of the Paderewski example. In addition it points to a natural language technology in which we think in terms of linguistic systems in a state of flux and have transparent representations of information states that can be reasoned about and modified.

Conversation as a Game

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Abstract. Paul Grice (1975) proposed a theory of implicature that consists of a cooperative principle and four maxims, namely *maxim of quality*, *maxim of quantity*, *maxim of relation*, and *maxim of manner*. Sperber and Wilson (1986/1995[2]) replaced these maxims by a single principle of relevance; the principle says that the speaker tries to be as relevant as possible in the circumstances. In this presentation, we propose to interpret a conversation as a game and analyze conversational games based on a modification of *Dynamic Normative Logic* (DNL). In other words, we show how to analyze scalar implicatures and conversational implicatures based on a framework that is an extension of DNL.

Keywords: Paul Grice, dynamic normative logic, scalar implicature, conversational implicature

1 Dynamic Normative Logic and Its Modification

Dynamic Normative Logic (DNL)[1] is based on *Logic for Normative Systems* (LNS) that was proposed in Nakayama (2010, 2011). We extend LNS and define *BOD Logic*. We use **not**, **&**, ***or***, $\Rightarrow$, and $\Leftrightarrow$ as meta-language expressions of logical connectives.

Definition 1. We use *cons*(Γ) as an abbreviation of "Γ is consistent". Let *BB*, *OB*, and *DB* be consistent sets of FO-sentences (*i.e.* sentences of *First-Order Logic*). Let $BOD = \langle BB, OB, DB \rangle$. *BB*, *OB*, *DB* are called *belief base*, *obligation base*, and *desire base*, repeatedly. *BOD* is called a *BOD- system*.

(1a) [Deductive closure] $Cn(\Gamma) = \{\varphi : \varphi \text{ deductively follows from } \Gamma\}$.
(1b) [Belief] $\mathbf{B}_{BOD}\, \varphi \Leftrightarrow_{\text{def}} \varphi \in Cn(BB)$.
(1c) [Possibility] $\mathbf{M}_{BOD}\, \varphi \Leftrightarrow_{\text{def}} cons(BB \cup \{\varphi\})$.
(1d) [Obligation] $\mathbf{O}_{BOD}\, \varphi \Leftrightarrow_{\text{def}}$
$(cons(BB \cup OB)\ \mathbf{\&}\ \varphi \in Cn(BB \cup OB)\ \mathbf{\&}\ \textbf{\textit{not}}\ (\varphi \in Cn(BB)))$.
(1e) [Prohibition] $\mathbf{F}_{BOD}\, \varphi \Leftrightarrow_{\text{def}} \mathbf{O}_{BOD}\, \neg\varphi$.
(1f) [Permission] $\mathbf{P}_{PB}\, p \Leftrightarrow_{\text{def}} (cons(BB \cup OB \cup \{\varphi\})\mathbf{\&}\ \textbf{\textit{not}}\ (\varphi \in Cn(BB)))$.

[1] DNL was proposed in Nakayama (2014, 2015).

(1g) [Desire] $\mathbf{D}_{BOD}\ \varphi \Leftrightarrow_{\text{def}}$

$(cons(BB \cup DB)$ **&** $\varphi \in Cn(BB \cup DB)$ **&** ***not*** $(\varphi \in Cn(BB)))$.

(1h) BOD is consistent $\Leftrightarrow_{\text{def}}$ $(cons(BB \cup OB)$ **&** $cons(BB \cup DB))$.

(1i) $mod(T) =_{\text{def}} \{M : M \text{ is a FO-model of } T\}$.

Now, we show how to read main BOD-sentences.

(2a) $\mathbf{B}_{BOD}\ \varphi$: It is believed in *BOD* that φ.
(2b) $\mathbf{M}_{BOD}\ \varphi$: It is believed to be possible in *BOD* that φ.
(2c) $\mathbf{O}_{PB}\ \varphi$: It is obligated in *BOD* that φ.
(2d) $\mathbf{F}_{PB}\ \varphi$: It is forbidden in *BOD* that φ.
(2c) $\mathbf{P}_{PB}\ \varphi$: It is permitted in *BOD* that φ.
(2f) $\mathbf{D}_{PB}\ \varphi$: It is desired in *BOD* that φ.

The following list shows some of valid BOD-sentences[2].

Proposition 1. The following sentences are main meta-logical theorems of BOD Logic. Here, we assume $BOD = \langle BB, OB, DB \rangle$.

(3a) $(\mathbf{X}_{BOD}\ (\varphi \rightarrow \psi)\ \&\ \mathbf{X}_{BOD}\ \varphi) \Rightarrow \mathbf{X}_{BOD}\ \psi$, when $\mathbf{X} = \mathbf{B}$ or $\mathbf{X} = \mathbf{O}$ or $\mathbf{X} = \mathbf{D}$.
(3b) $(\mathbf{X}_{BOD}\ (\varphi \rightarrow \psi)\ \&\ \mathbf{B}_{BOD}\ \psi) \Rightarrow \mathbf{X}_{BOD}\ \psi$, when $\mathbf{X} = \mathbf{O}$ or $\mathbf{X} = \mathbf{D}$.
(3c) $(\mathbf{X}_{BOD}\ \forall x_{1\ldots} \forall x_{\text{n}}\ (\varphi(x_1,\ldots, x_{\text{n}}) \rightarrow \psi(x_1,\ldots, x_{\text{n}}))\ \&\ \mathbf{B}_{BOD}\ \varphi(a_1,\ldots, a_{\text{n}})$ **&** ***not*** $\mathbf{B}_{BOD}\ \psi(a_1,\ldots, a_{\text{n}})) \Rightarrow \mathbf{X}_{BOD}\ \psi(a_1,\ldots, a_{\text{n}})$, when $\mathbf{X} = \mathbf{O}$ or $\mathbf{X} = \mathbf{D}$.
(3d) $(\mathbf{X}_{NS}\ \exists x_{1\ldots} \exists x_{\text{n}}\ (\varphi(x_1,\ldots, x_{\text{n}}) \wedge \psi(x_1,\ldots, x_{\text{n}}))\ \&\ \mathbf{B}_{NS}\ \varphi(a_1,\ldots, a_{\text{n}})\ \&$ ***not*** $\mathbf{B}_{NS}\ \neg\psi(a_1,\ldots, a_{\text{n}})) \Rightarrow \mathbf{X}_{NS}\ Q(a_1,\ldots, a_{\text{n}})$, when $\mathbf{X} = \mathbf{F}$.

Proof. See Nakayama (2015).

In DNL, the belief base can be updated. Here, we also update BOD-systems and call this framework BOD-DL. A BOD-system in BOD-DL involves information about its stage. We write a BOD-system of BOD-DL as follows: $BOD(n) = \langle BB(n), DB(n), DB(n) \rangle$. Note that any BOD-system in BOD-DL is a BOD-system in BOD Logic.

2 Ascription of Propositional States and Mutual Belief

In BOD-DL, we can imitate ascriptions of propositional states. We introduce the following notations.

[2] 'Valid' means here the validity in the First-Oder Logic (FOL).

Definition 2.

(4a) $BB(A>B,n) =_{\text{def}} \{\varphi : A$ believes in stage n that B believes that $\varphi\}$.

(4b) $OB(A>B,n) =_{\text{def}} \{\varphi : A$ believes in stage n that B believes that it is obligated that $\varphi\}$.

(4c) $DB(A>B,n) =_{\text{def}} \{\varphi : A$ believes in stage n that B desires that $\varphi\}$.

(4d) A BOD-system for ascription of A to B in stage n is defined as in Definition 1: $BOD(A>B,n) = \langle BB(A>B,n), OB(A>B,n), DB(A>B,n)\rangle$.

(4e) $\mathbf{B}_{BOD(A>B,n)}\ \varphi$, $\mathbf{O}_{BOD(A>B,n)}\ \varphi$, and $\mathbf{D}_{BOD(A>B,n)}\ \varphi$ are defined in the same way as in Definition 1.

(4f) $\mathbf{K}_{BOD(A>B,n)}\ \varphi \Leftrightarrow_{\text{def}} (\mathbf{B}_{BOD(A>B,n)}\ \varphi\ \ \&\ \mathbf{B}_{BOD(B,n)}\ \varphi)$.

(4g) $\mathbf{K}^{\mathbf{D}}_{BOD(A>B,n)}\ \varphi \Leftrightarrow_{\text{def}} (\mathbf{D}_{BOD(A>B,n)}\ \varphi\ \ \&\ \mathbf{D}_{BOD(B,n)}\ \varphi)$.

We read $\mathbf{K}_{BOD(A>B,n)}\ \varphi$ as follows: A knows in stage n that B believes that φ. Similarly, $\mathbf{K}^{\mathbf{D}}_{BOD(A>B,n)}\ \varphi$ means that A knows in stage n that B desires that φ.

We assume that a common belief implies a corresponding mutual belief in general.[3]

Assumption 1. Let $G = \{A, B\}$. Suppose that $\mathbf{X} = \mathbf{B}$ or $\mathbf{X} = \mathbf{O}$ or $\mathbf{X} = \mathbf{D}$. Then, the following sentence holds.

$$\mathbf{X}_{BOD(G,n)}\ \varphi \Rightarrow$$
$$(\mathbf{X}_{BOD(A,n)}\ \varphi\ \&\ \mathbf{X}_{BOD(B,n)}\ \varphi\ \&\ \mathbf{X}_{BOD(A>B,n)}\ \varphi\ \&\ \mathbf{X}_{BOD(B>A,n)}\ \varphi).$$

We can easily show the following proposition.

Proposition 2. Here, we assume that $BOD = \langle BB, OB, DB\rangle$ and that Assumption 1 is satisfied. Then, the following BOD-sentences hold.

(5a) $\mathbf{B}_{BOD(G,\ n)}\ \varphi \Rightarrow (\mathbf{K}_{BOD(A>B,\ n)}\ \varphi\ \&\ \mathbf{K}_{BOD(B>A,\ n)}\ \varphi)$.

(5b) $\mathbf{D}_{BOD(G,\ n)}\ \varphi \Rightarrow (\mathbf{K}^{\mathbf{D}}_{BOD(A>B,\ n)}\ \varphi\ \&\ \mathbf{K}^{\mathbf{D}}_{BOD(B>A,\ n)}\ \varphi)$.

3 Gricean Implicatures in BOD-DL

The Gricean Theory consists of a cooperative principle and four maxims.

(6a) [Cooperative Principle] Contribute what is required by the accepted purpose of the conversation.

(6b) [Maxim of Quality] Make your contribution true; so do not convey what you believe false or unjustified.

(6c) [Maxim of Quantity] Be informative as required.

[3] We consider that $\mathbf{O}_{(G,n)}\ \varphi$ and $\mathbf{D}_{(G,n)}\ \varphi$ are kinds of mutual beliefs. For example, we read $\mathbf{O}_{(G,n)}\ \varphi$ as "It is a common belief in G that it is obligated that φ".

(6d) [Maxim of Relation] Be relevant.
(6e) [Maxim of Manner] Be perspicuous; so avoid obscurity and ambiguity, and strive for brevity and order.

In this paper, we propose to interpret Gricean conversational rules as normative requirements for communication partners (see Assumption 2). Furthermore, we interpret the Gricean rules as rules in a conversational game. To express the maxim of quality within BOD-DL, we need a relation of *being more informative*.

Definition 3.
(7a) [Definition of Proper Subset] $X \subset Y \Leftrightarrow_{\text{def}} (X \subseteq Y \;\&\; X \neq Y)$.
(7b) $\textit{more-informative}(X, Y) \Leftrightarrow_{\text{def}}$
$(\textit{cons}(X)$ **&** $\textit{cons}(Y)$ ***&*** $\textit{Cn}(Y) \subset \textit{Cn}(X))$.
(7c) $\textit{more-informative}_{[T]}(\varphi, \psi) \Leftrightarrow_{\text{def}}$
not $(\psi \in \textit{Cn}(T))$ **&** $\textit{more-informative}(T \cup \{\varphi\}, T \cup \{\psi\})$.

Usually, we use *more-informative* with respect to a belief base, for example $\textit{more-informative}_{[BB(H)]}(\varphi, \psi)$. Note that the degree of information depends on the belief state of communication partners. If *S* knows that φ (thus, $\varphi \in \textit{Cn}(\textit{BB}(S))$), then φ is not informative for *S*. However, the same information φ can be informative for *H*, when *H* does not know that φ (thus, ***not*** $(\varphi \in \textit{Cn}(\textit{BB}(H)))$.

Let *tr* be a translation function that translates a FOL-sentence into an English sentence. Now, we can interpret Gricean maxims as obligations.

Assumption 2. Here, we assume that *OB*(*S*) denotes the obligation base of a speaker. In this paper, we call *S* a *Gricean speaker*, when *OB*(*S*), *BB*(*S*) and *BB*(*S*>*H*) satisfy conditions (8a), (8b), and (8c). We call *H* a *Gricean hearer*, when *OB*(*H*>*S*), *BB*(*H*>(*H*>*S*)), and *BB*(*H*>*S*) satisfy conditions (8d), (8e), and (8f). Furthermore, we call *A* a *Gricean communication partner*, when *A* is both a Gricean speaker and hearer. We consider a Gricean communication partner as a player of Gricean conversational games.

(8a) [Maxim of Quality 1 for *S*] $(\varphi \rightarrow \neg \textit{say}(S, \textit{tr}(\neg\varphi)))$ is an element of *OB*(*S*).
(8b) [Maxim of Quality 2 for *S*] $(\textit{say}(S, \textit{tr}(\varphi)) \rightarrow \varphi)$ is an element of *BB*(*S*).
(8c) [Maxim of Quantity for *S*] If $\textit{more-informative}_{[BB(S>H)]}(\varphi, \psi)$, then $((\varphi \wedge \psi) \rightarrow \textit{say}(\mathrm{S}, \textit{tr}(\varphi)))$ is an element of *OB*(*S*).
(8d) [Maxim of Quality 1 for *H*] $(\varphi \rightarrow \neg \textit{say}(S, \textit{tr}(\neg\varphi)))$ is an element of *OB*(*H*>*S*).
(8e) [Maxim of Quality 2 for *H*] $(\textit{say}(S, \textit{tr}(\varphi)) \rightarrow \varphi)$ is an element of

$BB(H{>}S)$.

(8f) [Maxim of Quantity for H] If $\textit{more-informative}_{[BB((H>(S>H))]}(\varphi, \psi)$, then $((\varphi \wedge \psi) \rightarrow \textit{say}(S, \textit{tr}(\varphi)))$ is an element of $OB(H{>}S)$.

Based on BOD-DL, the following sentences follow from these requirements.

Proposition 3. If both S and H are Gricean communication partners, then the following BOD-sentences hold.

(9a) $\mathbf{B}_{BOD(S)}\ \varphi \Rightarrow \mathbf{F}_{BOD(S)}\ \textit{say}(S, \textit{tr}(\neg\varphi))$. (If S believes φ, then it is forbidden for S to say $\textit{tr}(\neg\varphi)$.)

(9b) $\mathbf{B}_{BOD(S)}\ \textit{say}(S, \textit{tr}(\varphi)) \Rightarrow \mathbf{B}_{BOD(S)}\ \varphi$. (If S believes that S says $\textit{tr}(\varphi)$, then S believes φ.)

(9c) $\textit{more-informative}_{[BB(S>H)]}(\varphi, \psi) \Rightarrow (\mathbf{B}_{BOD(S)}\ (\varphi \wedge \psi) \Rightarrow \mathbf{O}_{BOD(S)}\ \textit{say}(S, \textit{tr}(\varphi))$. (If S believes that φ is more informative for H than ψ, then [if S believes $(\varphi \wedge \psi)$, then it is obligated for S to say $\textit{tr}(\varphi)$.)

(9d) $\mathbf{B}_{BOD(H>S)}\ \varphi \Rightarrow \mathbf{F}_{BOD(H>S)}\ \textit{say}(S, \textit{tr}(\neg\varphi))$. (If H believes that S believes φ, then H believes that it is forbidden for S to say $\textit{tr}(\neg\varphi)$.)

(9e) $\mathbf{B}_{BOD(H>S)}\ \textit{say}(S, \textit{tr}(\varphi)) \Rightarrow \mathbf{B}_{BOD(H>S)}\ \varphi$. (If H believes that S believes that S said $\textit{tr}(\varphi)$, then H believes that S believes φ.)

(9f) $\textit{more-informative}_{[BB((H>(S>H))]}(\varphi, \psi) \Rightarrow (\mathbf{B}_{BOD(H>S)}\ (\varphi \wedge \psi) \Rightarrow \mathbf{O}_{\ BOD(H>S)}\ \textit{say}(S, \textit{tr}(\varphi))$. (If H believes that S believes that φ is more informative for H than ψ, then [if H believes that S believes $(\varphi \wedge \psi)$, then H believes that it is obligated for S to say $\textit{tr}(\varphi)$.)

(9g) [Secondary Implicature[4] for Belief] $(\textbf{\textit{not}}\ \mathbf{B}_{BOD(S>H,n)}\ \varphi\ \mathbf{\&}\ (\mathbf{B}_{BOD(S>H,n)}\ \varphi\ \textbf{\textit{or}}\ \mathbf{B}_{BOD(S>H,n)}\ \neg\varphi)) \Rightarrow \mathbf{B}_{BOD(S>H,n)}\ \neg\varphi$.

Proof. (9a) follows from (8a) and Definition 1, and (9b) follows from (8b) and Definition 1, and (9c) follows from (8c) and Definition 1. (9d), (9e), and (9f) can be proved in the same way as (9a), (9b), and (9c). (9g) follows from the classical logic. Q.E.D.

The maxim of relation and the maxim of manner can be also interpreted as obligations for communication partners. In this paper, we do not intensively discuss them, because they are heavily context sensitive.

4 Scalar Implicature

The scalar implicature is a paradigmatic application case for the Gricean theo-

[4] The notion *secondary implicature* is proposed by Sauerland (2004). Here, we modified Saueland's definition. For Sauerland's approach, see Section 4.1 of this paper.

ry. In this section, we briefly summarize a standard neo-Gricean approaches to Scalar Implicatures, then propose to reformulate this problem as a game.

4.1 Sauerland's Approach to Scalar Implicatures[5]

In this section, we summarize Sauerland's approach to scalar implicatures. Sauerland's approach is based on an approach of Gazdar (1979: pp. 57-61).[6] Gazdar's approach is applicable only to sentences φ where an expression from a quantitative scale α appears not in the scope of any logical operator. This can be expressed as $\varphi = f(\alpha)$. If α is not the maximal item on its quantative scale Q, then Gazdar's mechanism predicts an implicature for φ. Let α' be the word hat is following α on Q. In this case, the mechanism forms $\varphi = f(\alpha')$ by replacing α with α' in φ. Finally, the scalar impliactures of φ generated are all expressions $\mathbf{B}\neg\psi$ for any ψ that can be derived in the way just described.[7]

Sauerland (2004) points out that Gazdar's approach predicts many wrong results when it is applied to logically complex sentences. Furthermore, Soames (1982: p. 521) and Horn (1989: p. 432) criticize the epistemic commitment assigned to the implicature by Gazdar's proposal. Both Soams and Horn argue that it only follows from Gricean maxims of conversation that ψ is uncertain, rather than that ψ is certainly false as Gazdar claims. In other worse, they point out that what is implicated is $\neg\mathbf{B}\psi$ rather than $\mathbf{B}\neg\psi$. $\mathbf{B}\neg\psi$ would follow from $\neg\mathbf{B}\psi$, only if some additional knowledge is assumed, for example the knowledge that $\neg\mathbf{B}\psi \vee \neg\mathbf{B}\psi$ holds (Sauerland 2004: p. 383).

Following Soams and Horn, Sauerland (2004) distinguish two kinds of implicatures. A *primary implicature* is expressed by $\neg\mathbf{B}\psi$ and a *secondary implicature* is expressed by $\mathbf{B}\neg\psi$. These two implicatures are characterized as follows.

(10a) If ψ is an element of the set of scalar alternatives of φ and $(\psi \Rightarrow \varphi)$ and not $(\varphi \Rightarrow \psi)$, then $\neg\mathbf{B}\psi$ is a *primary implicature* of φ.

(10b) If $\neg\mathbf{B}\psi$ is a primary implicature of φ and $\mathbf{B}\neg\psi$ is consistent with the conjunction of φ and all primary implicatures of φ, then $\mathbf{B}\neg\psi$ is a *secondary implicature* of φ.

As Sauerland (2004) points out, to reason for a secondary implicature $\mathbf{B}\neg\psi$, one has to assume that the speaker obeys all the Gricean maxims to derive

[5] This section is added after a suggestion given by one of the reviewers.
[6] See Sauerland (2004: p. 368f).
[7] In Gazdar's original text, $\mathbf{K}\neg\psi$ is used to express that t*he speaker is certain that* ψ *is false*. We replaced $\mathbf{K}\neg\psi$ by $\mathbf{B}\neg\psi$, because our paper uses $\mathbf{B}\neg\psi$ to express that t*he speaker is certain that* ψ *is false*.

$\neg\mathbf{B}\psi$ and furthermore that $\mathbf{B}\psi \vee \mathbf{B}\neg\psi$ or some stronger assumption is justified.

Spector (2007: p. 228f) points out that Sauerland's definition of the set of scalar alternative is *ad hoc*. To identify implicatures of (p *or* q) Sauerland introduced two binary connectors c_L and c_R such that ($p\ c_L\ q$) is equivalent to p, and ($p\ c_R\ q$) is equivalent to q, and then stipulates the following scale: $\langle$*or*, *and*, c_L, $c_R\rangle$. In the next section, we show that our approach does not need any such special binary connectors to explain implicatures of disjunctions.

4.2 Interpretation of Scalar Implicartures within BOD-DL

Let us begin with a simple example of quantification.

(11) S says to H in stage 1: "Some athletes smoke".

We translate this sentence as $\exists x\ (athlete(x) \wedge smoke(x))$ and abbreviate it as *some-smoker*. Here, we assume that both S and H are Gricean communication partners. After S's assertion of (11), we set: $BB(S, 2) = BB(S, 1) \cup \{say(S, tr(some\text{-}smoker))\}$ & $BB(H{>}S, 2) = BB(H{>}S, 1) \cup \{say(S, tr(some\text{-}smoker))\}$. Thus, we have $\mathbf{K}_{BOD(H>S,\,2)}\, say(S, tr(some\text{-}smoker))$. Then, because of (9b) and (9e), $\mathbf{K}_{BOD(H>S,\ 2)}$ *some-smoker*. Here, we abbreviate $\forall x\ (athlete(x) \rightarrow smoke(x))$ as *all-smoker*. Because (*all-smoker* $\rightarrow$ *some-smoker*) is a FOL-theorem, according to (7c) and (9f), $\mathbf{B}_{(H>S,\,2)}$ *all-smoker* $\Rightarrow$ $\mathbf{O}_{(H>S,\,2)}\, say$ (S, "All athletes smoke"). However, because H knows that S makes statement (11), ***not*** $\mathbf{B}_{(H>S,1)}$ *all-smoker*. Thus, H believes that S's statement (11) implicates that S does not know whether all athletes smoke. This is a primary implicature of (11). In a case that H is sure that S believes *all-smoker* or S believes $\neg$*all-smoker*, because of (9d), we obtain $\mathbf{B}_{(H>S,2)}\,\neg$*all-smoker*. This is a secondary implicature of (11).

Now, let us take S's statement of (p *or* q) as an example. Let us assume that both S and H are Gricean communication partners in sense of Assumption 2. We set: $BB(S, 2) = BB(S, 1) \cup \{say(S, (p\ or\ q))\}$ & $BB(H{>}S, 2) = BB(H{>}S, 1) \cup \{say(S, (p\ or\ q))\}$.Then, because of (9e), we can easily show that ($\mathbf{B}_{BOD(H>S,2)}$ $(p \vee q)$. Furthermoe, according to (9f), ***not*** $\mathbf{B}_{BOD(H>S,2)}\, p$ **&** ***not*** $\mathbf{B}_{BOD(H>S,2)}\, q$ **&** ***not*** $\mathbf{B}_{(H>S,2)}\,(p \wedge q)$). Now, suppose $\mathbf{B}_{(H>S,2)}\,\neg p$. Then, it holds: $\mathbf{B}_{(H>S,2)}\, q$, because $((p \vee q) \wedge \neg p) \rightarrow q)$ is a FOL-theorem. Then, according to (9f), $\mathbf{O}_{BOD(H>S,2)}\ say(S, tr(q))$. However, because S did not say $tr(q)$, we can infer ***not*** $\mathbf{B}_{BOD(H>S,2)}\,\neg p$. In the same way, we obtain ***not*** $\mathbf{B}_{BOD(H>S,2)}\,\neg q$. This means, ***not*** $\mathbf{B}_{BOD(H>S,2)}\, p$ and ***not*** $\mathbf{B}_{BOD(H>S,2)}\, q$ are primary implicatures, but $\neg p$ and $\neg q$ cannot be secondary implicatures of (p *or* q). We have these consequences purely from the Gricean maxims. In this inference, we had no need for special connectors such as c_L or c_R, which Sauerland (2004) introduced to block the possibility of secondary implicatures of $\neg p$ and $\neg q$. As a result, only $\neg(p \wedge q)$ can be a possible secondary implicature. When $\mathbf{B}_{BOD(H>S,2)}\ \neg(p \wedge q)$ ***or*** $\mathbf{B}_{BOD(H>S,2)}\ (p \wedge q)$, we obtain $\mathbf{B}_{BOD(H>S,2)}\ \neg(p \wedge q)$, which is a primary implicature of saying "(p *or* q)".

As the next example, let us consider the following sentence from Sauerland (2004: p. 374).

(12) It's not the case that Paul ate all of the eggs.

This sentence can be expressed as $\neg\forall x\ (egg(x) \rightarrow ate(Paul, x))$, which is abbreviated as *not-all-eggs*. We abbreviate $\neg\exists x\ (egg(x) \wedge ate(Paul, x))$ as *no-egg* and $\exists x\ (egg(x) \wedge ate(Paul, x))$ as *some-eggs*. Note that it holds in FOL: *no-egg* $\rightarrow$ *not-all-eggs*. Now, we assume that the speaker S of (12) and the hearer H are Gricean communication partners in sense of Assumption 2 and they know this each other. We set: $BB(S, 2) = BB(S, 1)\cup\{say(S, (12))\}$ & $BB(H>S, 2) = BB(H>S, 1)\cup \{say(S, (12))\}$. In this situation, because of (9e) and (9f), we obtain: ($\mathbf{B}_{BOD(H>S,2)}$ *not-all-eggs* **&** ***not*** $\mathbf{B}_{BOD(H>S,2)}$ *no-egg*). Thus, ***not*** $\mathbf{B}_{BOD(H>S,2)}$ *no-egg* is a primary implicature. In the case, in which ($\mathbf{B}_{BOD(H>S,2)}$ *no-egg* ***or*** $\mathbf{B}_{BOD(H>S,2)}$ $\neg$*no-egg*) holds, because of (9f), we have $\mathbf{B}_{BOD(H>S,2)}$ $\neg$*no-egg*. Then, we obtain $\mathbf{B}_{BOD(H>S,2)}$ *some-eggs*, because ($\neg$*no-egg* $\leftrightarrow$ *some-eggs*) is a FOL-theorem. Here, $\mathbf{B}_{BOD(H>S,2)}$ *some-eggs* is a secondary implicature of (12).

To obtain the same result, Sauerland (2004: p. 373f) distinguishes an upward and downward operating environment. As we have just seen, to explain implicatures in combination of negation and quantifiers, we do not need such an additional assumption.

4.3 Implicatures and Background Knowledge

As Sauerland (2004: p. 309) points out, Chierchia's criticism against Gricean approaches can be defended by taking a background knowledge into consideration. See the following example of Sauerland (2004: p. 309).

(13a) Every student wrote a paper or made a classroom presentation.
(13b) $\forall x\ (student(x) \rightarrow (wrote\text{-}paper(x) \vee made\text{-}presentation(x))$.

Sauerland (2004: p. 310) suggests that implicatures of (13a) depends on our background knowledge. Without special background knowledge, the following sentence seems to be implied by (13a).

(14a) Not every student wrote a paper and made a classroom presentation.
(14b) $\neg\forall x\ (student(x) \rightarrow (wrote\text{-}paper(x) \wedge made\text{-}presentation(x))$.

Indeed, according to our approach, because of (9f) and (9g), (14b) can be accepted as a secondary implicature, when (14b) or its negation is believed. According to Chierchia (2004: p.50), (13a) has the following implicature in a neutral context.

(15a) Every student wrote either a paper or made a presentation but not both.

(15b) We abbreviate $\neg\forall x$ ($student(x) \rightarrow \neg(wrote\text{-}paper(x) \wedge made\text{-}presentation(x))$ as *nobody-both*.

(15c) (13b) $\wedge$ *nobody-both*..

However, when we already know that some of students made both assignments, implicature of (15c) is canceled (Chierchia 2004: p.50).

These observations by Sauerland (2004) and Chierchia (2004) can be supported by our approach. Suppose that two persons *H1* and *H2* try to interpret sentence (13a) asserted by speaker *S*. We assume that these three persons are Gricean communication partners in sense of Assumption 2. *H1* does not know that some of students made both assignments, but *H2* knows it (We abbreviate $\exists x$ ($student(x) \wedge wrote\text{-}paper(x) \wedge made\text{-}presentation(x)$) as *both*). Furthermore, we assume that *H1* believes that all students are lazy and they do not do anything more than required (We abbreviate this statement *lazy-students*). Thus, we have: $lazy\text{-}students \in Cn(BB(H1, 1))$ & $both \in Cn(BB(H2, 1))$ & not ($both \in Cn(BB(H1, 1))$) & not ($lazy\text{-}students \in Cn(BB(H2, 1))$). We assume: $lazy\text{-}students \in Cn(BB(H1>S, 1))$ & $both \in Cn(BB(H2>S,1))$ & not ($both \in Cn(BB(H1>S, 1))$) & not ($lazy\text{-}students \in Cn(BB(H2>S, 1))$).

Firstly, we analyze a belief update of $BB(H1>S)$. We set: $BB(H1, 2) = BB(H1, 1) \cup \{(13b)\}$ & $BB(H1>S, 2) = BB(H1>S, 1) \cup \{(13b)\}$. We assume: ($lazy\text{-}students \wedge$ (13b)) $\rightarrow$ *nobody-both*. Thus, $\mathbf{B}_{BOD(H1>S,2)}$ ((13b) $\wedge$ *nobody-both*). Because (*nobody-both* $\rightarrow$ (14b)) is a FOL-theorem, we obtain: $\mathbf{B}_{BOD(H1>S,2)}$ (14b). Here, (14b) is not an implicatue but a result of a straightforward inference from belief base of *H1* at stage 2.

Now, let us analyze a belief update of $BB(H2>S)$. We set: $BB(H2, 2) = BB(H2, 1) \cup \{(13b)\}$ & $BB(H2>S, 2) = BB(H2>S, 1) \cup \{(13b)\}$. Here, we have: $\mathbf{B}_{BOD(H2>S,2)}$ ((13b) $\wedge$ *both*). Because ($both \rightarrow \neg nobody\text{-}both$) is a FOL-theorem, it holds: $\mathbf{B}_{BOD(H2>S,2)}$ $\neg$(15c). When ($\mathbf{B}_{BOD(H2>S,2)}$ (14b) *or* $\mathbf{B}_{BOD(H2>S,2)}$ $\neg$(14b)), we obtain $\mathbf{B}_{BOD(H2>S,2)}$ (14b), which is a secondary implicature of (13b).

This analysis shows that an analysis of implicature should be combined with a presupposition of background knowledge and that BOD-DL is an appropriate framework for dealing with this problem.

5 Conversation as a Game

Let us analyze some examples of short conversations. The following example is an example of speaker implicature from Davis (2014: sect. 1). In this example, Alan asks a question and Barb answers to it.

(16a) Alan: Are you going to Paul's party?
(16b) Barb: I have to work.

Let us assume that Alan and Barb are Gricean communication partners. Here, we introduce some additional assumptions which deal with expressions about desire.

Assumption 3. When S and H are Grican communication partners, then the following conditions are satisfied.

(17a) [Maxim of Quality 2 for S′s Desire] $(say(S, tr(\mathbf{D}_{BOD(S,\,k)}\,\varphi)) \to \varphi)$ is an element of $DB(S, k)$.

(17b) [Maxim of Quality 2 for H′s Attribution of S′s Desire] $(say(S, tr(\mathbf{D}_{BOD(S,\,k)})\,\varphi) \to \varphi)$ is an element of $DB(H{>}S, k)$.

From Assumption 3, we have the following consequence.

Proposition 4. If both S and H are Gricean communication partners and $BOD(S, k)$ and $BOD(H{>}S, k)$ are consistent, then the following BOD-sentence hold.

(18) $(\textbf{\textit{not}}\ \mathbf{B}_{BOD(S,\,k)}\,\varphi\ \mathbf{\&}\ \textbf{\textit{not}}\ \mathbf{B}_{BOD(H>S,\,k)}\,\varphi\ \mathbf{\&}\ \mathbf{K}_{BOD(H>S,\,k)}\ say(S, tr(\mathbf{D}_{BOD(S,\,k)}\,\varphi)))$
$\Rightarrow \mathbf{K^D}_{BOD(H>S,\,k)}\ \varphi$.

Proof. We assume that both S and H are Gricean communication partners and $BOD(S, k)$ and $BOD(H{>}S, k)$ are consistent. Then, (18) follows from (1b), (1g), (4f), (4g), (17a), and (17b). Q.E.D.

In BOD-DL, this conversation can be interpreted in the following way. Here, we assume A = Alan, B = Barb, and $G = \{A, B\}$. We assume that both Alan and Bob are Gricean communication partners.

The goal of this game is for Alan to know if Barb wants to go Paul's party. This goal can be formally expressed as follows: $\exists k\ (\mathbf{K^D}_{BOD(A>B,k)}\ going\text{-}party(B)\ \textbf{\textit{or}}\ \mathbf{K^D}_{BOD(A>B,k)}\ \neg going\text{-}party(B))$. In the first stage of the conversation, Alan does not know Barb's intention. Therefore, Alan asks question (16a). Here, we assume that Alan and Barb mutually believe that work and participation in a party are incompatible, formally: $\mathbf{B}_{BOD(G,\ 1)}$

$\neg\exists x$ (*work*(x) $\wedge$ *going-party*(x)). Then, from (5a) and Assumption 1, we have: $\mathbf{K}_{BOD(A>B,1)}$ $\neg\exists x$ (*work*(x) $\wedge$ *going-party*(x)). After Barb's answer (16b), we obtain: $\mathbf{K}_{BOD(A>B,\,2)}$ *say*(S, *tr*($\mathbf{D}_{BOD(B,\,2)}$ *work*(B))). Then, according to Proposition 4, we have: $\mathbf{K^D}_{BOD(A>B,\,2)}$ *work*(B))). Because we can assume that ***not*** $\mathbf{B}_{BOD(A>B,1)}$ $\neg$*going-party*(B)) **&** ***not*** $\mathbf{B}_{BOD(B,1)}$ $\neg$*going-party*(B)), we obtain: $\mathbf{K^D}_{BOD(A>B,\,2)}$ $\neg$*going-party*(B))). So, Alan knows in stage 2 that Barb is not going to the party. This game ends here, because the initial goal has been achieved.

Next, let us consider the following conversation from Sperber and Wilson (1995: p. 34).

(19a) Peter: Do you want some coffee?
(19b) Mary: Coffee would keep me awake.

The goal for Peter is serving coffee for Mary, if she wants. So, Peter needs to know if Mary wants some coffee. There are two possible implicatures of (19b) depending on their mutual belief. We assume P = Peter, M = Mary, and $G = \{P, M\}$. We assume that Peter and Mary mutually believe that being awake is incompatible with sleeping: $\mathbf{B}_{BOD(G,1)}$ $\neg\exists x$ (*awake*(x) $\wedge$ *sleep*(x)). Then, from Assumption 1 and Proposition 2, we have: $\mathbf{K}_{BOD(P>M,1)}$ $\neg\exists x$ (*awake*(x) $\wedge$ *sleep*(x)). Furthermore, we assume that P and M are Gricean communication partners.

[Case 1] Peter and Mary mutually believe that Mary wants to be awake: $\mathbf{D}_{BOD(G,1)}$ *awake*(M). Then, from Assumption 1 and Proposition 2, we have: $\mathbf{K^D}_{BOD(P>M,\,1)}$ *awake*(M). After Mary's answer (19b), $\mathbf{D}_{BOD(G,\,2)}$ *say*(M, (19b)). Then, because of (4f), (9b), (9e), and Proposition 2, we have: $\mathbf{K}_{BOD(P>M,\,2)}$ (*drink*(M, *coffee*) $\rightarrow$ *awake*(M)). Here, we assume that Peter and Mary mutually believe that drinking coffee is the best method for being awake in this situation.[8] Then, we have: $\mathbf{D}_{BOD(P>M,2)}$ *drink*(M, *coffee*). So, Peter ascribes to Mary a desire of drinking coffee. Thus, Peter will serve Mary coffee and the game ends.

[Case 2] Peter and Mary mutually believe that Mary wants to sleep: $\mathbf{D}_{(G,1)}$ *sleep*(M). Then, from Proposition 2, we have: $\mathbf{K^D}_{BOD(P>M,1)}$ *sleep*(M). After Mary's answer (19b), Peter ascribes to Mary the belief that coffee keeps her awake as described in case 1. Then, in BOD-DL, we can prove: ***not*** $\mathbf{D}_{BOD(P>M,2)}$ *drink*(M, *coffee*). This is a primary implicare of (19b). Because of the maxim of relation, it is likely that ($\mathbf{D}_{BOD(P>M,2)}$ *drink*(M, *coffee*) ***or***

[8] Here, we will need a formulation of abductive reasoning. However, for lack of space, we skip this part.

$\mathbf{D}_{BOD(P>M,2)}\ \neg drink(M,\ coffee))$. When this assumption holds, we obtain: $\mathbf{D}_{BOD(P>M,2)}\ \neg drink(M,\ coffee))$. This is a secondary implicature of (19b). So, Peter ascribes to Mary the desire of not-drinking coffee. The game ends here.

In this way, we can analyze conversational implicatures based on BOD-DL.

6 Concluding Remarks

Grice's theory of implicature is too strict and has many counterexamples (See Davis 2014: sect. 6~9). Our approach is more flexible than Gricean approach and it is more precisely defined than the Relevance Theory. In a ceremony, people use official expressions, because games of a ceremony have an official meaning. As this example shows, there are different types of conversational games. A game can contain some sub-games and these sub-games can contain other sub-games. In this way, we can introduce conversational structure in order to analyze complex conversations.[9]

References

1. Chierchia, G. (2004) "Scalar Implicatures, Polarity Phenomena and the Syntax/Pragmatics Interface, " In: A. Belletti (ed.) *Structures and Beyond*, Oxford University Press, pp. 39-103.
2. Davis, W. (2014) "Implicature," *The Stanford Encyclopedia of Philosophy* (Fall 2014 Edition), E. N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2014/entries/implicature/>.
3. Gazdar, G. (1979) *Pragmatics: Implicature, Presupposition, and Logical Form*, Academic Press.
4. Grice, H. P. (1975). "Logic and conversation," in P. Cole& J. Morgan (ed.), *Syntax and Semantics, 3: Speech Acts*, pp. 41–58, Academic Press.
5. Horn, L. R. (1989) *A Natural History of Negation*, University of Chicago Press.
6. Nakayama, Y. (2010) "Logical Framework for Normative Systems," In: S*OCREAL 2010: Proceedings of the 2nd International Workshop on Philosophy and Ethics of Social Reality*, Hokkaido University, Sapporo, pp. 19-24.
7. Nakayama, Y. (2011) *Norms and Games: An Introduction to the Philosophy of Society*, in Japanese. Tokyo: Keiso shobo.
8. Nakayama, Y. (2014) "Speech Acts, Normative Systems, and Local In-

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formation Update," In: Y. I. Nakano, *et al.* (eds.) *New Frontiers in Artificial Intelligence (JSAI-isAI 2013 Workshops, Kanagawa, Japan, Selected Papers from LENLS10, JURISIN2013, MiMI2013, AAA2013, DDS13)*, LNAI 8417, Springer Verlag, pp. 98-114.

9. Nakayama, Y. (2015) "Logic of Modalities and Updates of Propositional Attitudes" *Proceedings of the Twelfth International Workshop of Logic and Engineering of Natural Language Semantics 12 (LENLS 12)*, pp. 173-186. 978-4-915905-68-1 C3004(JSAI):LENLS, USB Memory.
10. Sauerland, U. (2004) "Scalar Implicatures in Complex Sentences," *Linguistics and Philosophy* **27**: pp. 367-391.
11. Soames, S. (1982) "How Presuppositions are Inherited: A Solution to the Projection Problem," *Linguistic Inquiry* **13**, pp. 483-545.
12. Spector, B. (2007) "Scalar Implicatures: Exhaustivity and Gricean Reasoning, " In: M. Aloni *et al.* (eds.), *Questions in Dynamic Semantics*, Elsevier, pp. 225-249.
13. Sperber, D., and Wilson, D. (1995) *Relevance: Communication and Cognition*, The second edition, Harvard University Press.